

Relationship between glacial isostatic adjustment and gravity perturbations observed by GRACE

A. Purcell,¹ A. Dehecq,^{1,2} P. Tregoning,¹ E.-K. Potter,¹ S. C. McClusky,¹
and K. Lambeck¹

Received 22 June 2011; revised 23 August 2011; accepted 24 August 2011; published 21 September 2011.

[1] The Gravity Recovery and Climate Experiment space gravity mission provides one of the principal means of estimating present-day mass loss occurring in polar regions. Extraction of the mass loss signal from the observed gravity changes is complicated by the need to first remove the signal of ongoing glacial isostatic adjustment (GIA) since the Last Glacial Maximum. This can be problematic in regions such as Antarctica where the GIA models are poorly constrained by observation and their accuracy is not well known. We present a new methodology that permits the GIA component to be represented mathematically by a simple, linear expression of the ratio of viscoelastic Love numbers that is valid for a broad range of Earth and ice-load models. The expression is shown to reproduce rigorous computations of surface uplift rates to within 0.3 mm/yr, thus providing a means of inverting simultaneously for present-day mass loss and ongoing GIA with all the accuracy of a fully detailed forward model. **Citation:** Purcell, A., A. Dehecq, P. Tregoning, E.-K. Potter, S. C. McClusky, and K. Lambeck (2011), Relationship between glacial isostatic adjustment and gravity perturbations observed by GRACE, *Geophys. Res. Lett.*, 38, L18305, doi:10.1029/2011GL048624.

1. Introduction

[2] One of the major contributions of space geodesy in the present era is in the facilitation of estimates of rates of melting of polar ice sheets such as Antarctica and Greenland. Changes in mass of ice on the continents can be measured through the associated change in the Earth's gravity field as detected by the Gravity Recovery and Climate Experiment (GRACE) space gravity mission [Tapley *et al.*, 2004]. However, GRACE detects not only present-day mass loss but also changes in the gravity field caused by ongoing glacial isostatic adjustment (GIA) [e.g., Wahr *et al.*, 2000; Velicogna and Wahr, 2006; Ramillien *et al.*, 2006] and it is not possible to separate these two signals from GRACE observations alone.

[3] A common approach to overcome this problem has been to remove the GIA signal by subtracting a modelled value derived from one (or more) ice load models [e.g., Velicogna and Wahr, 2006; Chen *et al.*, 2009]. In this case, any errors in the modelled GIA signal will propagate directly into estimates of present-day mass loss. The GIA

models are not well constrained in all polar regions, especially in Antarctica where the available geological observations to constrain the ice history models are particularly sparse [e.g., Ivins and James, 2005]. Consequently, the uncertainty that has been placed on the present-day mass loss estimates can be as large as the mass loss itself, simply because of the uncertainty in the modelled GIA signal [e.g., Velicogna and Wahr, 2006].

[4] Changes in hydrological surface loads (including continental water and cryospheric ice sheets) cause deformations of the surface and affect the Earth's gravity field [e.g., Farrell, 1972]. Following the work of Wahr *et al.* [1995], recent studies have used observations of these two phenomena to invert simultaneously for the GIA component and the present-day changes in surface loads [e.g., Rietbroek *et al.*, 2009; Wu *et al.*, 2010]. Simultaneous inversion is possible because the hydrological and GIA deformations occur on different time and length scales. However, in order to perform such an inversion one must use an assumed model to estimate GIA-induced effects.

[5] In this paper we test the validity of the Wahr and Han technique by evaluating numerically the ratio between surface displacement and geoid change for a range of Earth and ice models. From these results we derive an empirical expression for the ratio of viscoelastic load Love numbers for GIA-induced changes in present day vertical displacement and gravitational variation. Uplift rates obtained using these ratio values are then tested against the results of fully detailed forward modelling routines, and are found to agree to within 0.3 mm/yr, a significant improvement over the results of the Wahr and Han approximation. This increased accuracy holds for all models of Earth rheology considered in this study and is independent of ice sheet history provided that the major changes in glacier geometry ceased at least 6000 years ago (not including any present-day melting effects). In combination, these results establish empirically that GIA uplift rates estimated from a simultaneous inversion approach are not sensitive to uncertainties in ice or Earth models. Our new linear relation for the Love number ratios provides an improvement in accuracy of up to 15% over previously used approximations.

2. The Earth's Response to a Surface Load

[6] Physically, there is a strong correlation between the displacement caused by a load and the corresponding perturbation of gravitational potential. Assuming that the effects of material compression are negligible (which is not the case for the real Earth but is assumed here for the convenience of illustration), at any point in the Earth's interior the change in mass distribution caused by a load-

¹Research School of Earth Sciences, Australian National University, Canberra, ACT, Australia.

²Ecole Normale Supérieure de Cachan, Cachan, France.

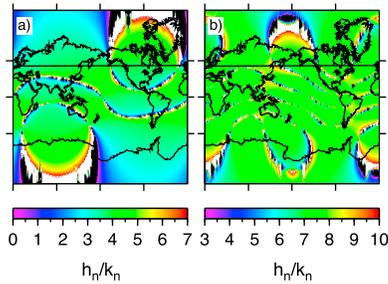


Figure 1. Ratio of the GIA-induced change in vertical displacement and corresponding change in gravitational potential over the last ten years. The values were obtained using the ANU group’s CALSEA program using a reference Earth model with their standard global ice model. The calculations were performed using the total calculated displacement and gravity change and are shown for (a) degree 3; (b) degree 6 summed over all orders.

induced deformation is, to first order, the product of vertical displacement and cross-sectional area (to give the volume of displaced material) and the radial gradient of density. The concomitant effect on gravitational potential at a point on the Earth’s surface may be calculated by dividing by the distance separating the two points and scaling by the gravitational constant. The total change in the gravity field may then be found by integrating this function through the Earth’s interior.

[7] Given a spherical harmonic load component of degree n , the characteristic length-scale of that component will be $\pi R/n$ where R is the radius of the Earth. This wavelength also characterises the depth to which the maximum load-induced stress occurs. For a given value of n there will be a depth-range in which the majority of the deformation due to the load occurs. Assuming a radially symmetric density profile, the relationship between vertical displacement and gravitational perturbation is a function of the change in density gradient within the depth range for which the vertical displacement is non-zero. The higher the value of n , the shallower the maximum vertical displacement occurs, and the more closely the relationship between vertical displacement and gravitational potential change approaches direct proportionality. Conversely, the smaller the value of n the deeper the load-induced deformation penetrates and the greater the proportion of the mantle sampled.

[8] *Davis et al.* [2004] generated estimates of elastic surface displacement, dH^e , from a combination of GRACE spherical harmonic coefficients and elastic load Love numbers h_n^e, k_n^e :

$$dH^e(\theta, \lambda) = \sum_{n=2}^N \sum_{m=0}^n R \frac{h_n^e}{1 + k_n^e} P_{nm}(\alpha) [\Delta C_{nm} \cos m\lambda + \Delta S_{nm} \sin m\lambda] \quad (1)$$

where α is the cosine of co-latitude, and λ is longitude, n and m are the degree and order of the spherical harmonic component, P_{nm} the normalised associated Legendre functions, and ΔC_{nm} and ΔS_{nm} the spherical harmonic coefficients that represent the change in the Earth’s gravity field ($\Delta C_{nm}/(1 + k_n^e)$ and $\Delta S_{nm}/(1 + k_n^e)$ are thus the spherical

harmonic components of the change in load, expressed as dimensionless Stoke’s coefficient anomalies).

[9] The ratio $h_n^e/(1 + k_n^e)$ introduces the relationship between vertical vertical displacement and geopotential change for the elastic case. The presence of the “1” in the denominator represents the change in the geoid produced by the direct gravitational attraction of the load while the k_n^e accounts for the change in gravity caused by the load-induced vertical displacement. In the case of ongoing GIA, the ice–water mass causing the deformation (i.e., the melt-water discharged from the major ice sheets since the LGM) has ceased to change; therefore, we can omit the “1” from the denominator. Applying this formulation to the case of viscoelastic vertical displacement yields:

$$dH^{ve}(\theta, \lambda) = \sum_{n=2}^N \sum_{m=0}^n R \frac{h_n^{ve}}{k_n^{ve}} P_{nm}(\alpha) [\Delta C_{nm} \cos m\lambda + \Delta S_{nm} \sin m\lambda] \quad (2)$$

which would provide a simple means of expressing GIA uplift in the time domain. It should be noted however that we have suppressed the full complexity behind this relationship (in the case of GIA, the Earth’s response to previous loading is obtained by convolving the load with the Love numbers rather than simple multiplication as in the elastic case). It remains to be shown that the algebraic shorthand adopted in this expression is in fact valid.

[10] For viscoelastic deformation, *Wahr et al.* [1995] argued that the dominant mode of relaxation, particularly for higher degrees, was the buoyancy force applied at the base of the lithosphere. The magnitude of the buoyancy effect depends directly on the amount of vertical displacement that has occurred at the base of the lithosphere which, in the case of an elastic lithosphere, is very nearly equal to the surface displacement. From this starting point they derived a direct proportionality between the viscoelastic Love numbers for vertical displacement and gravitational potential, being $h_n^{ve}/k_n^{ve} = (2n + 1)/2$. *Wahr et al.* [2000] found that this approximation of the Love numbers was able to recreate an uplift rate field for Antarctica to within ~ 2 mm/yr compared to a field generated by their ice sheet modelling program. This empirical approximation has been used by *van der Wal et al.* [2008], *Tregoning et al.* [2009] and *Wu et al.* [2010] for deriving GIA uplift rates from GRACE temporal spherical harmonic fields.

[11] Using the CALSEA software [*Nakada and Lambeck, 1987; Johnston, 1993; Lambeck et al., 2003*] we generated rigorous model estimates of the change in vertical displacement and the change in geoid height throughout the last ten years for each harmonic degree n . We then calculated the ratio between the two terms as a function of position to estimate the ratio h_n^{ve}/k_n^{ve} and explored its dependence on position. The reference rheological model used in this test comprised a radially symmetric Maxwell viscoelastic body with elastic parameters taken from PREM [*Dziewonski and Anderson, 1981*], a 65 km thick elastic lithosphere, an upper mantle viscosity of 4×10^{20} Pa s from the base of the lithosphere to depth 670 km, a lower mantle viscosity of 10^{22} Pa s from 670 km to the CMB, and a liquid core. Apart from small cusps where numerical instabilities dominate, the ratio was found to be nearly constant across the surface of the Earth for each degree (Figure 1 shows

results for harmonic degrees 3 and 6 but the results were similar for all harmonic degrees up to 256). Excluding numerically unstable regions, we were able to derive globally averaged values for the ratio for each degree for a range of combinations of ice history and Earth response.

3. Independence of h_n^{ve}/k_n^{ve} Ratio From Ice/Earth Model

[12] For each degree we calculated the ratio of GIA-induced changes in vertical displacement and gravity across a series of end members of Earth model parameter space spanning the most likely rheologies for a realistic Earth (lithospheric thickness was allowed to vary from 50 to 10 km, upper mantle viscosity from 10^{20} to 10^{21} Pa s, and lower mantle viscosity from 5×10^{21} to 5×10^{22} Pa s), for a series of ice models with plausible melting histories (and ice volumes increased and decreased by 20%). We found that the ratio is essentially the same for any ice/Earth combination providing that the ice sheet has been ‘stable’, (that is, has not undergone substantial change) for the last 6000 years. It should be noted that in this context ‘stable’ does not preclude present-day mass changes whose contribution is incorporated in the elastic signal. Attention should also be paid to the word ‘substantial’. While it is true that the Antarctic ice sheet has undergone some change throughout the last 6000 years, the effect of those changes will comprise only a small fraction of the overall GIA signal. That is, the Antarctic ice sheet has not changed significantly on this time-frame. The reference ice model used for this study incorporated an ESL change of 2 m for the Antarctic ice sheet over the period 6000–1000 years BP to test the validity of the method when faced with ice volume changes of this scale. Recent studies suggest that the Antarctic region is characterised by lower viscosity values for the upper and lower mantle than are considered in this study [e.g., *Ivins et al.*, 2011]. If this is correct then the long-term asymptote for the h_n^{ve}/k_n^{ve} ratio will be reached more quickly (on time scales significantly less than 6000 years). This will reduce the impact of any late-glacial melting of the Antarctic ice sheet on the derived value of h_n^{ve}/k_n^{ve} .

[13] We generated global averages of the h_n^{ve}/k_n^{ve} ratio (as per Figure 1) using a wide range of ice sheet histories and end member Earth rheology models. Figure 2 shows the asymptotic character of the h_n^{ve}/k_n^{ve} ratio after around 6000 years of the application of a Heaviside load for a series of Earth models. In all cases the ratio settles into a near-constant value that is only weakly dependent on Earth model parameters (and independent of the ice load). For each degree we took the global average of these ratios across the model space to derive an effective ratio between vertical displacement and gravity.

[14] Calculations extending out to harmonic degree 256 show that for large harmonic degrees the curve continues to climb away from the form proposed by *Wahr et al.* [2000] and becomes increasingly non-linear (see insert (a) of Figure 3). For low to intermediate degrees we observe a significant discrepancy between the Wahr and Han approximation and our derived models of about 15% (Figure 3, main panel), which might be expected since *Wahr et al.* [2000] based their analysis on the physics of the M0 mode which contributes 85% to 90% of the total rebound signal for intermediate harmonic degrees on longer time scales. At higher degrees the

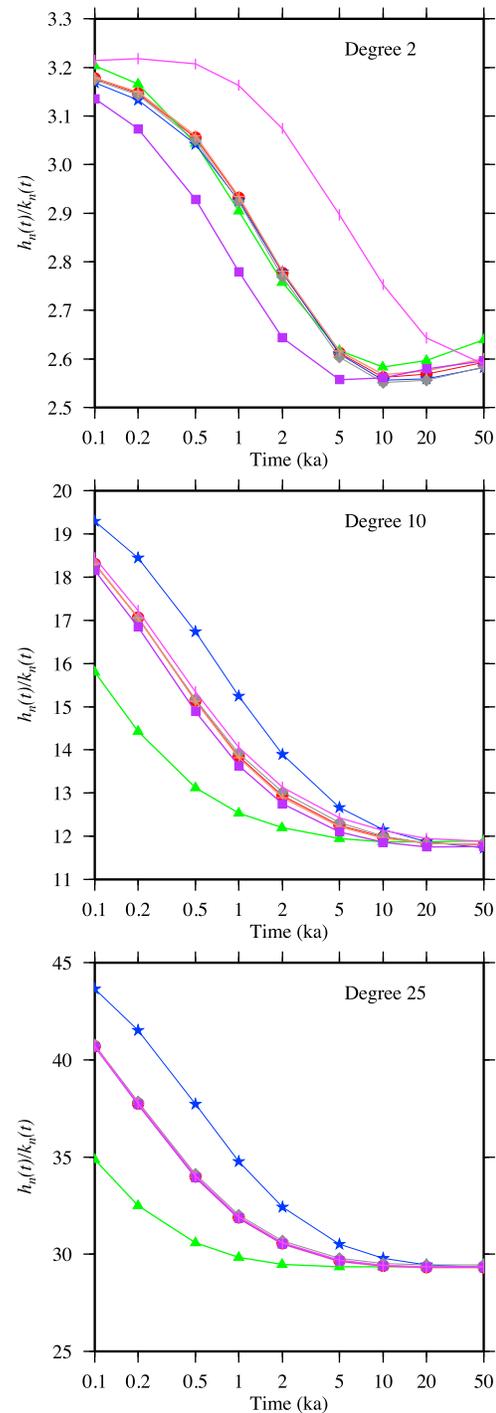


Figure 2. Plots of the ratio $h_n(t)/k_n(t)$ as a function of time for a range of Earth models for harmonic degrees 2, 10 and 25. These values were obtained using algebraic forms for both $h_n(t)$ and $k_n(t)$ for a Heaviside load applied at time zero. Note that the ratio is essentially stable for times more than ~ 6000 years after the load was applied.

Wahr and Han approximation does not seem to describe well the effect of the M0 mode, their theoretical values for the Love number deviating sharply from those we obtained numerically (the same holds true for various intermediate ratios such as that in Wahr and Han’s equation 9

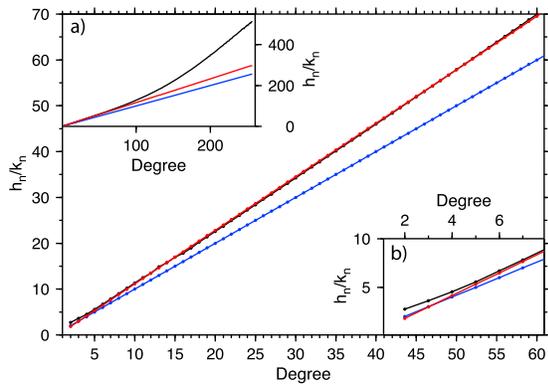


Figure 3. Ratio of h_n^{ve}/k_n^{ve} as a function of degree from CALSEA (black) and the expression of *Wahr et al.* [2000] (blue). The linear relation of equation (1) is plotted in red.

between r_{0n}^h and $(n+1)r_{0n}^k$ which in numerical calculations climbs away from 1 with increasing n).

[15] We found that the linear expression

$$\frac{h_n^{ve}}{k_n^{ve}} = 1.1677n - 0.5233 \quad (3)$$

represents well the average ratio values up to degree 60, but the globally averaged values diverge from linear at higher degrees. Thus, we recommend the use of the linear approximation to degree 60 and the actual averaged values for studies (such as GRACE mascons) requiring higher degree spherical harmonic evaluations (values are provided in Table S2 in the auxiliary material).¹ Substituting equation (3) into equation (2) and using a set of spherical harmonic coefficients that represent an ice load, we can therefore derive estimates of the present-day uplift rate from the change in gravitational potential.

[16] We compared the corresponding uplift rates derived using equations (2) and (3) with the values computed rigorously using the CALSEA program and found differences of <0.3 mm/yr (Figure 4). Substituting the *Wahr et al.* [1995] expression of $(2l+1)/2$ into equation (2) in place of our equation (3) results in uplift rate errors of >1 mm/yr. Our empirical expression (equation (3)) is thus more accurate in reproducing results derived from full ice sheet history models and accounts for all modes, not just Mode 0.

4. Discussion

[17] The consequences of the proportionality between vertical displacement and change in gravitational potential were immediately appreciated by *Wahr et al.* [1995, 2000] and *Wu et al.* [2010] but in those analyses the convolution of the load and response functions was simplified to a direct multiplication without any discussion of the complexities involved. They were also restricted to instances where the M0 mode was dominant, which is not the case for all degrees at all times. The results presented here identify the conditions under which such an assumption is valid. Given the properties of the Love number ratios shown in Figure 2 and the timing of the last deglaciation, the convolution of

load and response functions does not complicate the relationship between the change in gravitational potential and vertical vertical displacement. Further, rather than relying on a mathematical formulation restricted to the M0 mode, our approach allows the h_n^{ve}/k_n^{ve} ratio to be calculated to approximate closely the values derived from a robust numerical technique.

[18] The difference between the full model results and uplift values derived by applying the synthetic ratios to the change in gravitational potential are illustrated in Figure 4. The pattern of the misfit using the $(2n+1)/2$ approximation of *Wahr et al.* [2000] shows significant dependence on the load geometry, being largest in areas of former glaciation and in regions where the water load is of short wavelength. This is consistent with the behaviour observed in Figure 3 where the *Wahr et al.* [2000] response parameters diverge from the numerically-derived values for intermediate and high harmonic degrees, so that the disagreement between the two methodologies is greatest at points where the characteristic wavelength of the load is small. It should also be noted, however, that in regions far from the former ice sheets the Wahr and Han approximation produces a smaller error than our linearised ratios.

[19] Our linear regression representation of h_n^{ve}/k_n^{ve} yields the best agreement between the CALSEA model predictions of uplift rate and those derived from synthetic response param-

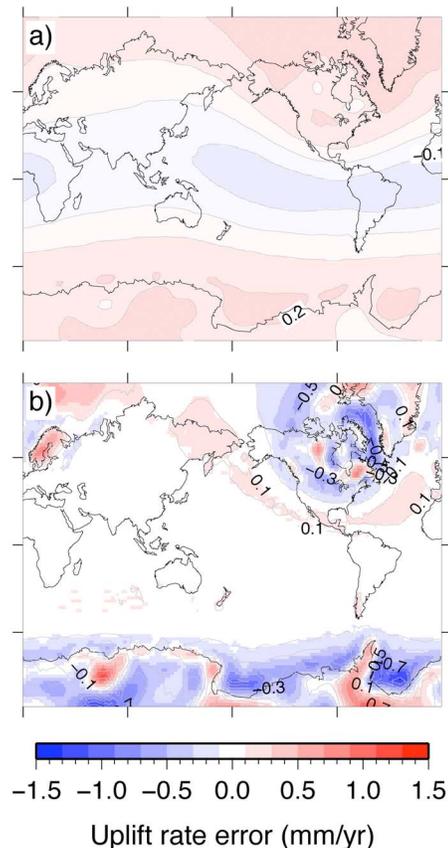


Figure 4. Differences between the present-day GIA uplift computed by the CALSEA program and those derived by (a) our empirical model and (b) the *Wahr et al.* [1995] model. All calculations were performed using the reference Earth model from Figure 1.

¹Auxiliary materials are available in the HTML. doi:10.1029/2011GL048624.

eters, with misfit ranging from 0.30 mm/yr to -0.16 mm/yr for all Earth models considered in this study. The pattern of the differences strongly suggests that they are due primarily to an error in the degree 2 coefficient, where the linear approximation does not perform well; however, replacing the linear approximation with the actual globally averaged values for the ratio produced errors of similar magnitude but reversed sign. The reason for the improved accuracy of the linearised ratios over the original globally averaged ratios is not well-understood. It is possible that the slight reduction in the coefficients at low degrees produced by the linear regression (see insert (b) of Figure 3) compensates for the effects of recent load redistribution in the last 6000 years. The low degree terms are particularly important in calculating hydro-isostatic effects. Since the water load is still being affected by the on-going collapse of the peripheral bulges around Antarctica, North America and Scandinavia, the Love number ratio for these terms is more strongly dependent on time and Earth model and may not be as well represented by the average values derived in this study as the higher degree terms. This remains an avenue for further investigation.

5. Conclusion

[20] The immense computational expense of analysing the relative tradeoffs between GIA and present-day mass changes requires faster and more efficient methods for relating geoid and displacement change fields. This paper represents a substantial improvement over existing methodologies of this type. Given an observed change in gravitational potential produced by GIA effects, our synthetic response parameters provide a computationally convenient and accurate technique for deriving a corresponding trend in vertical uplift, without reliance on an assumed ice model. It remains to apply the techniques developed in this study to an actual data set to establish their ability to reconcile GRACE and GPS observations. *Wu et al.* [2010] have already done as much using the approximation of *Wahr et al.* [2000], although they estimated the ratio for degrees 2 to 7 where they acknowledged that the approximation was not sufficiently accurate. The increased accuracy of our empirical expression for the ratio between vertical displacement and gravitational variations should yield an improvement in the inversions of these observational datasets.

[21] **Acknowledgments.** The authors gratefully acknowledge the review comments of E. Ivins and an anonymous reviewer, which helped to improve and clarify this manuscript. This research was supported under the Australian Research Council's Discovery Projects funding scheme (DP0985030) and the Australian Space Research Program. The numerical calculations undertaken in this study were performed on the Terrawulf cluster, a computational facility supported through the AuScope initiative. AuScope Ltd is funded under the National Collaborative Research Infrastructure Strategy (NCRIS), an Australian Commonwealth Government Programme. The figures in this paper were prepared using the Generic

Mapping Tools (GMT) software package developed by P. Wessel & W. H. F. Smith (<http://www.soest.hawaii.edu/gmt/>).

[22] The Editor thanks Erik Ivins and an anonymous reviewer for their assistance in evaluating this paper.

References

- Chen, J. L., C. R. Wilson, D. Blankenship, and B. D. Tapley (2009), Accelerated Antarctic ice loss from satellite gravity measurements, *Nat. Geosci.*, *2*, 859–862, doi:10.1038/ngeo0694.
- Davis, J. L., P. Elósegui, J. X. Mitrovica, and M. E. Tamisiea (2004), Climate-driven deformation of the solid Earth from GRACE and GPS, *Geophys. Res. Lett.*, *31*, L24605, doi:10.1029/2004GL021435.
- Dziewonski, A. M., and D. L. Anderson (1981), Preliminary reference Earth model, *Phys. Earth Planet. Inter.*, *25*, 297–356.
- Farrell, W. E. (1972), Deformation of the Earth by surface loads, *Rev. Geophys.*, *10*, 761–797.
- Ivins, E., and T. S. James (2005), Antarctic glacial isostatic adjustment: A new assessment, *Antarct. Sci.*, *17*, 541–553.
- Ivins, E. R., M. M. Watkins, D.-N. Yuan, R. Dietrich, G. Casassa, and A. Rülke (2011), On-land ice loss and glacial isostatic adjustment at the Drake Passage: 2003–2009, *J. Geophys. Res.*, *116*, B02403, doi:10.1029/2010JB007607.
- Johnston, P. (1993), The effect of spatially non-uniform water-loads on prediction of sea-level change, *Geophys. J. Int.*, *114*, 615–634.
- Lambeck, K., A. Purcell, P. Johnston, M. Nakada, and Y. Yokoyama (2003), Water-load definition in the glacio-hydro-isostatic sea-level equation, *Quat. Sci. Rev.*, *22*, 309–318.
- Nakada, M., and K. Lambeck (1987), Glacial rebound and relative sea-level variations: A new appraisal, *Geophys. J. R. Astron. Soc.*, *90*, 171–224.
- Ramillien, G., A. Lombard, A. Cazenave, E. R. Ivins, M. Llubes, and R. Biancale (2006), Interannual variations of the mass balance of the Antarctica and Greenland ice sheets from GRACE, *Global Planet. Change*, *53*, 198–208.
- Rietbroek, R., S.-E. Brunnabend, C. Dahle, J. Kusche, F. Flechtner, J. Schröter, and R. Timmermann (2009), Changes in total ocean mass derived from GRACE, GPS, and ocean modeling with weekly resolution, *J. Geophys. Res.*, *114*, C11004, doi:10.1029/2009JC005449.
- Tapley, B. D., S. Bettadpur, M. Watkins, and C. Reigber (2004), The gravity recovery and climate experiment: Mission overview and early results, *Geophys. Res. Lett.*, *31*, L09607, doi:10.1029/2004GL019920.
- Tregoning, P., G. Ramillien, H. McQueen, and D. Zwart (2009), Glacial isostatic adjustment and nonstationary signals observed by GRACE, *J. Geophys. Res.*, *114*, B06406, doi:10.1029/2008JB006161.
- van der Wal, W., P. Wu, M. G. Sideris, and C. K. Shum (2008), Use of GRACE determined secular gravity rates for glacial isostatic adjustment studies in North America, *J. Geodyn.*, *46*, 144–154.
- Velicogna, I., and J. Wahr (2006), Measurements of time-variable gravity show mass loss in Antarctica, *Science*, *311*, 1754–1756.
- Wahr, J., H. DaZhong, and A. Trupin (1995), Predictions of vertical uplift caused by changing polar ice volumes on a viscoelastic Earth, *Geophys. Res. Lett.*, *22*, 977–980.
- Wahr, J., D. Wingham, and C. Bentley (2000), A method of combining ICESat and GRACE satellite data to constrain Antarctic mass balance, *J. Geophys. Res.*, *105*, 16,279–16,294.
- Wu, X., M. B. Heflin, H. Schotman, B. L. A. Vermeersen, D. Dong, R. S. Gross, E. R. Ivins, A. W. Moore, and S. E. Owen (2010), Simultaneous estimation of global present-day water transport and glacial isostatic adjustment, *Nat. Geosci.*, *3*, 642–646, doi:10.1038/ngeo938.
- A. Dehecq, École Normale Supérieure de Cachan, 61, avenue du Président Wilson, F-94235 Cachan CEDEX, France. (amaury.dehecq@wanadoo.fr)
- K. Lambeck, S. C. McClusky, E.-K. Potter, A. Purcell, and P. Tregoning, Research School of Earth Sciences, Australian National University, Mills Road, Canberra, ACT 0200, Australia. (anthony.purcell@anu.edu.au)