Quasi-spherical approach for seismic wave modeling in a 2-D slice of a global Earth model with lateral heterogeneity

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[1] For iterative calculations of synthetic seismograms with limited computer resources, a fast and accurate modeling method is needed. Axisymmetric modeling has often been used in global seismology to restrict computational time and storage. This approach can correctly model 3-D geometrical spreading effects with computational times comparable to 2-D methods, but cannot treat asymmetric structures about the source axis. To overcome this problem, a new approach is proposed for seismic wave propagation in a 2-D slice through a global Earth model with lateral heterogeneity. The elastodynamic equation for spherical coordinates is not solved in the conventional spherical domain but instead in the “quasi-spherical domain” using the finite-difference method. The validity and efficiency of this technique is illustrated with numerical examples including subduction zone structures. Citation: Toyokuni, G., H. Takenaka, Y. Wang, and B. L. N. Kennett (2005), Quasi-spherical approach for seismic wave modeling in a 2-D slice of a global Earth model with lateral heterogeneity, Geophys. Res. Lett., 32, L09305, doi:10.1029/2004GL022180.

1. Introduction

[2] In order to understand the Earth’s dynamics and the mechanism of earthquake generation, it is necessary to obtain detailed pictures of the Earth’s inner structure. In recent years, lateral heterogeneities in deep parts of the Earth’s interior, such as stagnating slabs, superplumes, and the D" layer, have drawn much attention. Waveform modeling for global Earth models can make a significant contribution to investigate these heterogeneities (e.g., Igel [1999] for slabs, Ni and Helmberger [2003] for superplume, and Emmerich [1993] for the D" layer). For laterally heterogeneous, 3-D global Earth models, we need to use numerical procedures such as the finite-difference method (FDM). However, synthetic seismogram calculations by the FDM, for full 3-D situations are computationally intensive and costly even on parallel hardware. Therefore, in the field of global seismology, 2-D or axisymmetric modeling has often been employed. However, since 2-D modeling cannot correctly model the geometrical spreading effects in 3-D, axisymmetric modeling represents a reasonable compromise between realism and computational efficiency for global modeling. In the axisymmetric modeling approach the structural model is assumed to be axisymmetric with respect to the axis through the source, and the elastodynamic equation is then solved in spherical coordinates. Many authors have used the FDM for axisymmetric modeling: for acoustic waves [Thomas et al., 2000], for SH waves [e.g., Igel and Weber, 1995; Chaljub and Tarantola, 1997; Igel and Gudmundsson, 1997], and for P-SV waves [Igel and Weber, 1996]. Nevertheless the conventional axisymmetric modeling cannot be fully realistic because it cannot treat structures that are asymmetric about the source axis. Furthermore scattered and reflected waves from the symmetric continuation of the structure can be returned to the target zone as artificial numerical noise.

[3] Takenaka et al. [2003] developed the quasi-cylindrical approach for the 3-D seismic wave field in an arbitrary heterogeneous 2-D structure in cylindrical coordinates, to achieve 3-D spreading for the costs of 2-D calculations. However, the main target structures for this quasi-cylindrical approach are relatively shallow (the crust and the upper mantle) where the curvature of the Earth can be neglected. Further in the quasi-cylindrical FDM, since 2-D structures are discretized by rectangle grid configurations, curved surfaces are approximated by stairsteps of rectangle finite-difference grids. For global simulations of seismic wave propagation we need to be able to provide accurate representations of structures with large curvature and it is most effective to work in spherical.

[4] In this paper, we therefore extend the concept of the quasi-cylindrical approach to spherical coordinates and propose a new method that we shall call a “quasi-spherical approach”. This scheme can simulate seismic wave propagation in a 2-D slice of a global Earth model with an arbitrary lateral heterogeneity, with a similar computation time and storage to that of the corresponding 2-D modeling. We can correctly model 3-D geometrical spreading effects and so achieve a direct comparison of real and synthetic waveform data. The new method overcomes the limitations of the conventional axisymmetric modeling mentioned above, and improves solutions from the axisymmetric methods. The numerical implementation is through a
FDM scheme, and the approach is illustrated with a number of numerical examples.

2. Theory

[5] We consider the elastodynamic equation in spherical coordinates \((r, \theta, \phi)\) for an axisymmetric source located on the \(\theta = 0\) axis [e.g., Lapwood and Usami, 1981]. For example, the equation for the radial (vertical) component of particle velocity \(v_r\) is

\[
\rho \frac{\partial v_r}{\partial t} = \partial_r \sigma_{rr} + r^{-1} \partial_\theta \sigma_{\theta r} + r^{-1} (2 \sigma_{rr} - \sigma_{\theta \theta} - \sigma_{\phi \phi} + \sigma_{\theta r} \cot \theta) + f_r,
\]

where \(t\) is time, \(\rho\) is the density, and \(\sigma_{ij}\) are the components of the stress tensor. This paper is devoted to P-SV waves only.

[6] Let us now apply the idea of the quasi-cylindrical approach to spherical coordinates. The elastodynamic equation in spherical coordinates is usually solved in the conventional spherical domain \((0 < r < \infty, 0 \leq \theta \leq \pi, -\pi \leq \phi \leq \pi)\). However, we here consider a new domain \((0 < r < \infty, -\pi \leq \theta \leq \pi, -\pi/2 \leq \phi \leq \pi/2)\) that maps the sphere in a different way, and which we designate a “quasi-spherical domain”.

[7] In a conventional spherical domain we first have a semicircle with an infinite radius formed by rotation from \(\theta = 0\) to \(\theta = \pi\), then rotation of this semicircle in \(\phi\) through \(2\pi\) to cover the whole spatial domain. Thus a cross section along a great circle of the Earth is described by two semicircles located at \(\phi = 0\) and \(\phi = \pi\). When we assign a 2-D structure model in the \(\phi = 0\) plane, the structure on a \(\phi = \pi\) plane becomes the same because of the axisymmetry (Figure 1a).

[8] On the other hand, in the quasi-spherical domain we first have a circle with an infinite radius by rotation from \(\theta = -\pi\) to \(\theta = \pi\), and then rotate this circle in \(\phi\) through \(\pi\) to cover the whole spatial domain. In this new domain, a cross section along a great circle of the Earth is described by only one circle for \(\phi = 0\) and the directions of the transverse coordinates are unchanged across the source axis \(\theta = 0\). Hence, we can apply an arbitrary structure model in this plane (Figure 1b). We call “quasi-spherical approach” the method of solving the elastodynamic equation in spherical coordinates in the quasi-spherical domain. We are able to model seismic wave propagation in a 2-D slice of a global Earth model with an arbitrary lateral heterogeneity with similar computation time and storage as for the 2-D modeling, but with full 3-D geometrical spreading.

3. FDM Implementation

[9] To solve the elastodynamic equation in spherical coordinates we use a velocity-stress finite-difference scheme, with second-order accuracy in time, and fourth-order accuracy in space, with a staggered-grid formulation. We use the same staggered-grid distribution as Igel and Weber [1996]. The grids for \(v_r\) and the normal stress components lie on the source axis \(\theta = 0\). In the staggered-grid scheme the derivatives of a field quantity are naturally defined halfway between the grid points where the field quantity is defined. For the elastodynamic equation, some terms which do not have spatial derivatives are often evaluated by using linear interpolation although then the accuracy of these terms drops to second-order. Here, we re-write the elastodynamic equation by using identities such as

\[
2r^{-1} \sigma_{rr} = r^{-2} \partial_r (r^2 \sigma_{rr}) - \partial_\theta \sigma_{\theta r}
\]
followed by discretization. By this means all terms in the elastodynamic equation can be represented as derivatives and the accuracy of the terms is kept fourth-order in space.

[10] In the axisymmetric modeling, because of the singularity at $\theta = 0$, $\sigma_{r\theta}$, $v_r$, and $\tan \theta$ become 0 on the source axis, where we cannot evaluate terms including $\cot \theta$ in the elastodynamic equation. Since in the "quasi-spherical approach" we have also put $v_r$ and the normal stress grids on the source axis, we cannot calculate these components directly. However, following Takenaka et al. [2003], we exploit the formulae derived from limiting operations with the L'Hospital rule such as

$$\sigma_{r\theta} \cot \theta \rightarrow \partial_d \sigma_{r\theta} \quad (\theta \rightarrow 0),$$

(3)

to overcome this difficulty. We can then input the source terms at grid points on the source axis.

4. Comparison With the DSM

[11] The basic idea of the quasi-spherical approach is common to the quasi-cylindrical one, which has been validated for laterally heterogeneous structures by Takenaka et al. [2003]. Thus, we here check only the accuracy of the FDM implementation described in the previous section. We calculate synthetic seismograms for a spherically symmetric medium and compare the results with those obtained by the Direct Solution Method (DSM) [Takeuchi et al., 1996] that can give exact waveforms for spherically symmetric media. The velocity model employed in this modeling is the IASP91 model [Kennett and Engdahl, 1991]. The model is defined on a 678 ($r$) x 1800 ($\theta$) grid with an angular range of 180° and a maximum depth of 5321 km. We use an irregular grid configuration [Pitarka, 1999] in the vertical coordinate ($r$), whereas a uniform grid spacing is used in the angular ($\theta$) direction. Figure 2 illustrates the grid configuration in the radial direction that is chosen to have smaller spacing near the free surface and the core-mantle boundary.

[12] We consider a simple situation with a 638 km deep explosion source and a source time function as a phaseless bell-shaped pulse with width of 60 s. The time increment in the FDM is 0.1 s. Figure 3 shows the UD components (i.e. $r$-components) of the synthetic seismograms at four epicentral distances ($\Delta = 18^\circ$, $36^\circ$, $54^\circ$, $72^\circ$) for both the quasi-spherical FDM and the DSM. There is a very good agreement of the waveforms and the arrival times for all major phases except for the latest arrivals from about 1200 s at epicentral distance $72^\circ$. This discrepancy is mainly due to artificial reflections from the right-hand side edge of the model. For signals before the arrival of these artificial boundary reflections, the comparison with the DSM results suggests that our FDM scheme has sufficient accuracy and that the grid configuration we have used is reasonable for this frequency band.

5. Numerical Examples

[13] We illustrate the application of the “quasi-spherical approach” with numerical examples for two models with subducting slab structure superimposed on the IASP91 model that cannot be treated with the conventional axisymmetric modeling. The first model has a slab stagnating above the 660 km discontinuity (shown by solid lines in Figure 4) and the second has a slab penetrating through the 660 km discontinuity (shown by dotted lines in Figure 4). The difference between these two models occurs only below 500 km depth. The perturbation for the compressional wave velocity, the shear wave velocity, and the density in the 100 km thick slab structures are set at +10.0% above the IASP91 basis. The source is an explosion point source located at $\theta = 0$, 440 km deep inside the slab. The grid configuration, numerical scheme, and source time function are the same as used in the previous modeling.

[14] We calculate synthetic seismograms at various epicentral distances. The vertical components of the calculated seismograms at the Earth’s surface are displayed in Figure 5.
Differential seismograms in Figure 5c are calculated by subtracting the results for the stagnating slab model in Figure 5b from those for the penetrating slab model in Figure 5a. The amplitudes of the differential seismograms are much larger than the differences between results from the quasi-spherical FDM and the DSM shown in Figure 3. The variations in the seismic waveforms thus arise from the difference between the two slab structures. Since the angular range of the difference between the two slab models is small (<10°), the phases which pass through the slabs are restricted. In Figure 5c, we can see the following phase anomalies caused by the difference between the two slab structures: P (and p410P) in a wide range, PP (and p410PP) from Δ = 30°, p410S up to 30°, p410SS and p410SSP from 20°. Since the source is located under the 410 km discontinuity, the upgoing P wave radiated from the isotropic source is replaced by P-to-S conversion at the discontinuity and behaves like direct S.

6. Discussion and Conclusion

[15] We have proposed a new efficient approach to modeling wave propagation on a 2-D slice, embedded in a fully 3-D Earth model. This approach with a FDM implementation can reduce the computation time and storage just like 2-D modeling, but correctly model 3-D geometrical spreading effects for all phases. Comparisons with the DSM show the high accuracy of our FDM scheme.

[16] The “quasi-spherical” calculations require 184 Mbytes memory in single precision with CPU time of 12 hours on a pentium IV, 2.6 GHz clock speed machine, and so the quasi-spherical FDM allows simulation to be easily carried out on a well configured PC. This is very important in actual applications because numerical techniques such as waveform inversion require iterative calculations of synthetic seismograms. Moreover, in the conventional axisymmetric modeling the source terms had to be put at grid points close to but not on the axis of symmetry due to the singularity of the elastodynamic equation at the θ = 0 axis [Igel and Weber, 1996]. By exploiting the L'Hospital rule, we have succeeded in dealing with seismic sources as point sources located on the axis θ = 0.

[17] In this paper we have used only explosion sources, but the quasi-spherical approach can be used with other sources such as double couple sources and this will be implemented in the near-future. We have also not treated waves which propagate through the center of the Earth because of the singularity in the elastodynamic equation at r = 0. Thomas et al. [2000] and Wang et al. [2001] have shown how such waves can be modeled without experiencing the coordinate singularity and a similar approach should be possible to incorporate into the “quasi-spherical” scheme.

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References


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