Volcanic processes: Simulation of Lava Dome Growth

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Outline of talk

- Motivation for research: Volcanic processes
- Data available highly variable in quality and quantity:
  - Petrology to describe lava rheology
  - Observational data often qualitative
  - In-field instrumentation capturing surface features
  - Analogue models to capture morphology
- Computational models
  - Crude compared to other areas of Earth science.
Volcanic Processes: Motivation for Research

Why is it important to Australia?
- Ring of fire surrounds north and east Australia
- Over 30 active volcanoes could affect Australia
- Partial melting from subduction generates highly viscous and volatile rich lava

Hazards:
- Pyroclastic flows
- Tsunamis
- Ash clouds and coverage
- Explosions

Poorly understood
Volcanic Processes: A Generic Volcano

- Magma chamber forms at depth and governs long-term activity
- Different styles of eruption:
  - Explosive (Gas-particle dispersion flows out of the vent)
  - Extrusive (Lava domes and Hawaii)
- From different volcanoes:
  - shield volcanoes
  - cinder cones
  - composite volcanoes
  - lava domes.
Volcanic Processes: Natural laboratory

- For quantitative models we can’t use a generic volcano
- Soufrière Hills Volcano, Montserrat formed from a subduction zone and is a lava dome forming eruption
- Current eruption began 1995 and still on-going
- Best documented volcano of its type
Volcanic Processes: Lava Domes

- Mounds of viscous magma
- Collapse events can generate pyroclastic flows, tsunamis and explosions
- Statistics used for a general trend for collapse events
- Subtle modelling techniques required to fully understand the science but limited data
Data: Overview

- Volcanology is fortunate to have observables at the free surface but it isn’t trivial to relate these back.

- Data available:
  - Physical properties and petrology describing lava rheology
  - Morphology data often qualitative
  - Instrumentation describing surface features
  - Analogue models
  - Seismicity

- Highly multi-disciplinary

- Matching of patterns in the available data dominates the progress of models.
Data: Physical properties

- Magma 3 phase = melt + crystals + gas.
- Melt: Temperature 800-1300 °C
- Pressure: $10^3 - 10^{-1}$ MPa
- Crystals: size $10^{-7} - 10^{-1}$ m, number density up to $10^{17}$ m$^{-3}$, fraction up to 100%
- Gas: H$_2$O - 60-95%, CO$_2$ - 0-35%, mass fraction 0.1-7%
- Melt viscosity $10^2 - 10^{12}$ Pa s

Bulk viscosity depends upon:
- Chemical composition
  - more SiO$_2$ → higher viscosity
- Temperature
  - higher temperature → lower viscosity
- Water content
  - higher content → lower viscosity
- Crystal content
  - higher content → higher viscosity
Data: Empirical Equations

- **Crystals:**
  - Liquidus and solidus temperatures
  \[ T_{\text{liq, sol}} = a_T + b_T \ln(P) + c_T \ln(P)^2 + d_T \ln(P) / P^2 \]
  - Equilibrium crystal content
  \[ \phi_{\text{equilibrium}} = \frac{A(P)(T - T_{\text{liq}}(P))}{B(P) - T} \]
  - Phenocrysts (crystals formed in magma chamber)
  \[ \delta \phi_{ph} = S_{ph}(P, T)U(P, T) \delta t \]
  - Microlites (new crystals formed during ascent)
  \[ U(P, T) = \text{growth fn}, S_{ph}(P, T) = \text{surface} \]
  \[ \delta \phi_{mc} = \left( \frac{3\sigma}{\phi_{mc}} (1 - \phi)U(P, T) \right)^{\frac{1}{2}} \int_0^1 \int_\omega U(P(\lambda), T(\lambda)) d\lambda d\omega \delta t \]
  \[ \delta = \text{variation @ x = fixed, } I = \text{nucleation rate, } \sigma = \text{shape coeff, } \lambda, \omega = \text{integration parameters} \]

- **Volatile**
  \[ J = 4\pi a^2 nD\rho_m \frac{c - c_{eq}}{a} \]
  \[ c_{eq} = C_f \sqrt{P} \]
  \[ c = \text{water content} \]
  \[ D = \text{diffusion} \]
  \[ C_f = \text{solvability} \]

- **Viscosity**
  \[ \eta = \theta(\phi)\eta_{\text{melt}} \]
  \[ \log \eta_{\text{melt}} = [\alpha + \beta \ln(c)] + \frac{\chi - \varepsilon \ln(c)}{T - [\gamma + \zeta \ln(c)]} \]
  \[ \log \theta(\phi) = \theta_0 (\arctan(\psi(\phi - \phi_0)) + \pi / 2) \]
Data: Observational

- Volcanoes are often cloud covered
- AVTIS device is a radar/radiometer
- Ground imaging from up to 7 km away
- For SHV dome growth imaged over 10 days

AVTIS data supplied by G. Wadge, ESSC, University of Reading, UK
Data: Observational
Montserrat, Perche’s viewpoint

AVTIS
DEM

Photo

25 Oct., 2005

4 Nov., 2005
Data: Observational


4 Nov. 2005 thermal images of the Soufriere Hills lava dome
Analogue models capture generic morphology
Endogenous vs. Exogenous growth
No single model can replicate all processes
Important for computational validation
Computational Volcanology

Current state of the art models:
- Analogue models for Bingham slurries
- Analytical one-dimensional non-Newtonian models for fixed conduit-diameter magma flow
- Analytical non-Newtonian dome growth models

Require:
- Coupled dome and conduit models
- Variable conduit shape models
- Change in lava dome growth style
- Free-surface morphology
- Shear bands with a seismicity link
- Stability of the lava dome
Computational Volcanology: Level-set

- **Problem:**
  - Volcanology complicated by free-surfaces

- **Merits:**
  - Computationally very “light”
  - Easily implemented in 3-D
  - Handles topological changes naturally

- **General idea:**
  - Implicit representation of the interface
  - Interface tracked using the equation of motion

- **Different steps:**
  - \( \varphi \) is initialized as a “signed distance function” with respect to the closest interface.
  - At each time step, once the velocity is solved, \( \varphi \) is updated using:
    \[
    \varphi_{,t} + \mathbf{v} \cdot \nabla \varphi = 0
    \]

  - \( \varphi \) is then reinitialized to ensure that it remains a distance function, using:
    \[
    \psi_{,t} + w \cdot \nabla \psi = \text{sign}(\varphi) \quad \text{with} \quad w = \text{sign}(\varphi) \frac{\nabla \psi}{|\nabla \psi|}
    \]
Computational Volcanology: Level-set

- Rayleigh-Taylor instability benchmark
- Density driven instability
- Level-set compares well to other methods and is computationally lighter

### Results

<table>
<thead>
<tr>
<th>Method</th>
<th>mesh</th>
<th>$Y_0$</th>
<th>$(v_{rms})_{max}$</th>
<th>reached at $t$</th>
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<tr>
<td>van Keken</td>
<td>coarse</td>
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<td>0.003045</td>
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<td>Level set</td>
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</tbody>
</table>
Computational Volcanology: Endogenous

- Simple endogenous dome growth model
- Tested model against analytical and analogue dome models
- Robust and accurate
- Suitable for “real” lava dome growth events
Computational Volcanology: Real Rheology

- Quantitative realistic results need data constrained to one volcano
- Additional processes:
  - Shear heating
  - Crystallisation
  - Latent heat
  - Shear zones
  - Empirical viscosity
  - Shear thinning
  - Heat loss
  - Volatile loss
Shear stress equals magma shear strength
\[ \tau_s = \tau_{\text{max}} \]
where \( \tau_s = \eta \gamma \)

New viscosity:
\[ \eta = \min[\eta_N, \eta_S] \]
where \( \eta_S = \tau_{\text{max}}/\gamma \) &

Is the shear viscosity
Computational Volcanology: Exogenous

- Modelled:
  - Shear heating
  - Crystallisation
  - Latent heat
  - Shearing
  - Empirical viscosity
  - Shear thinning
  - Heat loss
  - Volatile loss

Shear bands from:
- Strain-rate plasticity
- Crystal growth
- Thermal feedback

Important for:
- Stability
- Flow properties
- Seismic signals
Conclusion

- There are Many Unsolved Problems in Volcanology:
  - Conduit shape at depth
  - A link to seismicity
  - Initial conditions in chamber
  - Linking dome growth style to stability
- Advances in observational technology and numerical modelling techniques now allows advancement of Computational Volcanology
- Requires a multi-disciplinary approach