Observational and theoretical constraints on crustal and upper mantle heterogeneity

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The effects of crustal and mantle heterogeneity are reflected in the relative complexity of short-period seismograms at regional to teleseismic distances. Observations of events at regional ranges from Indonesia and New Guinea at the Warramunga array in Northern Australia allow a separation of the propagation through the mantle by concentrating on the coherent signal across the medium aperture array. Most of the incoherent features on the seismograms arise from signal generated noise in the general vicinity of the array with scale lengths of the order of 1—2 km or smaller. Partially coherent phases with relatively rapid changes in waveform across the array are to be associated with organised heterogeneity. This is most likely to arise from features in the crust or at the crust—mantle boundary near the receiver, but could in some cases be due to structural effects near the source. The coherent signals show considerable complexity for phases travelling large portions of their path above 80 km. As the depth of sources increases this coherent portion of the seismic field tends to become simpler, which suggests that the scale-length of heterogeneity is larger at depth.

The use of horizontally stratified models has provided a convenient framework for the understanding of many seismic wave phenomena via the calculation of synthetic seismograms. However, further development is needed for calculating the response of the laterally heterogeneous Earth. The convenience of physical interpretation provided by working with the reflection and transmission properties of the medium can be retained if the structure can be split into a stratified reference model with superimposed heterogeneity. With a plane-wave decomposition of the wavefield, the effect of lateral variations can be introduced by forcing mixing of wavenumber components. A wide variety of phenomena can be simulated by varying the statistical description of the heterogeneity field in the wavenumber domain. This varies the cross-coupling between wavenumber components and so changes the nature of the resulting theoretical seismograms. Such an approach allows a quantitative study of the degradation of the major seismic phases compared to the predictions for stratified models, and also the characterisation of the codas of these phases.

1. Introduction

Short-period seismic wave observations at teleseismic distances show considerable complexity, which can in part be explained by the time history and character of the source (see, e.g. Douglas, 1981) but which have a significant component arising from wave propagation processes. At regional ranges the complications are probably at their greatest, since there is the possibility of multiple paths of propagation in the upper mantle followed by interaction with the heterogeneous crustal waveguide. Although early attempts to use the character of the seismic waveforms to define the velocity distribution in the Upper Mantle were based on short-period observations (Helmberger and Wiggins, 1971; Wiggins and Helmberger, 1973), most subsequent studies have used long-period observations with the aim of avoiding the contamination due to small-scale heterogeneity (see, e.g., Grand and Helmberger, 1984a, b).

However, the use of long-period waves imposes an inevitable sacrifice in the level of potential detail which can be resolved in the velocity distri-
bution. Structures which would imply very different constraints on mantle petrology, e.g. by having sharp or diffuse velocity ‘discontinuities’ or the presence of small velocity inversions, can have almost identical long-period responses (Ingate et al., 1986). Further, the constraint of interpretation in terms of radially stratified models runs into considerable difficulty in those regions where there is observational evidence for substantial horizontal gradients in the structure of the upper mantle (Rial et al., 1985).

We are now entering a period in which the essential heterogeneity of the Earth on a variety of scales has begun to be recognised and where we seek to enhance our understanding of Earth processes by characterising this heterogeneity in terms of its scale-length and location. The work of Woodhouse and Dziewonski (1984) has mapped out some of the large scale variations in the S-wave speed in the upper mantle by inversion of digital seismograms. The minimum scale-length they could study was of the order of 3000 km, but there are certainly heterogeneities on a much smaller scale.

Studies of upper mantle structure using the large aperture Norsar array (King and Calcagnile, 1976; England et al., 1977, 1978), showed that even where the spatial sampling was dense difficulties arose in delineating the mantle structure. There was noticeable variability between the results for different events and even across the array itself. Examination of the published record sections shows the difficulty in extracting the simple travel time branches predicted for radially stratified models. The actual behaviour for any individual event is of the style indicated by Nye (1985), lateral heterogeneity causes the simple caustics to break up into more complex structures which do not correlate well from event to event.

Many of the techniques, which can be applied to sets of recordings of natural events from individual stations, depend on making assumptions as to the likely character of the velocity distribution over a number of wavelengths. With a medium aperture array such as the Warramunga array in Northern Australia, on the other hand, we have a number of observations in a relatively small zone and are able to compare the coherent and incoherent portions of the wavefield as it traverses the array. The major source of the incoherent energy appears to be signal-generated seismic noise arising from structures in the vicinity of the array. The correlation length is less than the 2 km separation between the array sensors, and implies generation by small-scale features (cf. Key, 1968). The incoherent part of the wavefield reflects the deeper part of the propagation path and can provide valuable information on the character of the seismic parameter distribution in the mantle. We shall show that, for regional events in the Banda Sea and New Guinea recorded at the Warramunga Array, there is a progressive change in the character of the propagation pattern with increasing source depth. Shallow events give relatively complex behaviour. The initial onset is followed by continuing arrivals of short bursts of coherent energy, with azimuths not too far from the great-circle path but a range of slownesses. Whilst the deepest events generally have well-defined sets of coherent arrivals corresponding to the classical seismic phases P, pP etc.

The patterns of propagation that we observe are sufficiently complicated that we need to seek some aid by modelling the effect of likely heterogeneity. To do this we must recognise the nature of the propagation process. The outer 50–80 km of the Earth appear to be significantly more variable in its properties than the regions at greater depth and so should be separately treated. Whilst such a separation is not uncommon in techniques for generating synthetic seismograms at teleseismic distances (see, e.g. Douglas et al., 1973), it has surprisingly received little attention in the study of upper mantle phases at regional distances.

An effective and instructive way of understanding the effects of lateral heterogeneity is to work with a horizontally stratified base model and look at the way that wave propagation is distorted by the presence of the additional structure. In a stratified medium, plane-wave components characterised by a horizontal wavenumber vector propagate independently, and seismograms can be constructed by Fourier synthesis once the response to individual wavenumber components has been determined. In the presence of horizontal
velocity variations it is still possible to work with the wavenumber components, but now they couple due to the presence of the horizontal variations in properties (Kennett, 1972, 1986). If one can track the coupling the seismograms can again be generated via Fourier synthesis. In general this is a formidable task. But if we are prepared to work with statistical measures of the inhomogeneities via their wavenumber spectra, we can devise simulation schemes which can represent, in large measure, the effects of various classes of heterogeneity in different portions of the model. With an efficient development of the wave propagation in stratified media it is possible to look at a variety of statistical perturbations at modest computational cost.

2. Observations of heterogeneous seismic wave fields

Individual short-period seismograms at sensitive stations usually display complex wavetrains in which prominent seismic phases can be recognised with an accompaniment of smaller disturbances of similar period. However, when one has the opportunity to look at a number of quite closely spaced stations, as in a medium aperture array, the degree of dissimilarity in the waveforms can be surprising. Such an array allows the use of signal processing techniques to try to enhance the coherent part of the wavefield and so avoid the complications introduced by structure in the neighbourhood of the receivers.

As an example of the potential of this approach, we consider two groups of events with closely grouped epicentres but varying focal depth recorded at the Warramunga seismic array in Northern Australia. The first group of events is from the southern side of the subduction zone around the Banda Sea with epicentral distances around 13.2° (1470 km). The second group of events is in the neighbourhood of the border between Irian Jaya and Papua–New Guinea at an epicentral distance of 17.3° (1925 km).

The Warramunga seismic array (WRA) is situated about 35 km south of the town of Tennant Creek in the Northern Territory of Australia (Cleary et al., 1968). The array consists of an L-shaped deployment of 20 short-period vertical seismometers, five sites have in addition two orthogonal, horizontal-component seismometers (see Fig. 1). The spacing between pits is approximately 2.25 km and the signals are telemetered to a central site where they are recorded digitally at 20 samples per second. The relatively close spacing of the seismometers gives an effective local sampling of the wavefield and the layout, in approximately straight lines (blue and red arms), means that it is comparatively easy to follow the character of arrivals from receiver to receiver. The array lies in a region of generally low relief with some rocky outcrops near the centre of the array. There is therefore only a modest topographic component to the scattering of seismic waves incident at the array.

The locations of the events we will be discussing relative to WRA are shown in Fig. 2, in which we can see the grouping of the clusters of events with different focal depths. The events were chosen from the period 1981 to 1982 so that the operational characteristics of the array were closely comparable. For each group of events the azimuths are not too far from 000°, so the time shifts between receivers is greater for the northern blue
multichannel coherency is provided by using the idea of semblance (Neidell and Taner, 1971), which is frequently used in association with seismic velocity stacking in geophysical prospecting work. A stack is performed over a time gate \((t - \Delta t/2, t + \Delta t/2)\) with a smooth weighting function \(w(t)\), which fairly quickly drops off away from the target time \(t\). The semblance is then formed as the ratio of the total energy of the stack within the time gate to the sum of the energy of the component traces within the same time window. We denote the channels, via upper-case subscripts and the discrete time samples in the \(J\)th trace (after phase delays), by \(s\). The semblance at time \(t\) is then given by

\[
S_t = \frac{\sum_s w_s \left( \sum_J g_{sJ} \right)^2}{N \sum_s w_s \sum_J g_{sJ}^2}
\]

where \(N\) is the number of channels and the summation over time \((s)\) is carried out over the time gate about \(t\). We have used a seven-sample gate \((0.35\ \text{s})\) applied at four sample intervals and used four-point Lagrangian interpolation to generate semblance values for each discrete time point in the stacked trace. This arrangement gives a reasonable balance between computational effort and systematic data coverage.

The semblance is always a positive quantity and will be of the order of unity for strong coherent events. A plot of the semblance against time therefore gives a picture of the degree of data consistency across the array at a particular combination of slowness and azimuth, and an indication of the strength of the coherent events arriving at the array. However, we have found it more useful to use the semblance trace as a modulator to the linear stack, to produce a nonlinear stack in which the most significant coherent events are enhanced.

This 'semblance-enhanced' stack eliminates the clutter of partially coherent energy on the standard linear stack. The trace for a particular slowness and azimuth emphasises those signals that arrive at the array as a coherent plane wavefront and the details will therefore reflect on processes...
occurring well away from the array, in the upper mantle or at the source. No stacking technique is able to cope with interfering wavelets of similar character so that the composite wavelet changes character across the array thus invalidating the simple incident plane wave model.

2.1. Banda Sea events

The first group of events was on the southern side of the Banda Sea arc to the east of Timor, and spanned the depth interval from 45 to 180 km (see Table I). In Fig. 3 we show the recorded P-signals from the blue arm of the array for about 1 min at the onset of the three events. For each of these events we have complex wave trains, rich in high-frequency energy (2–3 Hz or greater). The P-codas are sustained, rather messy and vary noticeably from receiver to receiver.

The shallowest event (1) shows a very slow decline in the amplitude of the coda, but close inspection of Fig. 3 reveals the presence of an underlying pattern of arrivals crossing the array. These phases are masked by a substantial chaotic component which varies strongly from trace to trace. The complex behaviour in this case may be due in part to the possibility of direct coupling of the event into the bent lithospheric waveguide extending from the Australian continent into the subduction zone.

For the two deeper events there is a more clearly defined P-wave phase. The coda amplitude more rapidly declines and there is less indication of late phases 15 s or more after the P onset than for event 1. For each of events 2 and 3, the shape of the onset of the waveform is similar across the array but this is followed by features which do not correlate too well from trace to trace. A second arrival can be clearly seen for event 2 around 39 s on traces B7, B8 and B10, and can be less readily traced across the other traces.

Trace B5 consistently displays greater high frequency content than the others after the onset of the P wavetrain. However, there is no such amplification for the background noise. This suggests that we are getting particularly strong signal-generated noise in the neighbourhood of this site, although there are no obvious surface features in its vicinity. Surface seismic velocities are rather high in the Tennant Creek region (around 5.00 km s⁻¹) so that the surface wavelength for 2 Hz energy is about 2.5 km, which is of the same order as the interstation spacing. At the highest frequencies we can get effects from localised features which have very little correlation with what is seen on even the neighbouring traces.

The ‘semblance-enhanced’ stacks for the data shown in Fig. 3 are presented in Fig. 4 for a span of slownesses from 0.09 to 0.13 s km⁻¹, and azimuth along the great-circle path from the event to the centre of the array. The slowness range covers the likely span for P-wave energy travelling beneath the crust. No corrections have been applied for local structure. The number at the base of the slowness panel for each event is a measure of the largest semblance value encountered and hence the degree of coherence in the wavefield for the event.

A striking feature of Fig. 4 is the increasing simplicity of the coherent energy with increasing focal depth. Comparison with Fig. 3 shows that the later phases for event 1, whose presence was suspected from visual inspection, now clearly emerge. The wavetrain arriving at the array con-

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
<th>Lat.</th>
<th>Long.</th>
<th>Depth</th>
<th>(M_b)</th>
<th>(\Delta^o)</th>
<th>Azimuth</th>
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<td>2 Sep. 1981</td>
<td>7.80 S</td>
<td>128.80 E</td>
<td>49 ± 14</td>
<td>4.8</td>
<td>13.22</td>
<td>157</td>
</tr>
<tr>
<td>2</td>
<td>3 Aug. 1981</td>
<td>7.91 S</td>
<td>128.22 E</td>
<td>65 ± 18</td>
<td>4.9</td>
<td>13.35</td>
<td>154</td>
</tr>
<tr>
<td>3</td>
<td>6 Apr. 1981</td>
<td>7.49 S</td>
<td>129.50 E</td>
<td>182 ± 17</td>
<td>4.6</td>
<td>13.23</td>
<td>159</td>
</tr>
<tr>
<td>4</td>
<td>2 May 1982</td>
<td>3.58 S</td>
<td>139.94 E</td>
<td>17 ± 16</td>
<td>5.1</td>
<td>17.16</td>
<td>198</td>
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<tr>
<td>5</td>
<td>27 Nov. 1982</td>
<td>2.99 S</td>
<td>139.41 E</td>
<td>65</td>
<td>4.6</td>
<td>17.55</td>
<td>196</td>
</tr>
<tr>
<td>6</td>
<td>18 Mar. 1981</td>
<td>3.78 S</td>
<td>140.92 E</td>
<td>105</td>
<td>4.5</td>
<td>17.30</td>
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Fig. 3. Montage of seismograms for the blue arm of WRA for the Banda Sea events at 13.2° range. About 1 min around the onset of P is shown.

Fig. 4. Slowness–time display for the Banda Sea events. For each slowness the semblance-enhanced stack trace along the great-circle path derived from the full WRA array is shown.

sists of numerous arrivals with a wide range of slownesses, associated with surface reflections and multiples in the crust and uppermost mantle. An azimuth analysis shows that most of the energy has apparent azimuths within 5° either side of the great-circle path. Some late energy can deviate rather more from the nominal azimuth indicating involved multipathing, e.g., with refraction at the continental margin of northwestern Australia. The total pattern for event 1 is consistent with propagation with multiple reflections in an imperfect waveguide.

For event 2, the most prominent feature is the second arrival around 40 s with a smaller slowness
than the onset of the waveform. The timing and slowness are consistent with this phase being returned from the '400 km discontinuity'. After this feature the stack records are characterised by small amplitude arrivals with a character similar to that for event 1. These would correspond to propagation paths with surface reflection and so with much less penetration into the mantle.

The third event (3) is characterised by a very simple stack section in which the principal feature is the P phase. The secondary phase with about 10 s delay has larger slowness and therefore must be pP rather than arising from deeper in the mantle. This phase is barely discernible in Fig. 3, and has been very considerably enhanced by the processing. However, the 5 s interval, after P which contains significant energy on each array trace, makes almost no contribution to the coherent stacks. In this time interval, we see that the waveforms correlate quite well with those on neighbouring traces but there is substantial variation across the whole array. As a result the arrivals do not fit the coherent plane-wave model used in the stacking and so are suppressed. The waveforms are varying noticeably on a scale of 5–10 km and these variations are likely to be associated with structural features on comparable scales.

2.2. New Guinea events

In Figs. 5 and 6 we present the seismic traces for the blue arm of WRA and the enhanced stacks from the full array, for the group of events in central New Guinea. These earthquakes are in the rather complex seismic zone created by the collision of the Australian and Pacific plates.

The behaviour of the seismic wavefield with varying focal depth is similar to that seen for the Banda Sea events at somewhat closer range, although the signals are not so rich in high-frequency content especially the crustal event (4). The lower frequencies give rise to slightly less incoherent noise than for the Banda Sea events, confirming that much of this energy is signal-generated noise arising from scattering by small scale features, certainly smaller than the interstation spacing.

Event 4 shows a very weak initial P onset but then builds to significant amplitude. The pattern of arrivals across the blue line in Fig. 5 at around 3 s suggests that we have the interference of retrograde and prograde branches of the arrivals from the '400 km discontinuity' which is consistent with the difficulty of resolving the slowness in the stacked section (Fig. 6). On the nearly orthogonal red line, the records are generally simi-
showing a sequence of coherent events but with lower frequencies these are more evident in the raw data.

For events 5 and 6, the initial P phases correlate very well across the array. However, they are immediately followed by more variable waveforms on the different traces and indeed the later arrivals vary considerably from trace to trace. For event 5 at 65 km depth there is a complex pattern of small coherent events after 52 s in Fig. 6, corresponding to surface reflected phases with fairly shallow paths in the mantle.

A striking example of a partially coherent arrival is provided by the prominent later phase for event 6. This appears as a clear simple arrival on the red line at stations R6–R10 but is hardly visible on R1–R5. Across the blue arm, as we can see from Fig. 5, the waveform changes from trace to trace. On a linear stack section the arrival fragments into a sequence of subarrivals with similar properties, and the maximum stack amplitude tends towards larger slownesses as time increases. The poor coherence in the waveform means that the feature is barely visible in the enhanced stack section compared with the P phase. It is, however, as large as some of the features seen for event 1.

2.3. Discussion

The seismograms from these two groups of earthquakes show how different scales and levels of heterogeneity in various parts of the Earth can interact to produce complex seismograms.

Energy, which has penetrated below about 80 km in the mantle, e.g. the P phases from the events at 65 km and deeper, shows a coherent onset with some level of fragmentation in the later part of the phase. On the other hand, higher frequency energy which has travelled above 80 km, such as the direct phase from shallow mantle events and the surface reflected energy from deeper sources, shows a succession of small phases with fairly high coherence arising from interaction with the surface and the uppermost mantle. The relatively simple propagation at depth suggest that the scale length of any heterogeneities is rather greater below 80 km than in the uppermost mantle.

The highest frequency energy on the array

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<th>0.0940</th>
<th>0.0980</th>
<th>0.1020</th>
<th>0.1060</th>
<th>0.1100</th>
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<th>0.1180</th>
<th>0.1220</th>
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<td>5</td>
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</table>

Fig. 6. Slowness–time display for the central New Guinea events. For each slowness the semblance-enhanced stack trace along the great-circle path derived from the full WRA array is shown.

lar to those at sites B1 and B2, but there is a shift in energy towards the back of the wavegroup towards the end of the line. The general character of the stacked section for event 4 is similar to the shallowest event from the other group (I), in
seismograms shows very little correlation between nearby sites, about 2 km apart. Most of this energy is likely to have a local origin in the neighbourhood of the array and to be associated with spatial features on the 1–2 km scale and smaller. The sustained nature of this signal-generated noise suggests that there is substantial variation in the Precambrian rocks underlying the Warramunga array. The scale-length of heterogeneity of the order of 1 km for the uppermost part of the crust fits in well with a number of studies (see, e.g. Frankel and Clayton, 1986) and references therein). Larger-scale features would produce a more coherent field with greater azimuth anomalies than are observed.

We have in addition, the partially coherent arrivals at the end of the P phases and accompanying later phases. These signals seem to be composed of sets of overlapping arrivals of similar character, just as in the study of Mack (1969) on short-period phases at LASA in Montana, U.S.A. In a study of Tonga/Fiji events at WRA, Wright et al. (1974) suggested that apparent bifurcations in phases were associated with the structure near the base of the crust. These partially coherent features show strong similarity over scales of 5–10 km and generally appear across the whole array. For events in New Guinea, a similar pattern of coherence (especially on the red arm) is seen for events at a range of source depths and locations. This suggests that the origin of the effect in this case should be sought in the vicinity of the array. One possible location would be in the lower crust or at the crust–mantle interface, with multipathing induced to the various array sites. Scattered energy in subarrivals generated further away from the direct transmission path would then have a lower apparent slowness, as seen for event 6. The individual subarrivals are not too large and so can be swamped by more coherent energy as in the onset of the P phase, they will be most evident when the incident pulse is simple and isolated.

Partial coherency could also be induced by structure near the source, e.g. in the subducted slab, and then be ‘frozen in’ and not significantly affected during the passage through the weakly heterogeneous medium to the receiver. For the complex high frequency wavetrains of Banda Sea events it would be very difficult to try to separate near-receiver or near-source effects.

Theoretical studies of the effect of lateral heterogeneity

In the previous section we have seen how we can begin to examine the scale-lengths and locations of the various classes of heterogeneity in the Earth. To confirm these interpretations, we need to be able to study the propagation of seismic waves through theoretical simulations of such structure.

The technique we use is based on the use of a plane-wave decomposition of the seismic wave field with the effect of heterogeneity introduced by coupling between plane-wave components (Kennett, 1972). For a medium whose properties can be represented as a three-dimensional heterogeneity superimposed on a horizontally stratified reference medium, the construction of the propagation terms is simplified. It is possible to express the reflection and transmission response of portions of the medium as a combination of the effect of the reference structure and an additional term incorporating the coupling between wavenumber components introduced by the heterogeneity (Kennett, 1986).

Consider a region bounded by two horizontal planes and an incident plane wave described by horizontal wavenumber $k_0$. The reflection matrix at wavenumber $k$ can then be expressed as

$$R(k, k_0) = R_0(k_0) \delta(k - k_0) + \int d^2 \xi r(k, \xi)$$

where $R_0$ is the reflection matrix for the stratified medium and $r(k, \xi)$ depends on the wavefield inside the region generated by the incident plane wave. In general, the coupling terms in $r(k, \xi)$ can only be described via a matrix whose entries depend on the horizontal wavenumber spectrum of combinations of the elastic moduli. However, for modest heterogeneity, we can extract from $r(k, \xi)$ a common term depending on the wavenumber spectrum of the heterogeneity, with shifted argument $\tilde{a}(k - \xi)$. The wavefield inside the region will have wavenumbers concentrated in the vicin-
ity of \( k_0 \), and so the effect of the heterogeneity will be to spread energy over a band of slownesses about \( k_0 \) characterised by the inverse scale-length of the heterogeneity in the region. If we adopt a Born approximation, we would use the stratified medium representation within the heterogeneous zone and reduce the integral over wavenumber to a single term \( r(k, k_0) \) (Kennett, 1987). Although it is clear that multiple scattering processes can play an important role, we can get an effective indication of the coupling introduced by heterogeneity by studying the simpler first-order Born scattering.

Since it is rather difficult to carry out fully the wavenumber coupling calculations, we have adopted a statistical description of the heterogeneity. We have assumed a form for the normalised amplitude spectrum \( |\tilde{a}(k)| \) and then used a random phase component, thus simulating an ensemble of such scatterers randomly distributed in horizontal directions. To get an idea of the effect of various levels of heterogeneity without having to form the full \( r(k, k_0) \) coupling term, we have worked with a simplified representation for reflection

\[
R(k, k_0) = R_0(k_0) \delta(k - k_0) + c|\tilde{a}(k - k_0)|\phi(k)
\]

(3)

where \( c \) is an adjustable measure of the strength of the heterogeneity and \( \phi(k) \) is a (pseudo-)random phase component. By adjusting \( c \) and the shape of the wavenumber spectrum in different parts of the model, this gives a flexible and powerful scheme for simulating the effect of heterogeneity.

A prerequisite for implementation of this scheme is a versatile and efficient method for generating the wave propagation characteristics of the underlying stratified, reference model. In our discussions of the observations we have noted that the crust and uppermost mantle appear to be in a different heterogeneity regime than the deeper mantle. We therefore need to be able to separate out the effects of these two distinct regions. This has been done by using a new set of numerical algorithms based on the discussion of split stratification in section 9.2 of Kennett (1983). An arbitrary separation level is introduced, and the propagation above and below this level separately treated with coupling via surface reflections or multiples between the two levels. The source may be placed in either the upper or lower zones, but different forms for the response are required in each case.

The stratified model may consist of uniform layers or piecewise smooth zones, and the total response is built up from the reflection and transmission characteristics of portions of the stratification. For the piecewise smooth sections we employ the scheme discussed by Kennett and Illingworth (1981) based on a recursive treatment using the Langer approximation in each smoothly varying zone. Once the plane-wave response of the stratification is established, a variety of statistical perturbations can be introduced at modest computational cost. The final stage is an integration over slowness and Fourier inversion over time to produce a set of synthetic seismograms.

As an example of this approach we consider the generation of theoretical seismograms for the distance range from 1700—2900 km in which the stratograms involve the interaction of energy returned by the upper-mantle structure. As the horizontally stratified base model we have used the shear wave model SNA proposed by Grand and Helmberger (1984a) for the shield regions of North America and constructed a scaled P-wave distribution. The P- and S-wave velocities for this model as a function of depth are shown in Fig. 7, after the application of the Earth-flattening transformation. The model has a slight velocity inversion around 250 km depth and very pronounced discontinuities in wavespeed at flattened depths of 417 and 720 km. In the numerical experiments we shall illustrate, we have assumed that these two discontinuities remain plane. However, corrugations on the interfaces could be included within our wavenumber coupling scheme by using the sort of method described by Kennett (1977) to allow for redistribution of energy between slownesses via the shape of the surface.

A suite of three-component theoretical seismograms for the P wavefield, which were generated by an explosive source at a depth of 15 km in the model SNA, are shown in Fig. 8.
band from 0.05 to 3.0 Hz. At each distance we show the vertical (Z), radial (R) and transverse component (T) traces. For the horizontally stratified medium there is, as expected, no energy on the transverse trace but it is included for comparison with the effect of heterogeneity. The separation level between the upper and lower parts of the structure was set at 60 km. Up to five surface multiples were allowed in the upper part of the model, whilst only a single reflection from depth was included to avoid the need for excessively long time intervals in the computation. The inclusion of surface multiples gives a complex set of broad-band seismograms in which the multiples catch attention by their strong correlation from trace to trace.

We have taken different models for the heterogeneity pattern above and below 60 km. In the upper zone we have taken $|\tilde{a}(k)|$ to be flat over
the interval $|k/2\pi| < 0.03$ and then tapered to zero at $|k/2\pi| = 0.1$. This allows a broad range of scale-lengths in the heterogeneity down to around 15–20 km. When heterogeneity has been imposed in the lower region we have used a tighter distribution in wavenumber space corresponding to much broader scale phenomena than in the upper zone. For this mantle heterogeneity we have taken $|\hat{a}(k)|$ to be flat for $|k/2\pi| < 0.01$ and tapered to zero at $|k/2\pi| = 0.02$, so that the scale-length is greater than 50 km.

A number of theoretical seismogram calculations were made with differing levels of heterogeneity in both the upper and lower zones to investigate the effects introduced by wavenumber coupling. An example of the resulting seismograms is shown in Fig. 9, for a case in which the principal heterogeneity component is imposed in the upper zone but weak mantle heterogeneity was also included. The weighting factor for the crustal component was 0.1, so that up to 10% variation in reflectivity could be produced; whilst only 5% maximum variation in mantle reflectivity was allowed. The effect of the random phase distribution in each case is to substantially reduce the effective perturbation. With the simple technique we have used for introducing the wavenumber coupling, some aliasing of signal generated noise occurs and this can be seen at the onset of the seismograms. Once we allow for three-dimensional structure, we have to include the possibility of interaction between P, SV and SH waves (Kennett, 1986). We have made the assumption that as a result of interaction with the heterogeneity, wavenumber coupling will be most significant for the same wavetype, with an equal chance of cou-

Fig. 9. Theoretical seismogram record section with the inclusion of heterogeneity. As discussed in the text, most of the heterogeneity is concentrated above 60 km, but weak mantle heterogeneity is also included.
pling into the other two wavetypes but a coupling factor half the size. In consequence, in Fig. 9, we have been able to induce some transfer of energy onto the transverse component which was previously (identically) zero.

Comparison of Figs. 8 and 9 shows that the heterogeneity has significantly modified the appearance of the later parts of the seismograms, where signal-generated noise frequently swamps the surface multiples. The multiple arrivals are therefore much less easily correlated from trace to trace. In addition, the amplitudes of the multiples are diminished by the distortion of the surface waveguide imposed by the heterogeneity in the upper zone. At first sight, the main propagation branches from the upper mantle structure do not show much change. However, detailed examination indicates subtle changes in timing and interference of different wave groups which are beginning to modify the waveforms. More substantial effects can be produced by increasing the perturbation level in the mantle. The signal-generated noise component is principally crustal in origin and shows little spatial coherence at the 50 km spacing of the traces, as expected from the character of the scatterer field.

To show the way in which the waveform is modified by increasing levels of heterogeneity, we display in Fig. 10 a sequence of four vertical component seismogram calculations for two different distances and varying types of heterogeneity. As the level of crustal heterogeneity is increased, the later multiple arrivals begin to be swallowed up in the signal-generated noise, but the most significant waveform distortion occurs when heterogeneity is also introduced in the lower zone.

Whilst the theoretical seismograms we have been able to generate show considerable complexity, they are still very much simpler than the observations we have presented from WRA. No really small-scale heterogeneity (e.g. 1–2 km scale) has been included because of the computational difficulties imposed by spreading out energy over a very broad span of wavenumbers. Nevertheless, we have shown that a modest level of heterogeneity on a 15–50 km scale can create an involved wavefield with a resemblance to the real data. For propagation where the dominant path is in the mantle, the influence of heterogeneity is limited by the relatively short time that a wave spends in a particular region. Multiple interactions are possible in the crust, but will have most significance later in the records. With our random phase component we do not allow for organisation in the heterogeneity, such as appears to be responsible for the partially coherent arrivals observed at Warramunga (and other arrays). Such organisation is likely to arise from topography on interfaces or locally complex structures near the receiver, but could also occur near the source.

We have therefore been able to show that the superposition of modest levels of heterogeneity on different scales in selected parts of the model can have significant effects on the resulting seismograms and go some way towards explaining the character of observed seismic records. Topography on the Mohorovicic and other discontinuities would impose further wavenumber coupling and so enhance the signal-generated noise field, leading to more realistic seismograms.
References


