Lithosphere-Hydrosphere interactions: Stokes flow with a free surface

J. Braun[(1)], P. Fullsack[(2)], M. DeKool[(3)]

(1) Géosciences Rennes, Université de Rennes 1, F-35042 Rennes cedex, France

(2) Department of Oceanography, Dalhousie University, Halifax, NS, B3H 4J1, Canada

(3) Research School of Earth Sciences, The Australian National University, Canberra, ACT 0200, Australia

Abstract

The coupling between surface processes (erosion, transport and sedimentation) and tectonics has emerged as one of the dominant processes that determines the large-scale morphology of mountain belts. The transport of mass at the Earth's surface affects the state of stress within the Earth's interior and, consequently, its response to tectonic processes. That erosion, and thus climate, are important players in determining the dynamical behaviour of the solid Earth has been demonstrated by sophisticated numerical models of the coupled lithosphere-hydrosphere system. The coupling is, however, difficult to represent by traditional numerical methods as it requires the accurate tracking of the free surface through deformation events that can easily lead to strain accumulation of several thousands of percent and the relative motion of parts of the model by thousands of kilometers. We first describe here various methods that have been developed in recent years to address these difficulties. We subsequently present a newly developed numerical model designed to address these challenges in a three dimensional framework. To overcome the geometrical complexity of the problem, we have used an octree division of space in which arbitrary surfaces are embedded. These surfaces have a dual representation based on a dynamically evolving cloud of Lagrangian particles residing on the surface and a level set function (lsf) defined at the nodes of the octree. This dual approach allows us to combine accuracy and efficiency. The method has been tested against other numerical methods and analogue experiments.

Keywords: Challenges in computational simulation of natural processes ; Hydrosphere-Lithosphere-Mantle system

Introduction: existing methods

Modelling the deformation of the solid Earth over geological time scales is a challenging problem that requires accurate methods to compute the velocity/deformation field within the Earth's interior but also the geometry of interfaces, such as the free surface or the crust-mantle boundary (the so-called Moho discontinuity) that are advected by the deformation. Small variations in the geometry of the free surface lead to important perturbations of the stress field and impact on the nature of the flow in the underlying crust. For example, it is because the pressure gradients caused by topographic slope are capable of driving lower crustal flow that topographic slope never reaches values greater than a few degrees (when measured at the scale of an orogenic system) (Willett et al., 1993). Conversely, because it is characterized by a smaller density contrast than the free surface, the Moho discontinuity can be greatly deformed by tectonic processes. For the numerical modeller, the challenge is thus to accurately predict the geometry of these deforming surfaces, as well as the effect that their complex geometries can have on the flow. Most methods developed so far have been limited to two dimensions (Fullsack, 1995; Braun and Sambridge, 1994), mostly due to the complexity of the problem and computational cost.

Lithospheric flow is also characterized by very large displacements and deformation, in part due to the complex, non-linear and localizing nature of rock rheology, which justifies the use of an Eulerian approach in which the numerical mesh is fixed in space. A Lagrangian approach (in which the numerical mesh is advected with the flow) is, however, much better suited to the tracking a deforming surface. This is why mixed or Arbitrary-Lagrangian-Eulerian methods have been successfully used in the past (Fullsack, 1995), but were limited in their resolution due to the regular nature of the Eulerian meshes used.

New 3D method

In the past two years, members of several research teams have collaborated to develop a numerical method that builds on previous experience in methodological development, with the purpose to achieve sufficient accuracy and efficiency to permit three-dimensional analysis. The new method is based on an octree division of space, the tracking of deforming

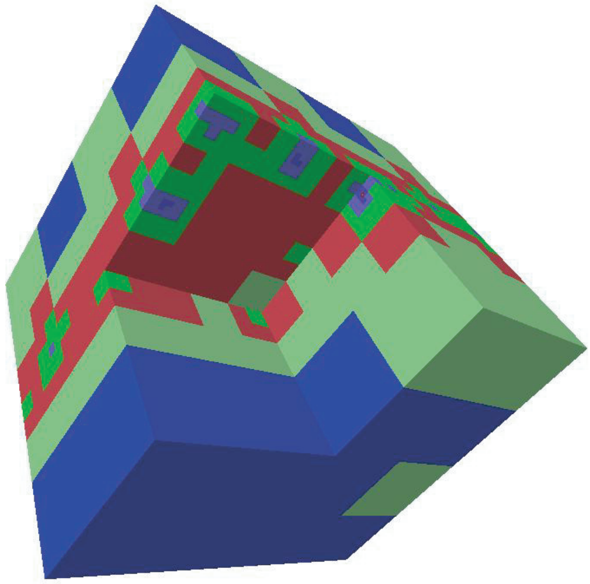

Figure 1: Non uniform, octree division of the unit cube; the leaves of the octree are drawn in a colour proportional to their size.

interfaces by using Lagrangian particles and level set functions, and a modified finite element approach based on an octree division of the element to perform volume integrals.

Octree division of space

We use an octree division of space to discretize a unit cube. The final discretization is made of cubes of non-uniform size and distribution (the 'leaves' of the octree) but allows for efficient 'navigation' through the leaves. For example, it is trivial to find the index of the leaf containing an arbitrary point. It is thus computationally efficient to interpolate a field onto the nodes of an octree. In Figure 1, we show a very simple octree that has been locally refined to level 10 (the size of the sides of the smallest leaves of the octree is 2^{-10}). We solve the three-dimensional form of Stokes equation using the finite element method in which the leaves of the octree are the elements. Where two adjacent leaves of the octree are of different level, hanging nodes do appear; these are nodes that are not connected to all neighbouring elements. To overcome this limitation of the octree discretization we resolve the geometric mismatch by imposing simple linear constraints forcing the flow to vary linearly across the faces where the mismatch occurs.

Surface Tracking

Surfaces are discretized by a series of particles that are strategically positioned on each surface and advected with the flow. The local density of the particles is maintained by injection and removal of particles according to a set of criteria, based on inter-particle distances and the curvature of the surface (measured by the divergence of the normals at the particle locations). To each surface is also associated an octree, the resolution of which is a function of the distance to the surface (smallest leaves in the vicinity of the surface). A level set function (lsf) is also computed on the nodes of the octree. Its value is defined as the 'signed distance' to the surface. A collection of octrees and lsf's are thus constructed, one for each of the surfaces to be tracked. A master octree is constructed as the union of each of the surface octrees and a complete set of lsf's is computed at each of its nodes.

Finite element representation

This master octree is used to construct the finite element matrices. The lsf's are used (locally) to determine the position of the nodes of each leaf (element) with respect to all of the surfaces. Elements can be of two types: those that are entirely within one 'medium' defined as the material comprised between two successive surfaces, and those that are cut by at least one surface. The contributions of those 'cut elements' are estimated by computing volumetric integrals of the finite element equations using an octree division of the element. As shown in Figure 2, the cut elements are divided in smaller cubes that are sequentially tested for intersection by the surfaces (using interpolated values of the various lsf's). This method, which we termed divFEM, insures that we can estimate efficiently and accurately volume integrals

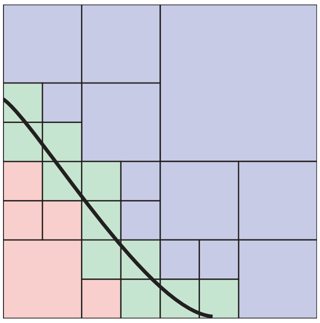

Figure 2: Division of the element/leaf illustrated by a simplified two-dimensional diagram. By successive octree division of the leave, the algorithm identifies sub-cubes that are entirely comprised in one of the two media on either side of the surface cutting the element. A simplified analytical expression is used to estimate the relative volume of the small cubes cut still by the interface.

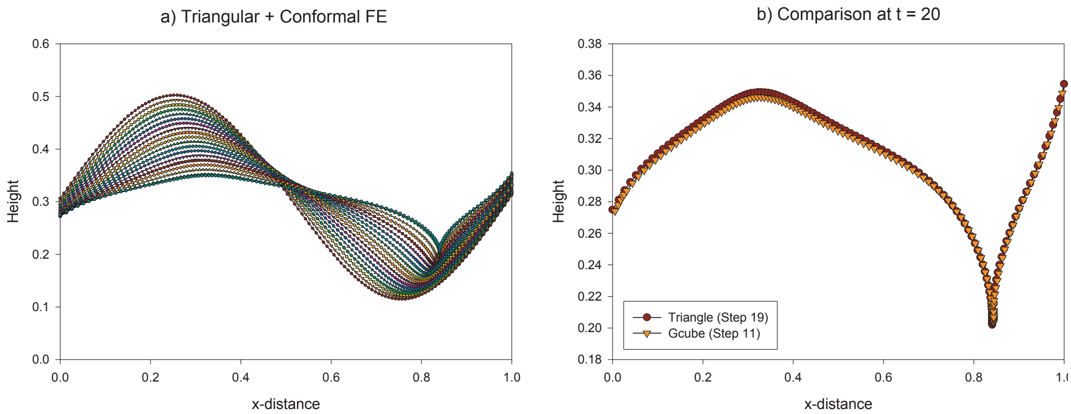

Figure 3: a) Evolution of the free surface of the fluid following a large amplitude, sinusoidal perturbation, as computed by our method. b) Comparison with a high resolution, 2D Lagrangian finite element solution.

of a function that may vary abruptly within an element, such as those generated at the free surface where the material properties vary greatly.

Comparison to other numerical methods

To verify the accuracy of our method, we computed the evolution of the free surface of a highly viscous fluid that was initially set to be a periodic sine function of amplitude comparable to its wavelength (Figure 3a). Gravitational forces drive the surface to a flat geometry but following a scenario that leads to the formation of a cusp on the initially low side of the surface. We also computed the solution of this inherently two-dimensional problem with a Lagrangian finite element method. The results are shown in Figure 3b and demonstrate that the octree/lsf/divFEM based method is very accurate. We also performed a series of analogue experiments designed to test the accuracy of our method to track the deformation of the free surface and its effects on the underlying flow.

References

J. Braun and M. Sambridge. Dynamical Lagrangian Remeshing (DLR): A new algorithm for solving large strain deformation problems and its application to fault-propagation folding. *Earth Planet. Sc. Lett.*, 124:211–220, 1994.

P. Fullsack. An arbitrary lagrangian-eularian formulation for creeping flows and its application in tectonic models. *Geophys. J. Int.*, 120:1–23, 1995.

S.D. Willett, C. Beaumont, and P. Fullsack. Mechanical model for the tectonics of doubly-vergent compressional orogens. *Geology*, 21:371–374, 1993.