

# The Impact of Solid Earth Tide Models on GPS Coordinate and Tropospheric Time Series

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Unmodelled sub-daily periodic signals can propagate into time series of daily geodetic coordinates and tropospheric estimates at various different frequencies. Geophysical interpretations of geodetic products, particularly at seasonal timescales, can therefore be affected by poorly modelled signals in the geodetic analysis. In this study, we use two solid Earth tide models (IERS2003 and IERS1992) and analyses of global GPS data to demonstrate how this process occurs. Aliased annual and semi-annual signals are evident in the vertical component of the GPS time series, with the amplitudes increasing as a function of latitude up to approximately 2 and 0.4 mm, respectively. Tropospheric zenith delay estimates show differences at the 2 mm level, with a dominant diurnal frequency. These results have significant implications in regard to the geophysical interpretation of GPS time series computed using the outdated IERS1992 model and, more generally, for any mis- or unmodelled periodic signals that affect geodetic sites.

## 1. Introduction

Recent studies have shown theoretically and through the use of simulated data how unmodelled periodic signals (such as ocean tide loading and errors in solid Earth tide models) propagate into GPS coordinate time series at various different frequencies (Penna and Stewart, 2003; Stewart et al., 2005 and Penna et al., GPS height time series: short period origins of spurious long period signals, submitted to *J. Geophys. Res.*, hereafter referred to as Penna et al., submitted manuscript). Such aliasing propagates sub-daily errors in modelling of periodic signals into erroneous effects at frequencies of geophysical interest, in particular, annual and semi-annual signals. This can lead to misinterpreting geodetic analysis ‘error’ as geophysical ‘signal’.

It is well known that the position of a point on the Earth’s surface varies over a range of temporal scales due to the elastic response of the crust to the external tide generating potential (TGP) (e.g., Melchior, 1983). The resultant response is called the solid Earth tide (also termed the Earth body tide), and can account for displacements up to  $\sim 0.4$  m at predominantly semi-diurnal and diurnal frequencies (Lambek, 1988). In any geodetic analysis, it is important to

consider how this process may affect the instantaneous site position, and hence influence the computation and interpretation of coordinate time series and related data products.

In this study, we demonstrate from the analysis of 5 years of global GPS data how ‘errors’ in the modelling of the solid Earth tide can propagate into coordinate and tropospheric estimates. We investigate the differences between two GPS time series generated using two different solid Earth tide models: IERS 1992 (McCarthy, 1992) and IERS 2003 (McCarthy and Petit, 2004), a minor revision of the IERS 1996 model. We analysed topocentric site coordinates derived from an analysis of a global network spanning a 5 year period using the GAMIT / GLOBK software suite (King and Bock, 2005; Herring, 2005). In the analysis we estimated station coordinates, orbital parameters, earth orientation parameters and 2-hourly zenith total delays. Two solutions were generated with the only difference being the selection of either the IERS 1992 or the IERS 2003 solid Earth tide model. We show the differences in the modelled displacements and the associated impact on both the coordinate and the tropospheric zenith total delay (ZTD) time series.

## 2. Periodic Signals and their Propagation

Typical strategies for GPS analyses involve processing 24 hours of data sampled at 30 second epochs (Herring, 1999). Given the high frequency response of the solid Earth tide, it has long been acknowledged that the displacement must be considered at the observation level in order to derive a representative estimate of site position for a specific processing session (see McCarthy, 1992 for an early example). This approach is particularly important for GPS analysis given the reliance on ambiguity resolution and tropospheric delay estimation over the duration of the 24 hour observation session.

It must be acknowledged that any given model incorporates uncertainty, and any complex geophysical process can never be modelled perfectly. The residual or unmodelled component of the solid Earth tide (or any periodic geophysical signal such as ocean tide loading) represents a signal of unknown amplitude and frequency which may vary spatially and temporally. A number of previous studies investigated the influence of unmodelled signals. King et al. (2003) investigated spurious periodic signals caused by unmodelled tidal signals present in GPS data acquired on a floating ice shelf. Using simulated GPS data, Penna and Stewart (2003) investigated low frequency aliased signals caused by both under sampling of small semi-diurnal and diurnal tidal signals and the effect of the sidereal repeat orbit of GPS satellites. Stewart et al. (2005) provided a theoretical development

with an investigation of the propagation mechanism behind systematic errors associated with unmodelled periodic signals and the least squares estimation process. Penna et al. (submitted manuscript) provides an analysis of GPS time series taken from a selection of globally distributed sites where controlled modelling errors were introduced into the analyses. They conclude that spurious low frequency signals (at fortnightly, semi-annual and annual periods) can be introduced into coordinate time series as a result of the truncated least-squares estimation process and the additional aliasing effects of sub-daily periodic mis-modelling.

Such spurious low frequency signals in geodetic time series have the potential to mask geophysical signals of interest including the seasonal signals associated with the hydrological cycle (van Dam et al. 2001), atmospheric loading (Tregoning and van Dam, 2005) and non-tidal ocean mass loading. Dong et al. (2002) were only able to attribute less than 50% of the power with an annual frequency in GPS time series to actual geophysical processes. This poses the obvious question regarding the source(s) of the unexplained signal(s).

To date, there has been little attention given to the investigation of differences in solid Earth tide models and their impact on GPS time series. Insight into the potential impact of solid Earth tide models was inadvertently made by Hugentobler (2004) when a coding error was discovered in the Bernese software suite. An error in computing the fractional hour of the day introduced significant errors in the modelled Earth tide displacement, effectively introducing a complex residual periodic signal. This error was found to introduce spurious diurnal and annual signals in coordinate time series which varied in amplitude as a function of latitude (Hugentobler, 2004).

By undertaking analysis on two time series differing only by the modelling of the solid Earth tide, we assess the actual impact of model differences and further explain the low frequency noise structure which remains unaccounted for in studies such as Dong et al. (2002).

### 3. Methodology

#### 3.1. Modelling the Solid Earth Tide

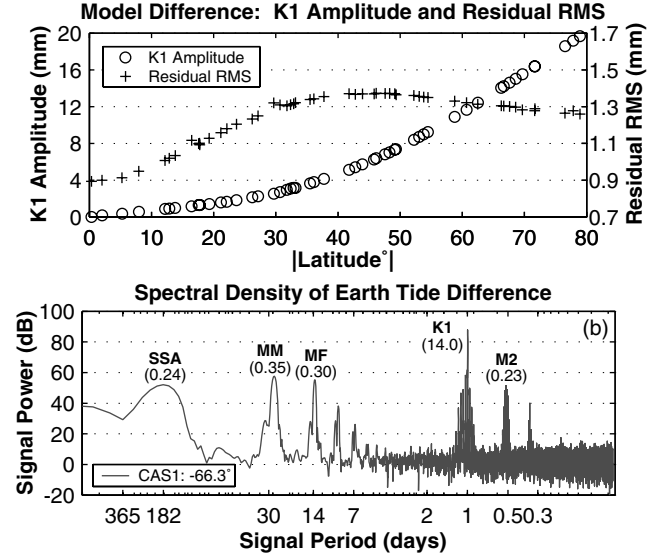
Modelling the displacement caused by the solid Earth tide is typically based on the Love number formalism (Love, 1944), further described in Munk and MacDonald (1975). This technique expresses the radial and transverse displacement of a point on the Earth's crust in terms of Love and Shida numbers ( $h$  and  $\ell$  respectively), in addition to the perturbation in the geopotential field using the Love number  $k$  (Mathews et al. 1997). Tidal displacements arise almost entirely from the degree 2 spherical harmonic components of the TGP, with the degree 3 harmonic component contributing a small effect. Higher degree harmonics are at the sub-mm level and can be considered insignificant (Mathews et al. 1997).

#### 3.2. The IERS 1992 and 2003 Models

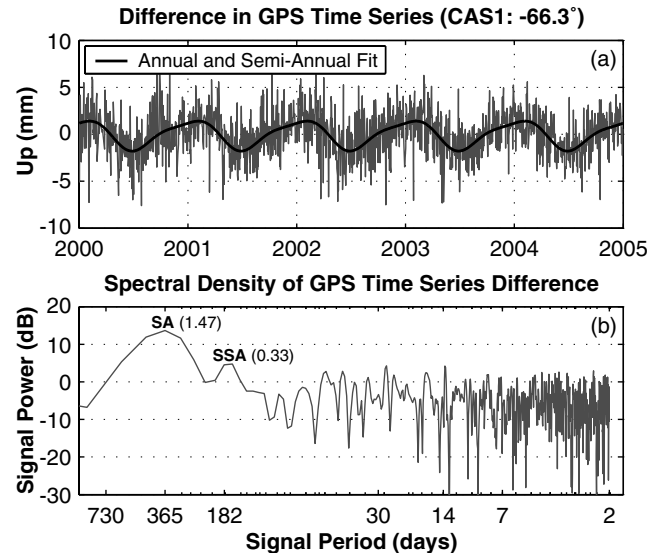
Early models of the solid Earth tide were based upon the Wahr model (Wahr, 1981), later incorporated into the IERS 1992 conventions (McCarthy, 1992). This model was implemented into the GAMIT software suite and has since been utilised as the default solid Earth tide model (see Morgan, 1994 for implementation details). With the exception of the K1 diurnal term, the IERS 1992 model adopts frequency and latitudinally independent Love and Shida numbers. The K1 frequency dependence is applied solely to the vertical component as a second step correction.

The IERS 2003 model (McCarthy and Petit, 2004) incorporates many enhancements including the effect of Love number dependence on tidal frequency (including long period terms) and station latitude (as discussed by Mathews

et al. 1995, 1997). The latitudinal dependence arises from the ellipticity of the Earth and the Coriolis force. The frequency dependence (particularly in the diurnal tidal band) is caused by the resonance of the nearly-diurnal free wobble associated with the free core nutation of the Earth (as discussed by Mathews et al. 1995). An additional consideration (at the few mm level) is the effect of taking mantle anelasticity into account. Anelasticity introduces a small imaginary component to the Love numbers that reflects a phase lag in the response of the Earth's crust to the TGP.



**Figure 1.** a) K1 amplitude and residual RMS of the solid Earth tide model differences (IERS 2003 - IERS 1992), in the vertical component as a function of the absolute value of site latitude. b) PSD of the vertical component of the model differences at the CAS1 site (note the signal amplitudes in mm within brackets).



**Figure 2.** a) Difference in the vertical component of the GPS time series (IERS 2003 - IERS 1992 solution) at CAS1. b) PSD of the difference time series at CAS1.

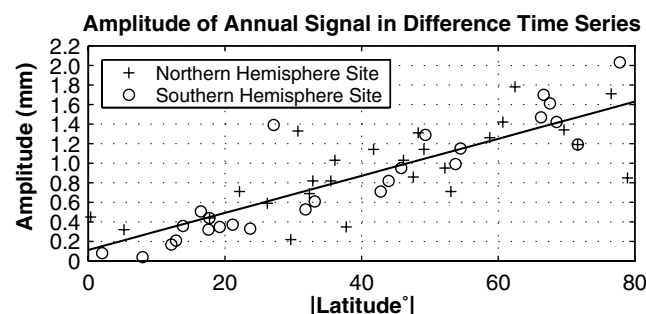
Each of these dependencies have been coded into the IERS 2003 conventions using the formulation described in Mathews et al. (1997).

### 3.3. Model Differences

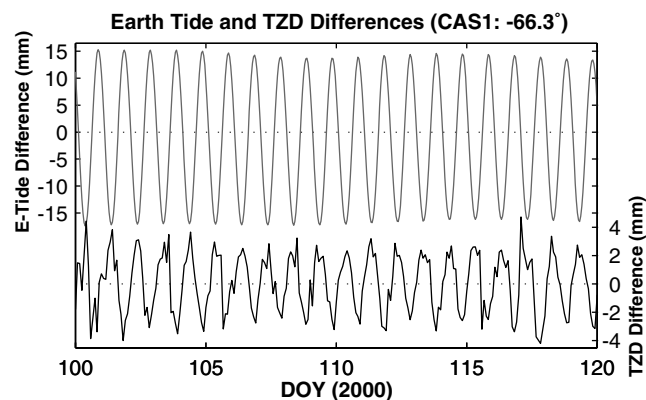
Tidal displacements (North, East and Up) were generated using both solid Earth tide models for a set of global IGS GPS sites spanning a five year period (2000.0–2005.0). 24 hour averages of the differences (IERS 2003 - IERS 1992) are less than 1 mm in each coordinate component for all sites but the instantaneous model differences in the vertical component exhibit a clear latitudinal dependence, with a dominant diurnal frequency corresponding to the K1 diurnal tide. Figure 1a shows the amplitude of the K1 term and the associated variation with latitude. Also shown is the residual RMS of the model difference following the removal of the K1 signal. The power spectral density (PSD) of the difference in tide models for the full 5-year series for the vertical component at an arbitrary IGS site, CAS1 (Lat:  $-66.3^\circ$ ), is shown in Figure 1b. Note the clear dominance of the K1 frequency term, with the remaining power spread throughout other tidal frequencies.

## 4. Results

The results presented in this paper summarise the analysis of the time series generated by differencing the IERS 1992 GAMIT/GLOBK solution from the IERS 2003



**Figure 3.** Amplitude of the annual signal in the difference time series (vertical component, IERS 2003 solution - IERS 1992 solution) as a function of the absolute value of site latitude.



**Figure 4.** Earth tide model and differences in ZTD estimates for an arbitrary 20 day period at the CAS1 site.

GAMIT/GLOBK solution for each coordinate component and 2-hourly ZTD parameter estimates.

### 4.1. Impact on Coordinate Time Series

The change in the solid Earth tide model causes a significant effect on the estimates of vertical coordinates with insignificant change to the north and east components. The vertical component shows a clear annual signal (and a smaller, yet significant, semi-annual term) with amplitudes that vary as a function of latitude. The PSD of these data shows significant peaks at the annual and semi-annual frequencies (Figure 2).

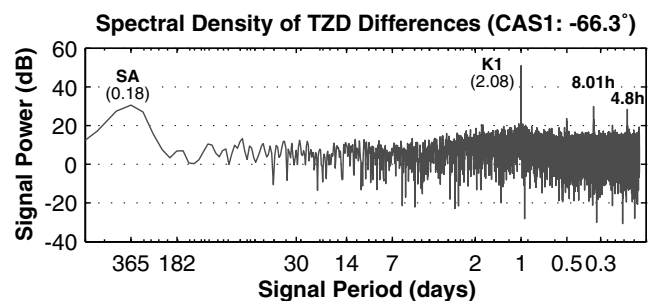
The amplitudes of the annual signals show a clear trend as a function of latitude (Figure 3), corresponding with the trend in the amplitude of the dominant K1 difference observed between the two Earth tide models (Figure 1a). The trend in the annual signal in the vertical component reaches a maximum of  $\sim 1.6$  mm at high latitudes.

The presence of the annual signals is consistent with the propagation of a mis-modelled K1 signal (Penna and Stewart, 2003). A component of the small ( $< 0.4$  mm) semi-annual signal is present in the model differences (Figure 1b). Additional contributions to this term are likely to arise from the aliasing of the mis-modelled K2 signal, in addition to aliasing caused by the satellites' orbital period and sampling of the mis-modelled S2 and P1 signals (Penna and Stewart, 2003). The relative magnitude of each of these mis-modelled K2, S2 and P1 signals (see Figure 1b) makes the exact source(s) of the semi-annual signals difficult to isolate.

At high latitudes, the difference in vertical velocity is  $\sim -0.35$  mm/yr increasing to  $\sim +0.2$  mm/yr at equatorial latitudes (symmetric about the equator). These differences are consistent with latitudinally dependent secular trends in the solid Earth tide models over the 5-year extent of the time series. The sub-sampling of the latitudinally dependent long period tidal signal implemented in the IERS 2003 model – but excluded from the IERS 1992 model – causes the differences in vertical velocity estimates. The magnitude of the velocity change depends on both the site latitude and which part of the 19.6 year periodic signal is sampled.

### 4.2. Impact on ZTD Estimation

ZTD differences also vary as a function of latitude. The ZTD differences exhibit a clear periodicity which is inversely correlated with the solid Earth tide model differences (Figure 4). At CAS1, the amplitude of the ZTD differences (at the K1 frequency) is at the 2 mm level. Analysis of ZTD differences from the entire network shows that this amplitude is consistently about 16-18% of the amplitude of the differences in the Earth tide model at the same K1 frequency.



**Figure 5.** Power spectral density of the ZTD differences at the CAS1 site.

The PSDs of the complete 5-year series of ZTD differences at each site highlight an interesting frequency distribution. The dominant frequency in the ZTD differences at CAS1 (and other high latitude sites) is clearly centred on the K1 tidal period (Figure 5). Subsequent peaks in the ZTD PSD estimates occur at 1 year, 12 hrs, 8.009 hrs and 4.797 hrs. With the exception of the semi-diurnal term, the relative power of the remaining signals decreases for sites closer to the equator.

These results clearly highlight that the choice of the solid Earth model is significant when estimating ZTD, and may contribute to the error budget in studies such as Humphreys et al. (2005). The exact mechanism causing the periodic signals in the ZTD differences is likely to be a complex interaction between the 2-hour sampling strategy, the periodic Earth tide model differences and the linearised functional model used in the least squares parameter estimation process (Stewart et al. 2005).

## 5. Conclusions and Implications

From an analysis of global GPS data we have shown that mis-modelled periodic signals do propagate into signals at a number of different frequencies, as predicted theoretically by Penna and Stewart (2003) and Stewart et al (2005). The differences between the IERS 2003 and IERS 1992 solid Earth tide models were used to demonstrate this; however, the conclusions will be valid for other sources of periodic signals, such as ocean and atmospheric tidal loading. Annual signals can be introduced into the vertical component with amplitudes up to  $\sim 1.6$  mm. In addition, diurnal signals in ZTD time series may be introduced with amplitudes at the  $\sim 2$  mm level.

There are clear implications relating to the geophysical interpretation of GPS time series data computed using the IERS 1992 solid Earth tide model. This has particular relevance to studies investigating seasonal geophysical signals (e.g., Dong et al. 2002; Williams et al. 2004) and for studies making use of GPS ZTD products derived using the approach presented here for investigating atmospheric tides (e.g., Humphreys et al. 2005) and integrated water vapour of the atmosphere (e.g., Rocken et al., 1997). More generally, any errors remaining in the current solid Earth tide model, ocean tide loading models and all other periodic models are likely to propagate into other frequencies; therefore, care must be taken when making geophysical interpretations from geodetic time series.

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