The Interaction of Seismic Waves with Horizontal Velocity Contrasts—III. The Effects of Horizontal Transition Zones

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Summary

The effects of some simple horizontal transition zones on incident plane seismic pulses is investigated by using an extension of the methods developed by the author for lateral discontinuities. A simple approximate approach is introduced which is shown to agree well with the results of wave theory calculations. This approximate method is then used in the calculation of theoretical seismograms for slight horizontal transitions in a representative crustal model.

Introduction

One of the major effects of tectonism is the concentration of rapid lateral variation in the Earth’s near surface physical properties into fairly narrow belts. These transition belts separate regions within which the physical properties are relatively much more uniform.

Whilst a fair amount of work has been done on the effect of horizontal discontinuities on seismic wave propagation, the problem of a horizontal transition zone in elastic properties has received less attention. Ghosh (1963) and Knopoff & Mal (1967) looked at surface wave propagation in the transition zone at the continental margin. More realistic models were later considered by Lysmer & Drake (1972) using the finite element method. Recently Smith (1975) has considered body wave propagation through the structure at a dipping plate boundary, again using the finite element technique.

In this paper we consider some very simple models of a horizontal transition by making an extension of the technique introduced by Kennett (1973, 1974, subsequently referred to as Papers I and II) for seismic wave propagation in the vicinity of a horizontal discontinuity in elastic properties. The results of this extension suggest a simple approximate method for estimating the response of a horizontal transition which has a small vertical extent for both small angle dips and linear transitions. This approximation is then used in the calculation of theoretical seismogram sections for models containing a linear horizontal transition and a locally dipping interface between representative crustal models.

A dipping transition

In Paper II we discussed a simple model of a horizontal discontinuity for $SH$ wave propagation. We there considered a plane pulse incident from a medium with shear modulus $\mu_1$, and density $\rho_1$ onto a layer of depth $H$ containing material with the

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properties \((\mu_-, \rho_-)\) and \((\mu_+, \rho_+)\) separated by a plane of hade angle \(\theta\). Following the technique introduced in Paper I we use the device of replacing the discontinuous layer by a layer of 'averaged' properties together with sources, both along the fault plane and distributed throughout the layer, which allows for the effect of the lateral discontinuity.

For large hades the lateral extent of the region containing the discontinuity is considerable, and if we choose the medium ' + ' to have the same properties as ' 1 ' we are able to model the transition from a flat to a dipping reflecting horizon. Alternatively, if ' 1 ' and ' + ' are different we can model a slow transition between the media ' - ' and ' + '.

The results for the first-order stress-displacement field \(\mathcal{B}(x, z, t)\) calculated as in Paper II (Section 2) may again be used for these large hades. The conditions on the use of this approximation may be relaxed slightly from those stated in Papers I and II. Thus if \(\beta_{\text{min}}\) is the smallest wave speed in any of the media and

\[
\eta = |2(\mu_+ - \mu_-)/(\mu_+ + \mu_-)|,
\]

i.e. a measure of the contrast across the fault plane, we now require that the depth of the layer \((H)\) should satisfy

\[
\frac{\omega H \eta}{\pi \beta_{\text{min}}} \ll 1
\]

for all significant frequencies \((\omega)\) in the incident pulse. We present below computations made using this simple model of a horizontal transition.

In our computations we have taken ' 1 ' and ' + ' to have an \(S\) wave speed of 1.5 km s\(^{-1}\) and density 2.0 Mg m\(^{-3}\) and ' - ' to have \(S\) wave speed of 2.0 km s\(^{-1}\) and density 2.5 Mg m\(^{-3}\), the thickness of the transition layer is 10 m. The reflected waveforms are presented for an incident pulse at normal incidence, consisting of a single cycle sinusoid of 20 Hz dominant period. These displacements are calculated at a height of 600 m above the reflecting horizons by the superimpositions of 256 plane waves at each of 64 frequencies using fast Fourier transformation in \(x\) and time.

As we noted in Paper II the behaviour of the reflected wave field from a thin transition structure of this type is relatively insensitive to the hade angle \(\theta\) for small hades. However once the hade becomes larger than 60° significant changes occur. In Fig. 2 we present results for hade angles of 80° and 85° (i.e. dips of 10° and 5°).
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The reflecting horizon is present for negative values of $x_0$ and in this region we see a strong reflection whose amplitude diminishes as the transition zone is approached. For positive $x_0$, beyond the solid marker on Fig. 2, there is no contrast in elastic properties to generate a reflection and we see that the numerical method has been quite successful in suppressing reflected energy. The main difference in the waveforms for the two hade models are seen in the diffracted waves from the discontinuity at the end of the reflecting horizon. Comparison of these reflected waveforms with those for small hades (Paper II, Fig. 3) shows that the diffracted wave amplitude for these more gentle transitions is considerably lower. For a 5° dip angle the horizontal transition zone extends for 114 m and here the diffracted energy appears to come from the lower edge of the transition zone. This effect is also found in seismic model experiments for similar configurations. The effects of the upper edge of the transition are mainly felt in the amplitude reduction in the reflected wave near the edge. The diffracted wave amplitude diminishes increasingly rapidly with increasing hade angle (i.e. decreasing dip) and, for example, for a 2.5° dip there is a smooth variation in reflected waveforms across the transition and very low amplitude diffracted waves.

The relative simplicity of the results for large hades shown in Fig. 2 suggests that the behaviour of the reflected wavefield across a transition region of very small dip should be smoothly varying, and also that it might be approximated by a simple composition of the structures bordering the transition region.
If in Fig. 1 the limits of the transition zone are at $x = a$ and $x = b$ we may para-
metrize the behaviour of the transition region in terms of
\[ \xi = (x - a)/(b - a). \]
For a very small dip angle it is reasonable to assume that in reflection the response at
a point $x$ is the same as if the local layering at $x$ should persist throughout. This
assumption has previously been used by Fuchs (1969) when looking at wedge-shaped
laminated transition zones. We may approximate the effect of local layering within
the dipping transition by taking a linear combination of the responses for the
bordering structures; provided that the thickness $H$ of the transition region is fairly
small compared with the dominant wavelength of the incident pulse ($\lambda$). If the layer
matrices for the depth range $0$ to $H$ for $\xi < 0$ and $\xi > 1$ are denoted by $P_<(0, H)$
and $P_>(0, H)$, the local layer response at a point $\xi$ will be
\[ P_\xi(0, H) = \xi P_>(0, H) + (1 - \xi) P_<(0, H), \quad 0 \leq \xi \leq 1, \]
with an error of $(H/\lambda)^2$. This approximation will also be valid for a linear horizontal
transition zone. This assumption of local layering will be valid provided that the
wavelength of the incident pulse is small compared with the horizontal dimensions
of the transition region.

In Fig. 3 we show a comparison of the results obtained using this simple local
layering approximation and those from the earlier wave calculations. The comparison
is performed for a $2.5^\circ$ dip transition zone for which the width of the transition zone

![Fig. 3. Comparison of relative amplitudes, \( \bullet \), measured from wave theory displa-
ments; -- --, simple local layering approximations.](image-url)
is slightly greater than twice the dominant wavelength and which therefore represents
the upper limit on dip angles to be used with the local layering hypothesis. Measured
computed amplitudes of the reflected waveforms for a 2.5° dip from the method out-
lined above are compared with those predicted from the linear approximation. We
see that there is quite good agreement in the relative amplitudes, the main differences
arise in the neighbourhood of the edges of the transition zone in a region about a
wavelength wide. The residual difference for large positive $x_0$ is due to the failure of
the wave theory calculations to achieve complete annihilation of the reflected waves
in this region. The agreement between the linear local layering approximation and
the wave theory improves as the dip angle reduces to zero.

Similar comparisons between the local layering results and the amplitudes of
reflected waveforms for plane pulses incident obliquely to the reflected horizon show
that the approximation works well for dip angles less than 2.5° and angles of incidence
less than 40°.

An approximate treatment of transition zones

The results we have obtained in the previous section suggest that, provided the
horizontal transition is of a limited spatial extent and that there is a slow change in
properties, comparatively simple approximations may be useful.

We have therefore investigated the use of the local layering approximation for the
calculation of theoretical seismograms for point sources in crustal models containing
slight horizontal transition. Thus for each source–receiver separation ($x$) we calculate
the response for a laterally homogeneous model with properties appropriate to the
structure beneath the receiver.

![Crustal models used in horizontal transition study](image)

**Fig. 4.** Crustal models used in horizontal transition study; ——RC1; ——RC2;
—RC3.
For this approximation to be effective, at refraction ranges, the structures beneath the source and the receiver must differ only in a limited depth range and be otherwise identical. Also, particularly in the case of a dipping transition, the calculation should be for ranges well away from the critical distance for the transition layer; since the reflection properties change markedly with angle in this region.

The evaluation of theoretical seismograms for a whole suite of very slightly different structural models is rather time consuming. Fortunately it was found that a simple linear approximation similar to that introduced in the previous section gave excellent agreement with the full local layering approximation.

We again consider a horizontal transition bounded by the planes \( x = a \) and \( x = b \) and introduce the parameter \( \xi = (x - a)/(b - a) \). We find that for thicknesses of the transition layer of less than a wavelength we may approximate the response within the transition by a composition of the responses for the bounding structures. If the response for the structure present in \( \xi < 0 \) evaluated at \( x \) is \( u_\xi(x, t) \) and that for the structure in \( \xi > 1 \) again evaluated at \( x \) is \( u_\xi(x, t) \), then the response within the transition is given by

\[
u(x, t) = \xi u_\xi(x, t) + (1 - \xi) u_\xi(x, t).
\]

Such \( u(x, t) \) agree very well with the results of direct computation, both for locally

![Fig. 5. Theoretical seismogram sections for the horizontal transitions between the models in Fig. 4 calculated using the simple transition approximations. The pointers indicate the extent of the transition regions. A reduction velocity of 7 km s\(^{-1}\) is assumed; (a) RC1\(\rightarrow\)RC2; (b) RC1\(\rightarrow\)RC3.](image)
dipping regions and for linear transitions in velocity, provided that the horizontal extent of the transition zone is large compared with the wavelength of the incident pulse.

The use of this simple approximation suffers from the same drawbacks as the full local layering approximation which, as we have previously seen, is most likely to be in error for locations within a few wavelengths of the ends of the transition zone. Even in this region it will however give an indication of the behaviour of the wavefield.

As an illustration of the use of this method we consider two simple horizontal transitions between representative crustal models. The structures which we shall be considering RC1, RC2 and RC3 are illustrated in Fig. 4. We have taken linear transitions between the structures RC1 and RC2, and between RC1 and RC3 over a distance of 110 km from a range of 100 km. The first configuration corresponds to the horizontal evolution of a gradient zone and the second to a dipping interface with a dip of 0.52°.

We have taken a common crustal structure for the three models so that the differences in their theoretical responses arise from the structure at or below the crustal-Moho interface near 30 km. For the models RC1 and RC3 where the material below this interface is uniform only a weak head wave is formed, the arrival times depending on the depth of the interface. Model RC2 has a sub-Moho gradient in velocity so that an interference head wave is formed with an amplitude about three times that for the structures RC1 and RC3. The responses computed for the two horizontal transitions are shown in Fig. 5; the markers on the distance axis indicate the extent of the transition. These results have been calculated for a pulse of dominant frequency 4 Hz and thus a wavelength in the vicinity of the transition of around 2 km. Since the transitions are taken to be spread over 110 km the conditions on the use of the simple linear transition approximation are well satisfied.

The linear transition between the models RC1 and RC2 models the horizontal development of a sub-Moho gradient zone. The effects of the transition are most clearly seen on the Moho head wave (the first arrival beyond 140 km) as this builds up gradually as the effect of the vertical velocity gradient becomes stronger with increasing range. The head wave amplitudes in the transition regions are diminished from those for a laterally uniform RC2 structure, especially near 160 km. For the dipping transition between RC1 and RC3 we find that the measured phase velocity of the head wave in the region of the dip agrees to within 0.01 km s⁻¹ with that expected from kinematic theory for an interface with the same dip. The most noticeable features introduced by the dipping interface occur in a change of interface pattern between various travel-time branches at a reduced time of 3 s, between the ranges 140 and 180 km.

This simple approximation has thus enabled us to obtain a good understanding of the effects of slight horizontal transitions without excessive computation.

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