Geophysical Inversion I

- We can only explore the Earth from, or close to, the surface
- In consequence all knowledge interior properties is derived from indirect inference, with typically an inverse problem
- Even apparently simple tasks such as the location of seismic events are actually highly non-linear inverse problems with data inputs of various types and quality.
Many problems involve either data dependency on multiple classes of parameters or many different sources of data associated with the same description of an Earth model. The result is that there has been a strong independent tradition of innovation in geophysical inverse problems, since conventional tools do not directly translate to the problems at hand.
Locating an Earthquake I

The distribution of seismic sources across the globe
Locating an Earthquake II

• We can represent the task of finding an event location as the determination of a 4-vector comprising origin time and 3-D position

• The data are the arrival times of seismic phases, read from a set of seismic traces from available stations

• Much location is performed with a 1-D reference model so 3-D effects introduce model errors as well as picking errors

• The residual distribution is long-tailed and definitely not Gaussian

• Many currently successful methods use direct search approaches e.g. Neighbourhood Algorithm
Locating an Earthquake III

Example of shakeNA convergence and definition of consistency region
A major problem is the description of the 3-dimensional interior structure of the Earth using observations of seismograms at the Earth’s surface, which can rapidly lead to large numbers of parameter and data inputs.

The dominant structure depends on radius and so progress has been made by developing reference models for the average radial structure of the seismic wavespeed in the Earth and then seeking the 3-D variations about this state.

I will illustrate the successes and problems associated with the generation of such reference models and the current state of imaging for 3-D structure.
Multi-data and Multi-parameter Inverse Problems

- The inverse problem for the internal structure of the Earth can be addressed in a number of ways, with multiple data sets in combination
- One set of data are the frequencies of free-oscillation of the Earth
  - Data depend on physical parameters of different dimensionality
- A second data set comes from the travel times of different seismic phases with different passage through the interior
  - Multiple data sets (of varying reliability) depend on the same physical parameters
- Information from different frequency ranges has ultimately to be combined
Global Structure

[Diagram showing the Earth's global structure with layers labeled as Core, Mantle, and Inner Core. The diagram includes a graph with depth in kilometers on the y-axis and various physical properties on the x-axis (AK135, β, α, and ρ in km/s and Mg/m³).]
The mode frequencies depend on P and S wavespeeds, and density through the influence of self-gravitation in the Earth.
Practical Methods for Multi-Parameters

- Linearised inversion techniques based on the application of derivatives of data values with respect to parameters frequently hit problems with the non commensurate size of different terms (with different dimensions).
- Such effects can partially be overcome using reweighting schemes, but require ad-hoc choices.
- An alternative is to partition the problem and use subspace methods:
  - Different physical parameters can be brought to common significance.
  - Efficient solution of large systems.
Fit to 18 travel time sets

Model generation by iterative improvement – based on multiple weighted misfits
Retrospective assessment of models

Monte Carlo sampling of models for core sensitive phases:
- CPw – weighted set of P phases, CSw – weighted set of S phases
- Base model – ak135
Global Seismic Tomography

- There are two major approaches to determining 3D structure for the whole globe.
- The first uses seismic waveform information dominantly for shear waves using a modal analysis based on a spherically symmetric model (PREM)
  - the model representations are normally via spherical harmonics or spherical splines.
- The second uses the arrival times of seismic phases recorded at stations across the globe
  - the model representations are generally cellular.
- Each system produces very large equation systems, e.g., 300,000 unknowns for a 2x2 deg model with 18 layers and over 1.5 million data values.
Inversion via spherical harmonic representation

SH wavespeed structure
High shallow heterogeneity
200 km

Moderate Heterogeneity
1300 km

High heterogeneity near CMB
2700 km

Inversion via spherical harmonic representation
Joint Body–wave tomography

The AK135 model is used as an initial reference. Residual band employed ±7 s deviations in P, 25 s deviations in S to account for Shields (+), LM (−).

Inversions on both regional and global scales using cellular representation with light damping.

- **Global**: 18 layers with 2×2 deg cells
- **Regional**: 19 layer model to 1500 km with 1×1 deg cells for the region of interest (e.g., NW Pacific) embedded in a 5×5 deg global model.

**Stage 1**: Linearised inversion for P and S separately.

**Stage 2**: Joint inversion for shear and bulk–sound speed with 3–D ray tracing.

Joint inversion uses a nested iteration with updating of cross–wavetype terms and iterative solution of the sets of linear equations for each wavetype using the LINBCG algorithm.
150–200 km depth
$\delta \phi / \phi$ and $\delta \beta / \beta$ at 1100–1300 km depth.
2000–2200 km depth
Cross section showing deep penetrating subduction in shear wavespeed signal