Reflection seismograms in a 3-D elastic model: an isochronal approach

Shunhua Cao and Brian L. N. Kennett

Research School of Earth Sciences, The Australian National University, GPO Box 4, Canberra, ACT 2601, Australia

Accepted 1989 March 29. Received 1989 February 27; in original form 1988 August 18

SUMMARY

The reflection and diffraction of elastic waves by surfaces in three dimensions and scattering by thin scatterers can be combined in a common formulation. This approach is derived by using an integral formulation of the elastic wavefield together with ray approximations for wave propagation between source or receiver and reflecting surface or scatterer. For a scatterer, first order scattering is assumed and at a reflecting surface, reflection and transmission effects are estimated using the assumption of a locally plane interface. With these approximations, the reflected seismic field can be represented as the convolution of an approximate source function with a weight function for a particular source and receiver configuration. The weight function at a particular time may be evaluated by a contour integral along the isochronal curve for which the total time from source to receiver via points along the curve on the reflecting surface or on the median surface of the scatterer is equal to the specified time. The kernel of this integral contains information on the scattering or reflection coefficients for the incident wave, the angular effects of the incoming and outgoing waves with respect to the surface normal and the speed of advance of the isochron on the surface. By transferring temporal derivatives to the source function in the convolution, a very stable numerical scheme can be formulated for the generation of synthetic seismograms. This method is illustrated by the calculation of P–P reflections for a seismic line oblique to an anticline and for a variety of parameter contrasts for a simple scatterer model. Reflections from multilayered models can be generated for one surface at a time and the effect of a number of scatterers will be additive within the first order approximation employed. The facility to calculate theoretical seismograms for both surface reflections and scattering is exploited to look at the nature of reflection from the crust–mantle boundary in three dimensions. Very similar seismic responses are obtained for the P-waves returned from an irregularly corrugated Moho surface and a model of the crust–mantle transition as a set of scatterers in the lowermost crust with mantle properties. Thus with current resolution there will be inherent ambiguities in interpretations of the character of the crust mantle transition.

Key words: Elastic wave, Green's function, isochron, ray theory, reflection, synthetic seismograms, three dimensions

1 INTRODUCTION

The modelling of reflection seismograms from subsurface structures is a valuable tool for interpreting seismic reflection profiles for both industrial exploration and seismic probing of the lower crust and the upper mantle. Such modelling is also a key ingredient in current attempts to develop iterative inversion schemes for the inversion of seismic waveforms (see, e.g. Mora 1987). As a result there have been many studies of the generation of synthetic seismograms in reflection with the object of achieving both high accuracy and high efficiency.

Two contrasting techniques are frequently employed in such modelling. The first approach based on direct numerical solution of the elastodynamic wave equations, such as finite-difference or finite-element methods can give accurate results for the full wavefield, but with a high computational cost. Thus it is not always feasible to use direct numerical solution on a large scale, especially in 3-D modelling and iterative waveform inversion. However, such modelling provides a reliable check for other approximate methods. The second approach is based on ray theory and provides a high frequency approximation to the seismic wavefield. The models are restricted to limited numbers of discrete interfaces but can be 3-D. Ray caustics give rise to singularities in the response which can be tamed by suitable modifications to the theoretical development (e.g. Gaussian beams (Červený 1983). A major disadvantage of the ray
An alternative style of modelling which includes both reflection and scattering phenomena can be developed in a form similar to the Kirchhoff–Helmholtz (KH) integral. This has been extensively used for acoustic waves (see e.g. Hilterman 1970, 1975; Haddon & Buchen 1981) and more recently for elastic waves (Frazer & Sen 1985; Sen & Frazer 1985, 1987; Frazer 1987). The wavefield in the standard KH method has to be evaluated by integrating over a surface, which can be very time consuming for a 3-D model especially for elastic waves. Hilterman (1975) transfers the surface integral into an expression involving the differentiation of the solid angle subtended at the shotpoint by the wavefront intersection with the surface in the coincident source-receiver case. For a non-coincident source-receiver case, Haddon & Buchen (1981) separate the source function and the propagation effects (weight function) by changing the coordinate system in the surface integral and evaluate the weight function by integrating the contributions of secondary sources over isochrons (curves of equal total travel time). However, the Jacobian of the coordinate transform may be difficult to calculate for a rough surface. This problem can be avoided by an alternative representation of the KH integral as a line integral over an isochron with an integrand containing the speed of advance of the isochron across the surface.

The propagation of seismic waves in this reflection problem can be partitioned into three stages. Firstly, seismic wave energy is generated at the source and then radiates away from the source region; secondly, waves are scattered from a heterogeneity or reflecting surface and finally, the primary (direct) wavefield and the secondary (reflected and/or scattered) wavefield are received at the seismic sensors. For a band-limited seismic source, the seismograms can be generated by obtaining the impulse response followed by convolution with a band-limited wavelet. With the assumption of a delta function source both in space and in time, the seismic impulse response can be obtained by using ray theory approximations for the incident waves from the source to heterogeneous regions and for the requisite Green’s tensor describing the radiation process from the secondary sources in heterogeneous regions to receivers, coupled with a wave theory implementation of the wave interaction with heterogeneity. To this approximation, the reflected impulse response at a given receiver can be evaluated by an isochronal approach, i.e. for each time step by calculating the contribution of an integral over the current isochronal curve. This impulse response gives a good representation of the wave propagation characteristics. The band-limited seismograms for this particular source and receiver configuration are then recovered by convolving the impulse response with the real wavelet.

This paper describes the derivation of the isochronal representation for both scatterers and reflecting surfaces and demonstrates the numerical implementation of the technique. The isochronal approach is then applied to compare the response of two rather different styles of 3-D reflectivity in the lower most crust. The first model consists of a corrugated crust–mantle interface which on a reflection profile gives rise to half a second or more of apparent reflectivity due to side scattered energy. The second model consists of a distribution of scatterers for which the principal contribution to the reflected seismograms comes from diffractions.

These two models have different petrological interpretations but give very similar reflection responses. In each case the character of the seismograms arise from the interaction of seismic waves in three dimensions. Interpretation of these results in terms of 2-D models will give rather misleading results.

2 SETTING OF PROBLEM

We consider a region \( D \) with density \( \rho(x) \) and elastic modulus tensor \( c(x) \) enclosed by a boundary \( \partial D \) consisting of three portions \( \Sigma \), usually a free surface, \( \Sigma_2 \), a cylinder with a large radius and \( \Sigma_3 \), a potential reflecting surface as shown in Fig. 1. We will consider the scattering effect of heterogeneity within \( D \) by changing the local medium properties and also the possibility of reflection from the lower surface \( \Sigma_3 \). When a force with volumetric density \( f \) and a traction with surface density \( T \) act at \( x_\circ \), there will be a displacement field in \( D \), \( u(x, t; x_\circ) \) which can be expressed by the following elastodynamic wave equations in the linear elastic theory:

\[
\rho(x) \frac{\partial^2 u(x, t; x_\circ)}{\partial t^2} - \nabla \cdot (c(x) \nabla u(x, t; x_\circ)) = f(x, t; x_\circ), \quad x \in D, \tag{1a}
\]

the equation of stress

\[
c(x)_{ijkl} \frac{\partial u_k(x, t; x_\circ)}{\partial x_j} n_j(x) = T_j(x, t; x_\circ), \quad x \in \partial D, \tag{1b}
\]

with associated initial conditions

\[
u_i(x, t; x_\circ) = 0, \quad t < 0, \quad \text{and} \quad \frac{\partial u_i(x, t; x_\circ)}{\partial t} = 0, \quad t < 0. \tag{1c}
\]

![Figure 1. An elastic medium in the domain D enclosed by the boundary \( \partial D \) consisting of the surfaces \( \Sigma_1, \Sigma_2, \) and \( \Sigma_3 \). A seismic source located at \( S \) generates seismic waves propagating through the medium. A seismic sensor located at \( R \) receives direct and reflected seismic signals built up by integration over isochronal curves.](image-url)
Here $\rho(x), c(x)_{ijkl}$ are the density and components of elastic modulus tensor at point $x$, $u_i(x,t;x_s)$ is the $i$th component of the displacement at point $x$ and time $t$ activated by a force $f_i(x,t;x_s)$, the $i$th component of the source located at $x_s$, and $n_i(x)$ the $j$th component of the unit normal at $x$ on $\partial D$.

Once we know the displacement field $u(x,t;x_s)$ to a particular situation, we can examine the parameters whilst maintaining the same source and initial conditions as before. With density $\rho_1(x)$ and elastic modulus tensor $c_1(x)$, the new displacement field, $u_1(x,t;x_s)$, is governed by the equations:

$$\rho_1(x) \partial_t u_1^i(x,t;x) - \partial_i c_1(x)_{ijkl} \partial_j u_1^k(x,t;x) = f_i(x,t;x), \quad x \in \partial D,$$

$$c_1(x)_{ijkl} \partial_i u_1^i(x,t;x_s)n_j(x) = T_j(x,t;x_s), \quad x \in \partial D.$$  \hspace{1cm} (2)

If we denote the density perturbation $(\rho_1(x) - \rho(x))$ by $\delta \rho(x)$, the elastic modulus perturbation $(c_1(x) - c(x))$ by $\delta c(x)$ and the displacement perturbation $(u_1(x,t;x_s) - u(x,t;x_s))$ by $\delta u(x,t;x_s)$ and insert equations (1) in (2), equations (2) may be rewritten as:

$$\rho(x) \partial_t \delta u(x,t;x_s) - \partial_i c(x)_{ijkl} \partial_j \delta u(x,t;x_s) = -\delta \rho(x) \partial_i [u_i(x,t;x_s) + \delta u_i(x,t;x_s)]$$

$$+ \partial_i \delta c(x)_{ijkl} \partial_j [u_k(x,t;x_s) + \delta u_k(x,t;x_s)],$$

$$c(x)_{ijkl} \partial_i \delta u(x,t;x_s)n_j(x) = -\delta c(x)_{ijkl} \partial_i [u_k(x,t;x_s) + \delta u_k(x,t;x_s)]n_j(x),$$

$$\delta u(x,t;x_s) = 0, \quad t < 0, \quad \partial_t \delta u(x,t;x_s) = 0, \quad t < 0.$$  \hspace{1cm} (3)

In reflection seismology, we are mainly interested in the perturbed field $\delta u(x,t;x_s)$ which includes reflections from both scatterers and reflecting surfaces. Therefore, the problem now becomes to get a solution for equations (3).

3 THE MODIFIED WAVEFIELD

By introducing Green's representation theorem (see e.g. Kennett 1983), the solution of equations for the modified wavefield (3) can be written as:

$$\delta u_p(x,t;x_s) = \int_D dV(x') G_{p,i}(x,t;x',0) \ast \delta f_i(x',t;x_s)$$

$$+ \int_{\partial D} dS(x') [G_{p,i}(x,t;x',0) \ast \partial T_i(x',t;x_s)$$

$$- \delta u_i(x',t;x_s) \ast H_{pi}(x,t;x',0)]$$ \hspace{1cm} (4)

where a star denotes convolution in time. $G_{p,i}(x,t;x',0)$ is the elastodynamic Green's tensor whose components are the displacement in the $i$th direction at time $t$ at $x$ generated by an impulsive unit force applied at time $t = 0$ in the $i$th direction at $x'$. $H_{pi}$ is the corresponding traction on $\partial D$.

The effective volume source $\delta f_i$ appearing in (4) due to the change in elastic properties is:

$$\delta f_i(x,t;x_s) = -\delta \rho(x) \partial_i [u_i(x,t;x_s) + \delta u_i(x,t;x_s)]$$

$$+ \partial_i \delta c(x)_{ijkl} \partial_j [u_k(x,t;x_s) + \delta u_k(x,t;x_s)]$$

$$+ \partial_i \delta c(x)_{ijkl} \partial_j \delta u_k(x,t;x_s)$$

and the traction imposed on $\partial D$ is:

$$\delta T_i(x,t;x_s) = -\delta c(x)_{ijkl} \partial_j [u_i(x',t;x_s) + \delta u_i(x',t;x_s)].$$

Thus we can explicitly express the displacement perturbation as:

$$\delta u_p(x,t;x_s) = \int_D dV(x') G_{pi}(x,t;x',0)$$

$$\ast \{ \delta \rho(x') \partial_i [u_i(x',t;x_s) + \delta u_i(x',t;x_s)]$$

$$+ \int_D dV(x') G_{pi}(x,t;x',0)$$

$$\ast \partial_i \delta c(x')_{ijkl} \partial_j [u_k(x',t;x_s) + \delta u_k(x',t;x_s)]$$

$$- \int_{\partial D} dS(x') G_{pi}(x,t;x',0)$$

$$\ast \delta c(x')_{ijkl} \partial_j \delta u_k(x',t;x_s) n_j(x')\}$$

$$- \int_{\partial D} dS(x') \delta u_i(x',t;x_s)$$

$$\ast \delta c(x')_{ijkl} \partial_j G_{pk}(x,t;x',0) n_j(x')$$ \hspace{1cm} (5)

In many cases the change in the displacement field is small compared to the original field, i.e. $\delta u(x,t;x_s) \ll u(x,t;x_s)$, and then (5) can be simplified by making a first order approximation. Alternatively, if the heterogeneity is a small thin scatterer, we may convert the displacement perturbation $\delta u(x,t;x_s)$ into structural parameter perturbations $\Delta \rho(x)$ and $\Delta c(x)$ in the integrals. More details may be found in Hudson (1977).

Regardless of the choice above, equation (5) can be simplified to:

$$\delta u_p(x,t;x_s) = \int_D dV(x') G_{pi}(x,t;x',0)$$

$$\ast \{ \delta \rho(x') \partial_i u_i(x',t;x_s)$$

$$+ \int_D dV(x') G_{pi}(x,t;x',0)$$

$$\ast \partial_i \delta c(x')_{ijkl} \partial_j u_k(x',t;x_s)$$

$$+ \int_{\partial D} dS(x') G_{pi}(x,t;x',0)$$

$$\ast \delta c(x')_{ijkl} \partial_j u_k(x',t;x_s)$$

$$- \int_{\partial D} dS(x') \delta u_i(x',t;x_s)$$

$$\ast \delta c(x')_{ijkl} \partial_j G_{pk}(x,t;x',0) n_j(x')\}$$ \hspace{1cm} (6)

where $\delta \rho$ and $\delta c$ are appropriate density and elastic modulus perturbations. Equation (6) for the modified displacement field can be simplified by exploiting the derivative property of a temporal convolution

$$f(t) \ast \partial_t g(t) = \partial_t f(t) + g(t).$$  \hspace{1cm} (7a)

the generalized divergence theorem

$$\int_D dV \partial_i F = \int_{\partial D} dS n_i F.$$  \hspace{1cm} (7b)
and the chain rule of spatial derivatives

$$\partial_i(G_{pi} \ast \delta \epsilon_{ijkl} \partial_i \mu_k) = G_{pi} \ast \partial_i(\delta \epsilon_{ijkl} \partial_i \mu_k) + \partial_i G_{pi} \ast (\delta \epsilon_{ijkl} \partial_i \mu_k).$$

(7c)

Applying the above properties to (6), we obtain

$$\delta u_s(x, t; x_s) = - \int_D \delta V(x') \partial_i G_{pi}(x, t; x', 0)$$

$$\ast [\delta \rho(x') \partial_i u_s(x', t; x_s)]$$

$$- \int_D \delta V(x') \partial_i G_{pi}(x, t; x', 0)$$

$$\ast [\delta \epsilon_{ijkl}(x') \partial_i u_k(x', t; x_s)]$$

$$- \int_D \delta S(x') \partial_i u_i(x', t; x_s)$$

$$\ast \epsilon_{ijkl}(x') \partial_i G_{pi}(x, t; x', 0) n_j(x').$$

(8)

Clearly, the first two terms represent scattered waves from the localized heterogeneities within the domain $D$, and the last term denotes reflections from elastic parameter contrasts across the boundary $\partial D$.

In equation (8) there will be no contribution to the surface integral over $\partial D$ from the cylinder $\Sigma 2$ since either there are no changes across the surface or, equivalently, we assume that $\Sigma 2$ is removed to an infinite distance from both source and receiver. In reflection modelling, we often neglect the effects of the free surface $\Sigma 3$ and only consider a jump in the structural parameters, $\delta \rho$ and $\delta \epsilon_{ijkl}$, across the surface $\Sigma 3$. In this case the surface integral can be confined to the surface $\Sigma 3$.

4 FURTHER APPROXIMATION FOR THE SCATTERED AND REFLECTED WAVEFIELD

When both sources and receivers lie well away from the volume and surface heterogeneities, we can approximate the form of the incident wave $u(x, t; x_s)$ and Green's tensor components in order to simplify the expression of (8). We suppose that the time dependence of the incidence wave $u(x, t; x_s)$ would be the same throughout the heterogeneous regions and the reflection surface, and that its amplitude varies only slowly, so that we may write for an incident P-wave

$$u(x, t; x_s) = n^0(x) A^P(x, x_s) f(t - T(x, x_s))$$

(9)

where the unit vector $n^0(x)$ lies along the gradient of $T(x, x_s)$, i.e.

$$\nabla T(x, x_s) = n^0(x) / \alpha(x)$$

(10)

where $\alpha(x)$ is the local P-wavespeed. This representation is equivalent to taking the first term in the ray expansion for $u(x, t; x_s)$.

In a similar way, we can apply ray theory to represent the Green's tensor components from the volume or surface heterogeneities to the receiver as a sum of two contributions whose wavefronts move out at the P- and S-wavespeeds. By using the symmetry of the Green's tensor we may express

$$G_{pi}(x, t; x', t')$$

in terms of the properties at the point $x$

$$G_{pi}(x, t; x', t') = g_p(x, x) B_p(x, x) \delta [t - t' - T^p(x, x)]$$

$$+ h_p(x, x) C_p(x', x) \delta [t - t' - T^S(x', x)]$$

(11)

where $g_p(x, x)$ and $h_p(x, x)$ are unit vectors related to the gradients of the phase functions $T^p(x', x)$ and $T^S(x', x)$ by

$$g_p(x', x) = \nabla T^p(x', x), \quad h_p(x', x) = \nabla T^S(x', x) = 0,$$

$$|b(x') V T^S(x', x)| = 1$$

(12)

and the vector $g_p(x', x)$ is directed away from $x$ to $x'$. $B_p(x', x)$ and $C(x', x)$ are the amplitude factors for P- and S-wave, respectively, incorporating the variations due to the far-field radiation patterns and geometric spreading.

4.1 Scattered waves

When we insert the approximate forms for the incident P-wave (9) and the Green's tensor (11) into the representation of the volume integral for the diffraction terms in (8) and ignore higher order terms, we obtain

$$\delta u_s(x, t; x_s) = \int d\tau \partial_\tau f(t - \tau) [F^P(x, \tau) + F^S(x, \tau)]$$

$$= \partial_\tau f(t) [F^P(x, t) + F^S(x, t)]$$

(13)

for the scattered wavefield from the heterogeneity within $D$ as a sum of P- and S-wave contributions. This temporal convolution depends upon the second derivative of the time dependence of the incident wave and so tends to emphasize higher frequencies. This second derivative can be clearly seen in the two differentiations in time associated with density perturbations in (8) and also arises from simplifying the spatial derivatives with the elastic modulus change. The dependence on the shape and properties of the volume heterogeneities arises through weight functions $F^P(x, t)$ and $F^S(x, t)$ which take the forms

$$F^P(x', \tau_p) = - \int_D \delta V(x)[\delta \rho(x)n^0(x)g_p(x, x')]$$

$$+ \delta \epsilon_{ijkl}(x)n^0(x)g_p(x, x')/\alpha^2(x)$$

$$\times A^P(x, x) B(x, x') \delta [\tau_p - T(x, x) - T^p(x', x')]$$

(14)

$$F^S(x', \tau_s) = - \int_D \delta V(x)[\delta \rho(x)n^0(x)h_p(x, x')]$$

$$+ \delta \epsilon_{ijkl}(x)n^0(x)h_p(x, x')/\alpha(x)\beta(x)$$

$$\times A^S(x, x) C(x, x') \delta [\tau_s - T(x, x) - T^S(x', x')]$$

(15)

where $\tau_p = T(x, x) + T^p(x, x')$ is the total travel time from the source to the receiver via a point $x$ within the heterogeneity for the scattered P-wave and $\tau_s = T(x, x) + T^S(x, x')$ the total travel time from the source to the receiver via point $x$ within the heterogeneity for the S-wave, whereas in (15) $g(x, x') = \beta(x') V T^S(x', x)$.

If the heterogeneous region is only a small thin body (scatterer), then the weight functions for the scattered P-
and S-wavefields (14) and (15) can be reduced to surface integrals, i.e.

\[ F'(x', t_p) = -\int_{S} dS(x)h(x)\left[ \delta \rho(x)n^2_0(x)g(x, x') \right. \\
+ \delta(\varepsilon_0)\rho_0 n^2_0(x)\delta(x, x')g(x, x')/\alpha^2(x) \]
\[ \times A^p(x, x')B(x, x')\delta[t_p - T(x, x) - T^p(x, x')] \]
\[ + \delta(\varepsilon_0)\rho_0 n^2_0(x)\delta(x, x')g(x, x')/\alpha(\beta(x)) \]
\[ \times A^p(x, x')C(x, x')\delta[t_x - T(x, x) - T^S(x, x')] \]
\[ \text{(16)} \]

where \( S \) is the median surface of the scatterer, \( h(x) \) the local thickness. The term \( t_p = T(x, x) + T^p(x, x') \) is the total travel time from the source to the receiver via a point \( x \) on the median surface of the scatterer for the scattered P-wave and \( t_x = T(x, x) + T^S(x, x') \) the total travel time from the source to the receiver via point \( x \) on the median surface of the scatterer for the scattered S-wave.

4.2 Reflected waves

For the surface integral in equation (8), we have to be able to estimate the change in the displacement field \( \partial u(x, t; x_s) \) on the surface \( \Sigma S \) associated with the introduction of structural contrast across the boundary. A full solution would require solution of an integral equation but when the curvature of the surface \( \Sigma S \) does not vary rapidly we can adopt a local tangent plane approximation for the generation of the reflected field. In other words, we treat the incident wave as plane and incident on a locally plane boundary in order to estimate the reflected field \( \partial u(x, t; x_s) \) to appear in the surface integral in (8). This contribution can be separated into different wavefield components via the action of the appropriate reflection coefficients on the incident field. The reflected and diffracted wavefield generated by the action of an incident P-wavefield on the surface \( \Sigma S \) can then be represented as

\[ \partial u(x, t; x_s) = \int d\tau \partial f(t - \tau)\left[ Q^p(x, \tau) + Q^S(x, \tau) \right] \]
\[ = \partial f(t) \ast \left[ Q^p(x, t) + Q^S(x, t) \right] \]
\[ \text{(18)} \]

where the first order derivative in time is associated with simplifying the single spatial derivative in the boundary integral in (8). The reflected P-wavefield from the surface \( \Sigma S \) is determined by a weight function

\[ Q^p(x, t_p) = -\int_{\Sigma S} dS(x)n_0(x)n_0^2(x)g(x, x')g(x, x')\delta(\varepsilon_0)\rho_0 \]
\[ \times A^p(x, x')R^{pp}(x)B(x, x') \]
\[ \times \delta(t_p - T(x, x) - T^p(x, x'))/\alpha(x) \]
\[ \text{(19)} \]

where \( n^p(x) \) is the direction vector of the reflected P-wave determined by Snell's law with the incident direction of P-wave, \( n'(x) \), and the local surface normal, \( n(x) \). \( R^{pp} \) is the reflection coefficient for P-waves. The term \( t_p = T(x, x) + T^p(x, x') \) is the total travel time from the source to the receiver via a point \( x \) on the reflecting surface for the reflected P-wave. The corresponding reflected S-wavefield for the incident P-wave is determined by a weight function

\[ Q^S(x, t_x) = -\int_{\Sigma S} dS(x)n_0(x)n_0^2(x)h(x, x')e(x)\delta(\varepsilon_0)\rho_0 \]
\[ \times A^p(x, x')R^{sp}(x)C(x, x') \]
\[ \times \delta(t_x - T(x, x) - T^S(x, x'))/\beta(x) \]
\[ \text{(20)} \]

where \( n^S(x) \) is the direction vector of the reflected S-wave determined by Snell's law with the incident direction of P-wave, \( n'(x) \), and the local surface normal, \( n(x) \). \( R^{sp} \) is the reflection coefficient for S-waves generated by a plane incident P-waves. The term \( t_x = T(x, x) + T^S(x, x') \) is the total travel time from the source to the receiver via a point \( x \) on the reflecting surface for the reflected S-wave. Again, \( g(x, x') = \beta(x)VT^S(x', x) \) is the same as in (15).

Although we have used a local approximation for \( \partial u(x, t; x_s) \) in the surface integral, we must remember that the total representation for the reflected field involves contributions from the whole surface and therefore will be more accurate than the internal approximation.

4.3 Nature of the approximations

The expressions we have derived for the scattered field (13, 16, 17) and the reflected field (18, 19, 20) have a great deal in common. In each case the wavefield is to be constructed by the convolution of a time derivative of the source–time function with a weight function which itself is a surface integral containing a delta function in time in its kernel. The delta function extracts from the surface integral only that portion which occurs at a time \( t \). This reduces each surface integral to a line contour integral along a locus for which the total time of propagation from the source to a point and from that point to the receiver is equal to the time \( t \), which we term an isochron.

As a result we can give a general representation of the reflected wavefields as a convolution of a wavelet related function \( g(t) \) with a line contour integral

\[ \partial u = g(t) \ast \int_{L(t)} dl \ G \cdot S \cdot D \]
\[ \text{(21)} \]

where \( D \) represents the incident wavefield, \( S \) the scattering (reflection) terms and \( G \) is the Green's tensor. The integration path \( L(t) \) is the isochron for the total travel time \( t \), which may well consist of a number of distinct loops.

In this form, (21) may be viewed as an approximate construction for a reflection operator \( R \) (Kennett 1984) which when applied to the incident wavefield \( D \) generates the reflected field

\[ \partial u = R[D] \text{ and } R = g(t) \ast \int_{L(t)} dl \ G \cdot S \]
\[ \text{(22)} \]

a combined temporal and spatial operator.

In order for the plane wave approximations we have employed to be reasonable the scatterers or reflecting surface should not be too close to the source, a distance of 10 times the dominant wavelength will usually suffice. For a velocity of 3 km s\(^{-1}\) and a 30 Hz dominant frequency the
scatterers should be at least 1 km deep. For crystalline rocks with higher velocities (up to 6 km s\(^{-1}\)) the development should be satisfactory for sources/scatterer separations greater than around 2 km. Such requirements are not very restrictive in practice. In addition, in order for the assumption of first order scattering to be appropriate for the localized heterogeneities, the product of the parameter contrast and the volume of the scatterer should not exceed the cube of the wavelength (Hudson & Heritage 1981). For a change of 5 per cent in properties this requires scatterers with volume less than around 0.02 km\(^3\), as for example a blob about 1 km\(^2\) in horizontal extent and 20 m thick.

We remark that the validity of equations (13) and (18) rests on the assumption that ray theory provides an adequate approximation for wave propagation from the source to heterogeneities (reflection interface or scatterer) and from the heterogeneities to receivers, respectively. It is not necessary that ray theory applies for wave propagation from the source to receivers via the heterogeneities. In many cases the domain containing the scatterers (reflectors) can be chosen so that the above assumptions are valid and also that the amplitude factors, \(A, B\) vary slowly through the scattering region. In this case \(A\) and \(B\) may, to a good approximation, be treated as constants so far as differentiation with respect to \(n^0\) or \(g\) is concerned, an approximation corresponding to the usual Fresnel approximation for monochromatic waves. The radii of curvature at any point on the surface should be greater than the wavelength because only the primary field is taken into account.

5 RESULTS FOR A HOMOGENEOUS REFERENCE MEDIUM

In order to illustrate the results for reflected and scattered waves more clearly, we will simplify to the case of a homogeneous and isotropic reference medium in \(D\), and for simplicity we concentrate on the \(P\)-wavefield. In other words, we assume an incident \(P\)-wave and only calculate the scattered and reflected \(P\)-waves. Furthermore, we will consider the scattered and reflected waves separately in order to make the situation much easier to handle. For more complicated cases, we may just add the reflected and scattered waves together with the inclusion of wavetype conversions.

5.1 Reflections from a thin scatterer

For modelling reflections from a thin scatterer, we assume there are no structural parameter differences across the boundary \(\Sigma 3\) and restrict ourselves to considering the scattering of a single thin scatterer. Within the scatterer, the structural parameters are uniformly different from the other region. From (16), (17) and (22), the scattered waves from such a thin scatterer can be written in terms of an isotropical integral as

\[
\delta u_d(x, t; x_0) = g(t) \ast F^P(x, t) = -\partial_t f(t) \ast F^P(x, t)
\]

\[
= -\partial_t f(t) \ast \int_{\mathcal{L}(t)} dl(x') h(x') [\partial \rho n'(x') g(x', x) \alpha^2 + \delta c_{\mu} n^0_0(x') n^0_0(x') g_0(x', x) g(x', x)]
\]

\[
\times A^P(x, x') B(x', x) [\alpha |n^0_0 + g_0|]^{-1}
\]

where \(dl(x')\) is a length element on the median surface of the scatterer, \(h(x')\) is the local thickness. The total travel time from the source to the receiver via points \(x\) on the median surface of the scatterer, \(t = T(x, x') + T^S(x, x')\), specifies the isochronal line \(L(t)\). \(r(x', n^0_0, g)\) is the local scattering coefficient, which controls the radiation pattern of diffracted energy. \(n^0_0\) and \(g_0\) are the projections of \(n^0\) and \(g\) on the median surface. When \(t\) is less than or equal to \(T_{\text{min}}\) or \(t\) is greater than \(T_{\text{max}}\), the line integral on the right side is defined as zero \((T_{\text{min}}\) and \(T_{\text{max}}\) are the minimum and maximum travel times, respectively, from \(x_0\) to \(x\) via a point on the median surface of the scatterer).

Under the assumption of an isotropic medium, the local scattering coefficient

\[
r(x, n^0_0, g) = \delta \rho n^0(x) g(x, x') \alpha^2 + \delta \lambda + \delta \mu [n^0_0(x) g(x, x')]^2
\]

\[
= \delta \rho P \cos \chi \alpha^2 + \delta \kappa \cos \chi \mu P_2(\cos \chi)
\]

where \(\cos \chi = n^0_0(x, x')\) governs the angular variation of the scattering through the Legendre polynomials \(P_1(\cos \chi)\) and \(P_2(\cos \chi)\). The angle \((\pi - \chi)\) is the angle of scattering. This local scattering gives rise to different energy radiation patterns with different constituents when an incident \(P\)-wave arrives. The density perturbation acts as a single force source in the incident direction; in terms of the Lamé parameters, the perturbation \(\delta \lambda\) contributes an isotropic explosion-type source, while the perturbation \(\delta \mu\) contributes a dipole source in the incident direction with \(\cos^2 \chi\) dependence. In terms of bulk and shear moduli, the variation in bulk modulus \(\delta \kappa\) contributes an isotropic explosion-type source and the variation in shear modulus gives rise to a radiation pattern governed by \(P_2(\cos \chi)\), the Legendre polynomial of order 2. These results are also obtained by Hudson (1977), and Wu and Aki (1985).

For a homogeneous and isotropic matrix, the Green's tensor in the far field is

\[
G_d(x, t; x', 0) = g(x, x) g(x', x) \delta [t - R(x', x)/\alpha] / [4\pi \rho R(x', x) \alpha^2] + \delta \nu - g(x, x) g(x', x)]
\]

\[
\times \delta [t - R(x', x)/\beta] / [4\pi \rho R(x', x) \beta^2]
\]

where

\[
R(x', x) = |x' - x|\quad \text{and} \quad g(x, x) = (x - x') / R(x', x).
\]

Thus the amplitude and time coefficients needed for the representation (13) of the Green's tensor are

\[
B_\nu = g_\nu / (4\pi \rho R \alpha^2), \quad T^P = R / \alpha.
\]

For an incident \(P\)-wave, if the source amplitude at a small distance \(r_0\) from the centre of the source is \(A_0^P\), the displacement at a distance \(R_s\) from the source will be

\[
A^P = A_0^P r_0 / R_s, \quad \text{with} \quad R_s = |x - x'|.
\]
Thus, the scattered waves from the thin scatterer under the assumptions can be written as:

$$\delta u_n(x, t; x_s) = \delta n_f(t) \int_{L(t)} dl(x') A_{\rho}^n h(x') r(x', u^n, g) g \times [4\pi \rho \alpha^3 R_{RR}(x', x) |n^0_0 + g_0|]^{-1}$$

(29)

and as before $n^0_0$ and $g_0$ are the projections of $n^0$ and $g$ on the median surface of the scatterer.

The amplitudes of the diffracted waves depend on the interaction of the shape of the scatterer and the radiation patterns associated with the contrasts with the surrounding medium. As time increases, the isochronal contour $L(t)$ on the median surface enlarges. The speed of its advance is given by

$$v = \frac{1}{|V(T + T^p)|} = \frac{\alpha}{|n^0 + g_0|}.$$  

(30)

Thus $v$ is infinite at the geometrical reflection point where $n^0_0$ and $g_0$ are oppositely directed, but will take lower values for the scattering regions at later time.

5.2 Reflections from a Surface

We now turn our attention to the reflected and diffracted waves returned from a reflecting surface at depth. We assume that the material inside the domain $D$ is homogeneous and isotropic but that there is a jump in properties at the interface $\Sigma$. The surface integral in the representation of the reflected field (18, 19) reduces to an isochronal contour integral on the surface $\Sigma$ and including the forms of the Green's tensor and incident field (25–28), the reflected $P$-waves from $\Sigma$ are given by

$$\delta u_n(x, t; x_s) = g(t) * Q^n(t) = \delta f(t) * Q^n(t)$$

$$= -\delta f(t) \int_{L(t)} dl(x') \langle \lambda n(x') n'(x') \rangle$$

$$+ 2\mu [n(x') g(x', x)] [n(x') g(x', x)]$$

$$\times A_{\rho}^n g(x, x) \{4\pi \rho R_{RR} \alpha^2 |n^0_0 + g_0|\}^{-1} R^p$$

(31)

where $n'$ is the reflection direction determined by Snell's law with the incident direction $n^0$, and the surface normal $-n$. $n^0_0$ and $g_0$ are the projections of $n^0$ and $g$ on the surface $\Sigma$ and $T = T(x, x') + T^p(x, x')$ is the total travel time from the source to the receiver via a point $x'$ on the surface.

The contour integral will be zero until $T_{min}$, the least travel time from source to receiver via the surface $\Sigma$. This will be the raypath defined by Fermat's principle and the apparent singularity because of the vanishing of $n^0_0 + g_0$, is avoided because the corresponding isochron reduces to a point and by definition the integral in (31) will be zero.

At later times, the isochron moves across the surface with a speed $v$ determined as in (30) and encloses progressively more of the surface $\Sigma$. When multiple arrivals from the surface are possible the integral (31) has to be taken over a number of disconnected loops.

The reflected field for a delta function source will be the derivative of the isochronal integral as a function of time. In the ray theory approximation there will only be a contribution from each geometric raypath, but the integral form (31) includes the non-geometrical phenomena such as diffraction. Within the integral the incident angle on the surface has a strong effect on the reflection coefficient $R^p$; if the local angle goes beyond critical $R^p$ becomes complex and it is then necessary to treat (31) as a representation for an analytical function of time with the real part convolved with the Hilbert transform of the wavelet.

6 Numerical Implementation for a Reflecting Surface

From equation (31), we have a representation for the reflected displacement field as the convolution of the first derivative of a source wavelet with a weight function $Q$ depending on the location of source and receiver. The value of the weight function $Q$ at any time can be obtained by a line integral along the current isochron.

The isochronal curve needed corresponds to constant total travel time between source and receiver via the reflecting surface. Two approaches are available for the construction of the isochrons. Firstly, for simple media, rays are traced from both source and receiver to regular gridpoints on the surface by an iterative ray-tracing method. At each gridpoint the travel times $T_S$ (from the source) and $T_R$ (from the receiver) are added to give the total travel time. The isochrons can then be constructed by linear interpolation between neighbouring points. Secondly, for more complex media, two point ray tracing can be avoided by shooting a spray of rays from either source or receiver so that each surface element contains at least one endpoint of a ray. The $T_S$ and $T_R$ fields can then be interpolated from the irregular set of endpoints on to a regular grid and the isochrons can be constructed as before.

One merit of the isochronal approach is that only one-way ray tracing to the reflecting surface is required. Once the one-way time field for a particular surface point has been calculated on the regular grid it can be stored and combined later with the one-way times from another point to set up the isochrons. The situation is particularly simple for the case of coincident source and receiver when the total time is just twice that for one-way travel.

Once the isochrons have been constructed, the weight function is to be evaluated by integrating along the contour. For a finely sampled surface we can treat the integrand on the segment of the isochron in each surface element as uniform. For each element we need to construct the local reflection coefficient, the angular effect of the incoming and outgoing waves with respect to the local normal to the surface and the local speed of advance of the isochron across the surface.

As an illustration of the numerical procedure, we calculate a synthetic seismogram section for a model of an anticline illustrated in Fig. 2. A horizontal surface lies at a depth of 15 km and the anticline has a maximum deviation of 150 m. We first consider the isochronal patterns for a specific source–receiver geometry (S, R in Fig. 2) Fig. 3 shows the projection of the isochronal distribution on to a horizontal plane with a time separation of 10 ms between contours. For this source/receiver geometry, the major feature is the reflection from the horizontal plane with circular isochrons about the geometric reflection point marked by a square. However, there is a slightly faster path.
Figure 2. An anticline model in three dimensions. A reflecting surface comprises three parts: two half planes separated by an anticline ridge. Two half planes are horizontally placed at a depth of 15.0 km with 0.15 km maximum perturbation for the ridge. A source is fired at (8.0, 14.0) and seismic waves are recorded at (16.0, 6.0).

by way of the anticline marked by the solid triangle. The isochrons in this region are elongated along the axis of the anticline and give rise to diffracted arrivals. About 40 ms after the main reflection, diffractions arrive from the transition zone between the ridge and the plane. In Fig. 3 this zone is indicated by the diamond and open triangle and corresponds to a saddle in the surface of total travel time.

The relative significance of the various arrivals can be roughly judged from the separation of the isochrons since the weight function $Q$ depends on the speed of advance of the isochrons. The actual contributions to $Q$ depend on the surface reflectivity and the geometry.

Figure 4 displays the synthetic seismograms generated from the isochronal pattern in Fig. 3. From left to right we display the source wavelet (asymmetric Gaussian), its first derivative, the weight function $Q_3$ for a $P$-wave source at $S$ received by a vertical component geophone at $R$ in Fig. 2, and the final seismograms generated by the convolution of the weight function with the first derivative of the source wavelet. The features of the seismograms are annotated with the markers used to denote the features on the isochronal plot (Fig. 3). The primary reflection from the plane associated with the main step in the weight function (solid square) has the form of the original wavelet. The
Reflection seismograms in a 3-D elastic model

Figure 3. The projection of the isochronal distribution onto a horizontal surface for the case described in Fig. 2. The projections of the source and receiver are indicated by a star (S) and a triangle (R), respectively. The contour interval is 10 ms.

diffracted phases from the ridge indicated by the triangles and diamond arise from cuspatc features in time of the weight function and so have a waveform close to the Hilbert transform of the source wavelet (Burridge 1963). The need for 3-D modelling is clearly illustrated by this simple case. The source-receiver line is inclined at 45 degrees to the axis of the anticline. However, the isochronal features from the ridge (Fig. 3) follow closely the geometry of the basic model. Therefore, 2-D modelling which would assume that the reflection features follow the shot-receiver line would not give a correct result.

The weight function only varies rapidly around stationary
points on the travel-time surface, and so fine steps in time are only needed in those stationary areas. Coarse steps can be taken in other areas, which saves a lot of computational effort. The accuracy of the synthetics depends mainly on the time sampling rate and the surface parameterization. To achieve high efficiency and high accuracy, the time sampling must be linked to the spatial sampling rate.

The Fresnel zone, following Sheriff (1980), can be defined as an annular area bounded by two isochrons with a time difference of half the dominant period of the waveform. The first Fresnel zone is the area bounded by the isochron with a time difference of half period from the centre of isochrons. The energy from a Fresnel zone will add constructively to produce reflections. For the above case, the first Fresnel zone is a square-like area on the ridge with a length of about 2.0 km and a width of about 0.4 km. As can be seen, reflected energy does not necessarily come from only the first Fresnel zone. Significant contributions can come from the second and even higher order Fresnel zone (e.g. reflections from the northern part of the plane marked by a square in Fig. 3).

Figure 5 illustrates a seismic reflection profile for vertical component geophones with coincident shots and receivers along a line extending between R and S. The first trace is for a shot point at R and trace 41 corresponds to a shot point at S. The main phases are indicated by symbols whose meanings match the usage of Fig. 3. The most prominent feature is the reflection from the horizontal surface to the north of the ridge (marked by a solid square). The reflection is truncated at the ridge and grades horizontally into a diffraction marked by the solid diamond. A stronger diffraction of opposite polarity follows the reflections arising once again from the abrupt change of the reflecting surface at the ridge. The diffractions from the northern flank of the ridge are mirrored by equivalent arrivals from the southern flank marked by the open diamond. Over much of the section the first arrival comes from the crest of the anticline (which lies beneath shot point 6). This arrival approximates a diffraction curve for a line diffractor at the crest.

In illustrating the method we have assumed a single layer model. In reality, especially in exploration seismology, we need to be able to consider multipe reflecting surfaces. For a multilayered model we are able to use the method directly for reflections from surfaces, for which the incident wave and Green’s tensor can be adequately described by a ray approximation. Each reflected phase must be dealt with separately but both surface and internal multiples can be considered within this scheme. Even if the ray approximation for the incident wave or Green’s tensor components breaks down it is usually possible to choose an intermediate control surface (Haddon & Buchen 1981; Frazer & Sen 1985) to alleviate the problem. However, before introducing any intermediate surfaces we must recall that the computation cost increases rapidly with the number of control surfaces. Thus, if a problem can be treated with a single isochronal surface, this will be the most efficient approach.

7 NUMERICAL IMPLEMENTATIONS FOR SCATTERERS

From (29) and (31), we know the expressions for reflections from thin scatterers and subsurfaces have similar structures. Indeed, the numerical implementation procedure for each of them is essentially the same, i.e. line contour integrals along isochrons are first evaluated to give a weight function, and then the first derivative (for subsurfaces) or the second derivative (for scatterers) of a chosen wavelet is convolved with the weight function. The difference between the evaluations of the weight function \( \mathbf{F} \) for a scatterer and \( \mathbf{Q} \) for a reflecting surface lies in the form of the integrand with heterogeneity terms replacing reflection coefficients.

Figure 6 shows an ellipsoidal thin scatterer model in three dimensions. The three main axes of the scatterer are 0.050 (vertical), 0.200 (N-S) and 1.000 km (E-W) with its centre located at \((1.0,1.0)\) underneath the ground with a depth of 15.0 km. A shot is fired at \((0.0,0.0)\) on the ground and two lines of receivers are laid along the N-S (line A) and E-W (line B) axes on the ground, respectively. The receiver line A starts at \((0.0,-1.0)\) and spacing is 0.2 km; the line B starts at \((-1.0,0.0)\) and spacing is also 0.2 km. The total number of receivers on each line is 31 and the reflection seismograms are shown in Fig. 7. The upper three panels
show reflections on the receiver line A while the bottom three panels show reflections on the receiver line B. From top to bottom, the first panel shows reflections from a scatterer of contrast in just the Lamé modulus $\lambda$, the second shows reflections from just the shear modulus $\mu$, and the third shows reflections from a pure density contrast. The parameter contrasts in each case represent a 10 per cent deviation in density and 40 per cent deviation in elastic modulus tensor from the surrounding matrix. The radiation patterns from the local scattering coefficients (described in 5.1) cannot easily be observed but they can still be recognized by a careful examination of lines A and B. For example, it is found that amplitudes in a pure $\mu$ scatterer decrease more rapidly than those in a pure $\lambda$ scatterer. The angular variations for scattering in all cases are fairly small. As can be seen, the radiation patterns on the line A for all scatterers are dominantly controlled by the shape of scatterers whereas the radiation patterns on the line B are much less influenced by the shape of scatterers. In the N–S direction, the scatterer is fairly long and so it behaves like a reflector. However, in the E–W direction, the size is small, especially in the area underneath the line and so it tends to behave like a point source.

From geometrical ray theory, there would be no reflections on the above two survey lines. 2-D modelling would not give any reflections from the scatterers.
Therefore, a 3-D modelling approach must be taken to generate the scattered wavefield. For the generation of the scattered field we have used the assumption of single scattering via a Born approximation. With a model consisting of a number of scattering centres, we would just add the contributions from each scatter to the total synthetic seismograms. In this approximation, we neglect interaction between scatterers and also between scatterers and reflecting surfaces, which would be most important when large parameter contrasts are involved.

8 REFLECTIONS FROM THE CRUST–MANTLE TRANSITION

Deep reflection surveys have been carried out in many parts of the world (see e.g. Mathews & Smith 1987). On those profiles extending to long enough reflection time to include reflections from the crust–mantle transition, there is often a band of reflections from 0.5–1 s long at the expected time for return from the crust–mantle boundary. Good examples of such features are shown in the summary by McGeary (1987) of the BIRPS surveys around the British Isles. The individual reflections within the band are coherent over relatively short distances (a few km) even on stacked sections. Such short reflectors are difficult to reconcile with simple models of reflection from an interface. Strong variations in seismic parameters in the near surface zone may lead to a reduction in the coherence of individual reflections but cannot account for the duration of the ‘Moho’ reflections.

In an attempt to explain the character of these reflections from the crust–mantle boundary, Blundell & Raynaud (1986) proposed a model consisting of a single reflecting surface which is periodic in the two horizontal coordinates (an ‘eggbox’ model—Fig. 8). Theoretical seismograms were initially calculated using ray theory but subsequently Raynaud (1988) used a variant of Kirchhoff method to include diffractions. The resulting seismograms show a substantial band of apparent reflections with a duration around 1 s (cf. Fig. 9).

For comparison with later results, we illustrate the Blundell & Raynaud model in Fig. 8(a). The reflecting surface is 2-D periodic with wavelengths of 12 km in the x direction and 10 km in the y direction varying about a mean depth of 26 km with a vertical amplitude of 1 km. The synthetic seismograms calculated with the isochronal method are in good agreement with those presented by Raynaud (1988). A merit of this procedure is that the isochrons themselves provide considerable insight into the nature of the reflection process. In Fig. 8(b) we show the projection of the isochronal pattern on to a horizontal surface with a contour interval of 25 ms for a coincident source–receiver at the position (20, 20) as indicated by a circle enclosing a star. The earliest reflection point is

---

**Figure 6.** An ellipsoidal thin scatterer model. Two profiles A and B are placed on the surface to record seismic waves excited by a shot at O. The projection of the profiles on the median plane of the scatterer is indicated by dashed lines.

**Figure 7.** Calculated seismograms reflected from the scatterer. Two-way times are shown on left of each seismic section. The top three sections are synthetic seismograms on receiver line A and the bottom three sections are synthetic seismograms on receiver line B. Each section shows reflections from different type of scatterers indicated on the top-right corner.
Figure 8. The Blundell & Raynaud (1986) 'eggbox' model. (a) Schematic diagram of two-dimensional sinusoidal reflecting surface with a mean depth of 26 km.

marked by a circle and is surrounded by secondary reflection points indicated by triangles which show the periodicity of the surface. Note that the first reflection point is well displaced from the source-receiver location. The seismogram at (20, 20) will include return from a substantial area of the reflecting surface. The enclosed contours will contribute direct reflections and the saddle points will give rise to diffractions. The contributions from the enclosed contours with an open triangle inside have 180° phase delays with comparison to the contributions from those with a solid triangle inside.

The character of the reflection seismograms for the eggbox model can be well illustrated by a record section of coincident shot-receiver pairs (simulating a CMP profile). In Fig. 9, we show a record section, calculated using the isochronal technique, extending from (8, 32) to (32, 8) at an angle of 45° to the x-axis with 1 km station spacing. The very complex seismograms extend over a band a second long, with quite rapid changes in the character of reflectors arising from the interference of many reflections from out of the plane of the section. Although the nature of the synthetics has similarities to the observed bands of reflections from the crust–mantle transition, it is very unlikely that such a regular periodicity would be sustained over any significant horizontal distance. However, the Blundell & Raynaud model does draw attention to the likely significance of reflections from out of the plane of the section.

In an attempt to construct a more realistic model for a single reflector at depth, we have constructed a smooth but irregular surface (Fig. 10a). The surface was generated by first constructing a 4 km mesh at a mean depth of 26 km and then applying random perturbations of up to 1 km at each mesh point. The smooth final configuration was then produced by interpolating between mesh points with bicubic splines. This process gives rise to a surface with a range of scales of variation. As a result, the isochronal pattern for
coincident source and receiver at (20, 20) (Fig. 10b) shows features with a range of sizes rather than the regular character of Fig. 8(b) for the periodic surface, with the same observation point.

In Fig. 11, we show a record section of synthetic seismograms along a profile with the same geometry and configuration as Fig. 9. Now that the variation in the reflector relief is more subdued, the structure of the seismogram section is simpler and individual events can be more clearly identified. The complexity of these seismograms is increased quite dramatically if the underlying mesh is reduced to 2 km spacing since this generates steeper facets and enhances opportunities for out of plane scattering. This model of a smoothed random surface is rather versatile since the complexity varies roughly inversely with the size of the underlying mesh. Cao, Kennett & Goleby (1988) have shown that a model of this type gives a good representation of the observed Moho reflections under the Northern Arunta Block in central Australia. In this case, the irregularity of the crust–mantle boundary is associated with listric termination of major crustal faults.

An alternative class of models for the lowermost crust is one in which the transition from crust to mantle arises from the interlamination of crustal and mantle materials. Sandmeier, Wilde & Wenzel have proposed both 1- and 2-D models of this type with vertical scales of around 100 m and horizontal scales of hundreds of metres. With the scattering formulation of the isochronal approach, we are able to set up a 3-D model of a scattering zone with many scatterers distributed in a small interval in depth. To allow comparison with the irregular surface model we have constructed a model with an array of thin ellipsoidal scatterers whose depths are randomly perturbed from a mean level of 26 km by up to 1 km. Thirty scatterers are distributed horizontally within the model region on a distorted grid with up to 0.5 km random perturbation in the

Figure 8. (b) Horizontal projection of isochronal patterns on the reflecting surface for a coincident source–receiver at (20, 20). The shot–receiver location is indicated by a circle enclosing a star. The two-way times are schematically illustrated by the darkness of the isochrons. The isochrons get lighter as the two-way time increases.
Reflection seismograms in a 3-D elastic model

Figure 9. A synthetic CMP section with the isochronal method ranging from (8, 32) to (32, 8) for the eggbox model.

Figure 10. An irregular single smooth reflecting surface model. (a) Schematic diagram of the reflecting surface at a mean depth of 26 km.
Figure 10. (b) Horizontal projection of isochrons—the symbols have the same meaning as in Fig. 8.

Figure 11. A synthetic CMP section for the irregular reflecting surface model with the same source-receiver geometry as in Fig. 9.
Figure 12. Schematic representation of scatterer model. The actual scatterers are ellipsoids with axes 2, 1 and 0.5 km distributed through a zone 2 km thick with a mean depth of 26 km.

x and y directions. The scatterers are taken to be roughly aligned with $\pm 10^\circ$ deviations in the orientations of the principal axes of the ellipsoids (Fig. 12).

The record section of synthetic seismograms for a profile along the same line as in Figs 9 and 11 is shown in Fig. 13. With the three principal axes of scatterers being 2.0, 1.0 and 0.3 km, there is a very strong resemblance between the record section from the scatterers which will be principally comprised diffractions and the reflections from an irregular surface of comparable scale lengths. Multiple scattering effects which we have not included would tend to enhance the complexity of the later parts of the records.

These two classes of 3-D models have very different character and petrological implications but cannot easily be resolved with present deep reflection techniques. In each case the major contributions arise from out of the plane of the record section and so even 2-D interpretation would give a highly misleading result.

REFERENCES


Figure 13. A synthetic CMP section for the multi-scatterer model with the same source–receiver geometry as in Figs 9 and 11.


