Synthetic reflection seismograms in three dimensions by a locked-mode approximation

G. Nolet*, R. Sleeman‡, V. Nijhof‡,
and B. L. N. Kennett§

ABSTRACT

We present a simple algorithm for computing the acoustic response of a layered structure containing three-dimensional (3-D) irregularities, using a locked-mode approach and the Born approximation. The effects of anelasticity are incorporated by use of Rayleigh’s principle. The method is particularly attractive at somewhat larger offsets, but computations for near-source offsets are stable as well, due to the introduction of anelastic damping. Calculations can be done on small minicomputers.

The algorithm developed in this paper can be used to calculate the response of complicated models in three dimensions. It is more efficient than any other method whenever many sources are involved. The results are useful for modeling, as well as for generating test signals for data processing with realistic, model-induced “noise.” Also, this approach provides an alternative to 2-D finite-difference calculations that is efficient enough for application to large-scale inverse problems. The method is illustrated by application to a simple 3-D structure in a layered medium.

INTRODUCTION

Current algorithms for obtaining the acoustic response of a structure have, in general, only limited accuracy for larger offsets. However, true inverse problems are gaining in importance in seismic interpretation. Modeling is often a prerequisite to solving the inverse problem (e.g., Tarantola, 1987; Mora, 1987; Duijndam, 1987). Efficient methods to solve the forward problem have limited accuracy: signals at large offsets are often not adequately modeled nor are multiples taken into account in many methods.

For a homogeneous background model with weak reflecting heterogeneities, Tarantola (1984a, b), using the Born approximation, established the correspondence between seismic migration and inversion. Mora (1987) and others have extended Tarantola’s results to more complicated background models, at the expense of a large increase in computation time. With the computation speed of present-day computers, it would seem that only homogeneous starting models, where Green’s functions are of the simple form exp (ikr)/r, can be used in practical applications. This is an undesirable situation.

This paper develops a method that produces accurate synthetic seismograms for reflecting heterogeneities in a layered background model, either in two or three dimensions. In contrast to methods based on parabolic approximations, our method is accurate at large offsets. It is efficient enough to produce synthetics on moderate size computers. The algorithm in this paper takes all multiples into account and, therefore, can accurately model waveform distortion because of thin-bed multiples and the echoes from the sea bottom or the basement.

Classical Sturm-Liouville theory is at the heart of the method: the Green’s function is developed as a sum of the eigenvibrations of the layered medium. Eigenvalues are forced to be real by locking the energy into the medium through the introduction of a perfect acoustic mirror at depth. The Green’s function in this form has the particular advantage that it is a simple function of the source location. This sets the algorithm apart from more conventional reflectivity methods (e.g., Kennett, 1983) in which a full response calculation is needed for each new source depth. Since we use the Green’s function to calculate the reflections from heterogeneities in the medium by Born’s approximation, in which each heterogeneity acts as a separate source, this form of the Green’s function leads to very high efficiency.

Modal summation is a well-known technique in global seismology, especially in the study of surface waves and very low-frequency eigenvibrations of the Earth. Nolet and Kennett (1978) established the correspondence between ray arri-
vals and stationary phases in the normal mode sum. However, at first sight, this does not seem to be a very practical way to calculate the wave field in reflection seismology; since ray-paths are practically vertical, energy is not confined to a wave channel, and most of the wave field is not contained in the normal mode expansion except at very large offsets. Harvey (1981) encountered this problem when trying to calculate refracted $P$ arrivals close to the source. His solution was to force the seismic energy to remain confined by introducing an artificial high-velocity layer at some depth into the model. This produces an approximation to the exact wave field, but with the advantage that the errors can be physically understood and unmasked: the high-velocity bottom produces artificial reflections with predictable time delays. Most importantly, if the bottom is placed deep enough, these reflections will arrive outside the time window of interest. Nolet (1983) applied this “locked-mode approximation” to calculate seismic risk for oil platforms in the North Sea region and showed that a large number of crustal reflections can be included into the modal sum, provided the velocity of the reflecting bottom layer is chosen large enough. Recently, Haddon (1986, 1987) developed a method based on the summation of leaky modes.

This paper develops a new mode summation formalism for the acoustic case. In order to lock even the rays at zero offset, we adopt a radical approach to mode locking: our bottom layer is a perfect reflector (Figure 1). This way we avoid multivalued functions, which forced Haddon (1986, 1987) to use integrations over complex frequency and wavenumber. Although we introduce damping to overcome at small offsets numerical difficulties caused by a logarithmic singularity in the Hankel function, damping is done by a perturbation approach; and all reflectivity calculations are in real arithmetic. Finally, we introduce heterogeneities in the layered medium as scatterers.

We note that the locked-mode approximation is purely artificial: if one changes the depth $H$ of the bottom reflector by a small amount, the eigenvalues and eigcnvectors of normal modes change; nevertheless, no change in the early time signal is visible.

In the following section we briefly introduce some useful results from Sturm-Liouville theory and establish the notation used subsequently. Some of the results in the following two sections are well known in earthquake seismology for elastic waves (Aki and Richards, 1980) and in ocean acoustics for sound waves (Tolstoy and Clay, 1966). However, the method presented in this paper differs fundamentally from these other modal approaches in that it solves the problem for the near-source waves as well, including the vertical waves that give rise to organ pipe modes.

**STURM-LIOUVILLE THEORY**

In most of this paper we shall assume that the background wave field satisfies the equations of motion for acoustic waves in a perfectly elastic fluid. The acoustic approximation simplifies the mathematics, but is not essential; later we show how to deal with elastic waves as well. Approximations that are commonly used in exploration seismology, such as neglecting density gradients or using the parabolic approximation to the differential equations, are not needed in our approach.

The displacement $u(r, t)$ satisfies the equations of motion

$$p(r, t) - (\frac{\partial^2 u(r, t)}{\partial t^2}) = -\nabla p(r, t) + f(r, t),$$

(1)

where $p(r, t)$ is the local density, $\rho(r, t)$ the pressure field, and $f(r, t)$ a force field. After application of Hooke’s law $-p = k\nabla \cdot u$ for incompressibility $k$ and transformation to the frequency domain, we find

$$\nabla \cdot (p^{-1} \nabla p) + \omega^2 k^{-1} p = \nabla \cdot (p^{-1} f),$$

(2)

where $\omega$ is the angular frequency. Application of the Hankel transform

$$p(k, z; \omega) = \int_0^r p(r, z; \omega) J_0(kr)r dt,$$

(3)

where $k$ is the horizontal wavenumber, gives the differential equations for the pressure field in a fluid excited by a point source at $r = r_0$, which is suitably normalized in strength to give $\nabla \cdot (p^{-1} f) = r^{-1} \delta(r) \delta(z - z_0)$ or

$$\left[-k^2 p^{-1} \frac{\partial}{\partial z} + \frac{\partial}{\partial z} \left(\frac{\partial}{\partial z} p^{-1} \frac{\partial}{\partial z} \right) \right] + \omega^2 k^{-1} p = \delta(z - z_0).$$

(4)

The solutions to this equation satisfy $p = 0$ at the stress-free surface $z = 0$ and a radiation condition at depth $z \to \infty$. The pressure field in time and space is given by the inverse Hankel and Fourier transforms,

$$p(r, z; t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega t} \int_0^r p(k, z; \omega) J_0(kr)k dk d\omega.$$  

(5)

Evaluation of this integral is time-consuming; $p(k, z; \omega)$ must be calculated for a large number of combinations of $k$ and $\omega$. Poles, on the real axis for a fully elastic model and close to it if we incorporate anelasticity, create numerical problems in the quadrature. These poles are caused by the surface wave (ground roll in the elastic case) part of the wave field. In the classical reflectivity method, one would circumvent the problems by limiting the integration range over the wavenumber axis to exclude any pole contributions. However, there is
much to say for the opposite viewpoint: lock all energy into the medium, such that the signal at low \( k \) (steep incidence) is represented by poles on or near the real \( k \) axis as well. To this end we replace the radiation condition at \( z = H \) with the condition that \( \frac{\partial p}{\partial z} = 0 \) at \( z = H \) to simulate a rigid reflector at \( z = H \).

We rewrite equation (4) in Sturm-Liouville form as

\[
\frac{\partial}{\partial z} \left( p^{-1} \frac{\partial p}{\partial z} \right) + (\omega^2 \kappa^{-1} - k^2 \rho^{-1}) p = \delta(z - z_0). \tag{6}
\]

The homogeneous variant of this equation is an eigenvalue problem:

\[
Lp = k^2 p^{-1} p, \tag{7}
\]

where \( L \) is an operator such that

\[
Lp \equiv \left[ \frac{\partial}{\partial z} \left( p^{-1} \frac{\partial p}{\partial z} \right) + \frac{\omega^2}{\kappa} p \right] p. \tag{8}
\]

To solve numerically this second-order differential equation, we introduce the vertical displacement \( w(z) \) as variable, which, from the equations of motion, satisfies

\[-p(z)\omega^2 w(z) = -\frac{\partial p}{\partial z}. \tag{9}\]

This enables us to rewrite equation (7) as a system of two coupled differential equations of the first order:

\[
\frac{d}{dz} \begin{bmatrix} w \\ p \end{bmatrix} = \begin{bmatrix} 0 & k^2/\omega^2 \rho - 1/\kappa \\ \omega^2 \rho & 0 \end{bmatrix} \begin{bmatrix} w \\ p \end{bmatrix}, \tag{10}
\]

or

\[
\frac{\partial w}{\partial z} = Ay. \]

We start the integration at \( z = H \) with \( y = (0, 1)^T \) and use a Runge-Kutta or predictor-corrector scheme to integrate up to the surface. Alternatively, we may specify our model in terms of a large number of homogeneous layers and use a propagator scheme (Gilbert and Backus, 1966),

\[
y(0) = \mathbf{P}(0, H)\mathbf{y}(H) = \mathbf{P}(0, z_1)\mathbf{P}(z_1, z_2) \cdots \mathbf{P}(z_{n-1}, H)\mathbf{y}(H), \tag{11}\]

where \( \mathbf{P} \) is constructed from the solution in a homogeneous layer

\[
\mathbf{P}(z_1, z_2) = \begin{bmatrix} \cos q(z_1 - z_2) & -\xi \sin q(z_1 - z_2) \\ \xi^{-1} \sin q(z_1 - z_2) & \cos q(z_1 - z_2) \end{bmatrix}. \tag{12}\]

In equation (12), \( q = (\omega^2 \rho / \kappa - k^2)^{1/2} = (\omega^2 / \alpha^2 - k^2)^{1/2}; \) \( \xi = q/\rho \alpha^2. \) A solution to equation (7) is an eigenfunction \( p_n(z) \) if it satisfies boundary conditions at both \( z = 0 \) and \( z = H. \) At fixed \( \omega, \) this can only happen for selected values of the wavenumber \( k. \) We have the usual orthogonality conditions for modes \( p_n \) and \( p_m \) with eigenvalues \( k_n \) and \( k_m \) at frequencies \( \omega_n \) and \( \omega_m \), respectively,

\[
\int_0^H p_n(z)p_m(z)dz = (\omega_n^2 - \omega_m^2) \int_0^H k(z)^{-1} p_n(z)p_m(z)dz. \tag{13}\]

We normalize \( p_n \) such that

\[
\int_0^H p_n(z)p_n(z)^{-1}dz = \delta_{nm}. \tag{14}\]

Equation (13) implies two useful results. First, let us suppose that \( p_n \) is the complex conjugate of \( p_n \) with eigenvalue \( k_n^* \) and real \( \omega_n = \omega_n^* \). We then find

\[
\text{Re} (k_n) \text{Im} (k_n) \int_0^H p(z)^{-1} p_n(z)p_n(z)^* dz = 0, \tag{15}\]

or \( \text{Re} (k_n) \text{Im} (k_n) = 0; \) either \( k_n \) is on the real axis or it is on the imaginary \( k \) axis (for a perfectly elastic solid). Since only \( k^2 \) enters into the differential equations, we have a real eigenvalue problem with \( -\infty < k^2 < \infty. \) For imaginary \( k, \) we have standing or organ pipe modes, which damp away exponentially from \( r = 0. \)

Standing modes offer no special computational problems. One should define \( k^2, \) not \( k, \) as the eigenvalue and search the negative \( k^2 \) axis for eigenvalues as well. However, although there is a cutoff value for the traveling waves with \( k^2 > 0, \) there is none for negative \( k^2 \) and this raises the question of the convergence of the modal sum. We shall take a pragmatic approach to this problem and avoid calculations at \( r = 0. \) It is evident that even a small offset \( \Delta r \) leads to a strongly damped contribution \( \exp (-|k|\Delta r) \) as \( k^2 \rightarrow -\infty. \)

If we allow for anelastic damping by allowing \( \omega \) to become complex, equation (13) shows that \( \text{Re} (k_n) \text{Im} (k_n) \) and \( \text{Re} (\omega_n) \text{Im} (\omega_n) \) must be of the same sign. To ensure causality, the integration path in the inverse Fourier transform runs slightly above the real axis. Hence, \( \text{Im} (\omega_n) > 0, \) which implies \( \text{Im} (k_n) > 0 \) for positive real frequency and wavenumber.

The second property of the Sturm-Liouville equation permits us to expand the Green’s function into eigenfunctions by means of the bilinear formula, using the orthogonality of the \( p_n. \) In other words, for the source \( \delta(z - z_0), \) the medium response is

\[
G(z | z_0) = \sum_n \frac{p_n(z)p_n(z_0)}{k_n^2 - k^2}. \tag{16}\]

The full solution for an explosion at \( r = 0, z = z_0, \) is

\[
G(r | r_0; \omega) = \sum_n \int_{-\infty}^{\infty} \frac{p_n(z)p_n(z_0)}{k_n(\omega)^2 - k^2} J_0(kr)k \, dk; \tag{17}\]

or, making use of

\[
J_0(kr) = \frac{1}{2} \left[ H_0^1(kr) + H_0^3(kr) \right], \tag{18}\]

and

\[
H_0^3(kr) = -H_0^1(-kr), \tag{19}\]

we find that

\[
G(r | r_0; \omega) = \frac{1}{2} \sum_n \int_{-\infty}^{\infty} \frac{p_n(z)p_n(z_0)}{k_n(\omega)^2 - k^2} H_0^1(kr)k \, dk, \tag{20}\]

with poles located in the first and third quadrants of the complex \( k \) plane. The contour may be closed in the positive imaginary \( k \) plane (Figure 2), since

\[
H_0^1(kr) \approx (2ikr)^{1/2} \exp (i(kr - \pi/4)) \quad \text{for} \ kr \gg 1. \tag{21}\]
The addition of damping not only avoids numerical instabilities for values of \( k^2 \) near the origin, but we may also exploit damping by giving the bottom layer a low \( Q \), which acts to reduce the speed of relaxation times. The perturbed operator is now

\[
\delta L = i \omega \kappa - 2 \gamma,
\]

and for the perturbation of the wavenumber due to anelasticity, we find

\[
\delta k^2 = i \omega \int_0^H \kappa(z)^{-1} Q(z)^{-1} p_n(z)^2 \, dz.
\]
suppress the later arrivals from the reflecting bottom.

Finally, we note that Rayleigh's principle may also be invoked to calculate the modal group velocity $d\omega/dk$, since with 

$$\delta L = 2\omega(x)z^{-1}\delta\omega,$$

we find

$$\frac{d\omega}{dk} = \frac{k}{\omega} \int \kappa(z)^{-1} p_n(z)^2 \, dz.$$ 

(32)

This expression may be used to filter late reflections with very low group velocities out of synthetic sections for larger offsets, an additional safeguard against artifacts that arise from the perfect reflector.

We checked the theory and the software for a simple two-layered model on top of a half-space (Figure 3). Comparison with arrival times of reflections and head waves shows excellent agreement: the onsets of the waves in Figure 3 are 8 ms fast due to the fact that the signal is a pulse which is band-limited to 128 Hz, producing an acausal waveshape. The signal maxima correspond exactly to the calculated arrival times. Even for offsets as small as 20 m, there is no sign of instability. The artificial reflection from the rigid bottom near 500 ms has been damped away very effectively.

**SCATTERED WAVES IN A HETEROGENEOUS MEDIUM**

Equation (23) gives us the response of a horizontally layered medium. This is already much more realistic than the response of a homogeneous background model, since it includes multi-
with \( R' = |(x - x')^2 + (y - y')^2|^{1/2} \) and \( R_s = |(x' - x)^2 + (y' - y)^2|^{1/2} \). There are no problems when \( R' = 0 \) or \( R_s = 0 \) because the logarithmic singularity of the Hankel function is integrable, and, as we stated before, we avoid zero offsets (with the complication that for vertical scatterers \( R' = R_s = 0 \)) to achieve fast convergence of the modal sum. Since \( k \) is complex because of the effects of anelasticity, \( k \) always has an imaginary component and a double singularity because of vanishing \( k_s \) is avoided as well.

**EXTENSION TO ELASTIC WAVES**

Especially for large offsets, the acoustic approximation is unsatisfactory. In this section, we shall briefly indicate how acoustic theory is extended to elastic waves. The displacement \( u \) in an elastic medium satisfies:

\[
-\rho \omega^2 u_j = \sum_{i,k} \frac{\partial}{\partial x_i} \left( C_{ijk} \frac{\partial u_k}{\partial x_i} \right) + f_j,
\]

where \( C_{ijk} \) is the tensor of elastic constants. In an isotropic medium,

\[
C_{ijk} = \lambda \delta_{ij} \delta_{kr} + \mu (\delta_{ik} \delta_{jr} + \delta_{jr} \delta_{ik}),
\]

with Lamé parameters \( \lambda \) and \( \mu \). In a horizontally layered half-space, the Green's function is now a sum over Love (SH) and Rayleigh (PSLJ) modes. Explicit expressions for the Green's function can easily be derived by redoing the analysis of Aki and Richards (1980, chap. 7) without introducing the far-field approximation. The solution to the elastic problem can now be written as

\[
\mathbf{u}(\mathbf{r}, \omega) = \int \int \mathbf{G}^{SH}(\mathbf{r} - \mathbf{r}') \cdot \mathbf{f}(\mathbf{r}') \, d^3r',
\]

and the equivalent of equation (36) is, with some algebra,

\[
\delta \mathbf{u}(\mathbf{r}, \omega) = \int \int \left\{ \delta \rho \omega^2 \mathbf{G}^{SH}(\mathbf{r} - \mathbf{r}') \cdot \mathbf{f}(\mathbf{r}') \right\} d^3r'.
\]

for the far-field case, expressions like equation (40) can be simplified considerably, since the Green's function can be written in dyadic form (Snieder, 1986). In a field experiment, Snieder (1987) has shown how such scattered modes can be used to image near-surface structures far away from the source.

**NUMERICAL DETAILS**

In this section we apply the theory of locked modes to compute Green's functions for some layered models and investigate some of the numerical aspects of the method in more detail.

We first note that we cannot satisfy the boundary conditions if the pressure field behaves exponentially in every layer of the model, i.e., \( \omega^2 a_i - k^2 < 0 \) for every \( i \). This implies an upper limit for the horizontal wavenumber \( k \) for which we may find modes: \( k_{upper} \leq \omega a_{min}^2 \), where \( a_{min} \) is the lowest velocity found in the model for \( 0 < z < H \). This implies that there is—at a fixed frequency \( \omega \)—only a finite number of traveling modes. For organ-pipe modes \( k^2 < 0 \), however, there is no limit on the number of eigenvalues. In practice, we calculate modes for values of \( |k| \) such that \( \exp (-|k| r_{min}) < \epsilon \) where \( r_{min} \) is the minimum offset and typically \( \epsilon = 0.005 \).

To avoid numerical overflow in depth ranges of exponential behavior, it is necessary to transform the propagator scheme (11). To this end, we scale the pressure-displacement vector \( \mathbf{u} \) in each layer where \( q^2 < 0 \)

\[
\mathbf{y} = \tilde{\mathbf{y}} \sinh \left[ q(z_i - z_{i+1}) \right],
\]

so that \( \mathbf{Py} \) transforms to \( \mathbf{Py} \) with

\[
\mathbf{P} = \begin{bmatrix} 1 & \xi \tanh \left[ q(z_i - z_{i+1}) \right] \\ \xi^{-1} \tanh \left[ q(z_i - z_{i+1}) \right] & 1 \end{bmatrix},
\]

where \( \xi = -i \omega a_i \). The propagation is carried out for the transformed vector \( \tilde{\mathbf{y}} \), and the scaling factors are stored for later restoration of the original vector \( \mathbf{y} \).

Since we construct the synthetics in the frequency domain, we have to transform back to the time domain. For this we use the fast Fourier transform (FFT). Because the FFT assumes a periodic signal, this has the disadvantage that signals which arrive outside of the time interval \( 1/\Delta \), with \( \Delta \) the frequency spacing, will fold over into this time window as constructed by the FFT. It is thus important to remove late arrivals before the inverse Fourier transform is applied. To do this in the frequency domain, we have essentially two tools. First, we may give the bottom layer a very low \( Q \). Late reflections from the perfect reflector will then be attenuated more than the others, essentially because the eigenfunctions that contribute to this arrival have much of their energy in this layer, and equation (31) will yield a large imaginary component for their wavenumbers \( k_\gamma \). Second, as has been shown by Nolet and Kennett (1978), the group velocity of each mode indicates at which time and distance this mode contributes constructively to the time signal. We use this property to remove modes from the sum, if their group velocity, calculated with equation (32), indicates a late arrival.
We investigated the convergence of the modal sum in a series of numerical tests. In general, the sum in equation (23) settles abruptly on its final value when $k^2$ changes sign. In the case of near-zero offset, organ pipe modes must be included, however.

The total number of modes needed in the sum depends on the total thickness of the layer. For example, in a 1000 m thick model, 51 modes are needed to obtain convergence at 150 m offset, whereas for an offset of 15 m we summed more than 100 modes to obtain convergence. The number of modes needed for convergence increases approximately linearly with the model depth. This could be demonstrated more rigorously using the asymptotic spacing of the modes, which goes like $1/\xi_{max}$.

On a Gould PN6080 minicomputer, it generally took a few hours to calculate the modes for $\sim 100$ frequency values in 1 km thick models. The time needed to calculate the response to one source, once these eigenvectors are known, is trivial in comparison. This makes the method an ideal algorithm for application in a Born scattering method.

To demonstrate the power of the method, we calculated the response of a pyramid structure in a model with five major interfaces (Figure 5 and Table 1) using the Born scattering equation (37). The pyramid was located in a layer with a velocity of 2150 m/s and had itself a velocity of 1700 m/s (a rough attempt to model a fluid or gas inclusion). Figures 6a–6c show the response of the laterally homogeneous model, the scattered field, and the sum of these two, respectively. The primary reflection from the “oil trap” arrives between 350 and 400 ms. When this signal is added to the unperturbed response, the wave character of the primary reflection from the 320 m interface changes markedly: multiple interference is seen to extend the length of the wavelet, and the shape of the wavelet changes with offset—see, for instance, the change in wave character near 300 m offset.

Table 1. Parameters used to construct the model of Figure 5.

<table>
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<th>Velocity (m/s)</th>
<th>Density (kg/m$^3$)</th>
<th>$Q$</th>
<th>Thickness (m)</th>
<th>Time (ms)</th>
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Fig. 5. Multilayered structure and pyramid-shaped oil trap.

CONCLUSIONS

In this paper we present a new algorithm for the calculation of the 3-D response to heterogeneous structures. At this stage, we have limited ourselves to testing the theory with a few simple models. The simplicity of the models is, however, not a factor that influences the accuracy of the results; and more realistic examples can be calculated if software to describe more complicated geometries is available.

The most important approximation in the method is the Born approximation. Essentially, we ignore multiple scattering within a heterogeneous body or between scatterers themselves. We feel justified in doing this, since reflection coefficients are generally small. We note that much of the methodology used in seismic exploration is in one way or another based on a Born approximation.

The approximation involved in the mode truncation is not important. Except for near-zero offsets, mode sums can be truncated at the onset of the organ-pipe mode regime. For near-zero offset, one has the choice between less efficient or less accurate computation.

The calculation times reported in this paper show that the method makes it possible to generate 3-D synthetics for complicated reflectors in layered media on moderately powerful minicomputers. A large number of applications for such calculations can be envisaged. Probably the most useful application is in testing with a high degree of reality the effects of processing steps prior to common-midpoint (CMP) stacking on synthetic signals. Another application is the generation of model-induced noise, so that one can do away with generating random noise for signals that are essentially deterministic, even if extremely complicated.

To do the 3-D calculations described in this paper, a finite-difference algorithm would have to employ a model with at least $10^7$ grid points: even with the continued growth in computing power, we do not think such calculations will be feasible in the near future. The embedded finite-element technique described by Van den Berg (1984, 1987) may offer some more hope for actual application in three dimensions with fast machines. His method would not suffer from the limitations posed by the Born approximation, but the price to be paid for this in terms of computing time is large. Since Van den Berg also employs a simple medium outside the heterogeneity, both methods are equivalent in their treatment of multiples.

An attractive feature of the method is the presence of all the multiples from the regular layering. Most standard processing aims at removal of these multiples. However, multiples contain valid information on the structure and should, if possible, be included in the interpretation. The inverse problem approach to seismic interpretation (Tarantola, 1984a, b, 1987; Mora, 1987) allows for this. Since this approach asks for the repeated solving of the forward problem for seismic scattering, the application using finite-differences leads to computer times that have to be counted in days, even for the relatively simple 2-D case (Tarantola, Pers. comm.). The locked-mode
Fig. 6. Common shotpoint gathers of (a) the response from the unperturbed layered model shown in Table 1, (b) the perturbation induced by the oil trap, and (c) the total response.
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approach should, in our opinion, be a good candidate for application in such inverse problems, since the 2-D forward problem takes only a minute or less per iteration on a fast machine.

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