Approximations for surface-wave propagation in laterally varying media

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SUMMARY
A number of techniques which exploit the waveforms of seismic surface waves depend on simple approximations for the character of the propagation process from source to receiver based on the representation for a stratified medium. Commonly the propagation path is assumed to lie along a great circle and to be representable by a path-averaged structure. The influence of structure near the source and near the receiver is included by using local modal formulations. However, the terms that depend on source depth and receiver depth in the stratified medium results are not purely local in character, and so care has to be taken to ensure a simple mapping between the modal shapes for the different structures.

For frequencies less than 0.03 Hz, different crustal structures can be used at the source, near the receiver, and along the propagation path, provided that the change in crustal thickness is not more than 10 km between contiguous structures. Furthermore, for frequencies up to 0.035 Hz, it should be possible to use a single modal set in non-linear waveform inversions for perturbations of up to 5 per cent in lithospheric velocities along the propagation path.

For propagation paths of length from 1000 to 4000 km, typical of a continental scale, the path-averaged structure approximation should be suitable for waveform fitting for frequencies in the range 0.01–0.03 Hz. The lower limit depends on the use of asymptotic approximations and the upper on the influence of heterogeneity on the modal content of the seismograms.

Where surface waves cross a major structural boundary such as the continent–ocean transition, some aspects of the wavefield can still be represented using the path-averaged approximation.

Key words: lateral heterogeneity, surface waves, waveform inversion.

1 INTRODUCTION
Surface waves form a very prominent part of the seismic wavetrain and their character is dominated by the influence of the shear-wave distribution in the outer layers of the Earth. Fortunately, the longer period features of the surface wavefield for a stratified earth model can be described by a limited number of modal contributions. As a result, the computational effort required to generate a theoretical seismogram for this portion of the wavefield is modest, and so it is possible to devise waveform-fitting schemes based on non-linear inversion. The result of such an inversion is a 1-D model that provides the best fit to the observed seismogram. However, it should be noted that the propagation path of the observed arrivals will have been influenced by 3-D structure encountered during passage from source to receiver.

The partitioned waveform inversion scheme of Nolet (1990) uses a set of 1-D model fits as representations of the average structure along the paths to each station, and then derives a laterally heterogeneous structure from the set of linear constraints provided by the path averages. This method was originally introduced to examine the 2-D structure beneath the linear array of long-period NARS instruments in western Europe and has subsequently been extended to 2-D structures by Nolet & Zielhuis (1994).

For a stratified medium, surface-wave contributions can be represented asymptotically in terms of a source contribution which depends on the modal eigenfunction at the source depth, and a receiver contribution which involves the modes at the receiver level. These excitation and receiver terms specify the amplitude of the seismogram components in the frequency domain, whilst the dominant phase contribution comes from the influence of the
propagation path. However, although the amplitude terms depend on the source and receiver depths they are not localized at the source and receiver. The source contribution in particular involves reflections from the structure beneath the source level. When the surface-wave representation is extended to a laterally varying medium, care has to be taken to ensure compatibility between the different elements used to synthesize the surface-wave portion of the seismogram.

This paper explores the representation of surface waves in laterally varying structures. We will see that, for frequencies less than 0.03 Hz, it is possible to employ approximations in which different crustal structures are used at the source, near the receiver, and along the propagation path, provided that the change in crustal thickness is not more than 10 km between contiguous structures. In order to reduce computation costs in waveform fitting, it is common to employ only the modes of the initial structure. This reference structure is chosen to give a general match to the dispersion characteristics of the surface-wave contribution. In some regions, with large contrasts in lithospheric structure, substantial velocity perturbations can be inferred in the course of the waveform inversion procedure. The use of the original modal set will be satisfactory if the shapes of the modes for the base structure and the perturbed model are similar between the source depth for the deepest earthquake considered and the surface. For perturbations of up to 5 per cent in lithospheric velocities, the use of a single modal set should represent an adequate approximation for frequencies up to 0.035 Hz.

For propagation paths on regional-continental scales (1000–4000 km), the use of 1-D models based on a simple path-averaging procedure should be suitable for waveform fitting in the frequency range 0.01–0.03 Hz. The lower limit arises from the nature of the path-averaging approximation, and the upper from the influence of structure on the modal content of the seismograms.

A further complication arises when the available sources and receivers lie on different structures. For example, in the SKIPPY experiment (van der Hilst et al. 1994), a mobile array of broadband instruments on the Australian continent is used to record events from the major earthquake belt to the north and east, from Indonesia through to Fiji and New Zealand. The propagation paths for many events include passage across a substantial oceanic segment before impinging on the continent. The relatively sharp transition from oceanic to continental structures leads to an apparent redistribution of energy between modes for higher frequency waves so that these cannot be satisfactorily modelled using the path-averaged approximation.

2 SURFACE-WAVE REPRESENTATION

We start by considering the nature of the surface wavefield for structures that depend only on depth, and show how this portion of the seismogram can be expressed by terms depending on the source depth, receiver depth and propagation path. We then explore the physical significance of the different terms in order to examine the basis for further approximations which can be used to describe propagation in media that depart weakly from stratification.

For a stratified medium, the representation of the seismic response to a source as a function of angular frequency ($\omega$) and slowness ($p$) has singularities in the form of poles in the complex $\omega$–$p$ plane corresponding to normal modes of the stratified structure. For a model underlain by a uniform zone with shear velocity ($\beta_s$), such poles occur for slownesses greater than $\beta_s^{-1}$ and less than the Rayleigh-wave slowness for a uniform half space with the properties at the surface. The normal modes correspond to those combinations of frequency and slowness for which it is possible to satisfy simultaneously the conditions on the wavefield for vanishing traction at the surface and decay at great depth. The residue contributions from these sets of poles constitute the surface wavefield. We will present the development for Rayleigh waves. Love wave results can be obtained in an analogous fashion.

In the notation of Kennett (1983, Chapter 11), the modal contribution to the surface displacement at a range $r$ and an azimuth $\phi$ from the source for Rayleigh waves can be represented as

$$u_\delta(r, \phi, t) = \frac{1}{2} \int_0^\infty d\omega e^{-i\omega t} \sum_m \sum_{n \in N(\omega)} \left( \frac{2\pi}{\omega} \right) p_j \text{Res} \left[ w_m^{(p, m, \omega)}(p, m, \omega) \right] R_{m,n}^{(1)}(\omega p r).$$

(2.1)

Here, $w_m^{(p, m, \omega)}(p, m, \omega)$ represents the spectral response for the coupled $P$–$SV$ wave system in the $\omega$–$p$ domain, and $N(\omega)$ is the number of Rayleigh modes for frequency $\omega$. The angular variation in radiation from the source is represented through the summation over the angular order $m$ of the vector surface harmonics contained in the tensor field

$$T_m^{(1)}(\omega p r) = [R_m^{(1)}(\omega p r), S_m^{(1)}(\omega p r)]^T.$$

(2.2)

The azimuthal variation in the spectral representation of the response comes from the expansion of the source contribution, which can be conveniently represented in terms of discontinuities in displacement $S_m^{(p)}(p, m, \omega, z)$ and traction $S_m^{(t)}(p, m, \omega, z)$ across the source level $z = z_s$.

The locations of the pole singularities in the $\omega$–$p$ domain arise when a single eigen-wavefield $w_\delta$ can simultaneously satisfy the boundary conditions of vanishing traction at the free surface and decaying evanescent waves in the underlying uniform medium. In these circumstances, the reverberation operator for the stratified structure is singular and the pole locations are to be found from

$$\det [I - R_m^{(p)} R_m^{(t)}] = 0,$$

(2.3)

where $R_m^{(p)}$ is the reflection response for down-going waves incident on the entire zone below the source, and $R_m^{(t)}$ represents the reflection response for the zone above the source including the influence of the free surface. The residue at such a pole can be extracted in terms of the source jumps and the eigen-wavefield:

$$\text{Res} \left[ w_m^{(p, m, \omega)}(p, m, \omega) \right] = \frac{g_j}{2\omega L} \left[ T_j^{(1)}(z_s) S_m^{(p)}(p, m, \omega, z_s) - w_m^{(p, m, \omega)}(p, m, \omega) \right] w_j(0),$$

(2.4)

where $g_j$ is the group slowness for the particular mode, and
The residue contribution has a convenient separation into a part that is linked to the source depth dependences can be written as

$$\mathbf{u}_s(r, \phi, t) = \int_0^r dw \exp(-i\omega t) \sum_m e^{im\phi}$$

$$\times \left[ \sum_{j=1}^{N_m} \frac{E_j}{4l_j} \mathbf{F}_m^{(1)}(p_j) \mathbf{A}_n(p_j) \right].$$

The source and receiver contributions to the displacement dependence on frequency $\omega$ for the phase and group slowness $p_j$, $g_j$ and all the terms such as $\mathbf{F}$, $\mathbf{A}$ that depend on the properties of individual mode branches. Although we have identified source and receiver terms above, we have already seen that these have non-local properties.

In (2.9), the major phase contribution comes from the horizontal propagation term but there will also some influence from the source and instrumental response.

### 3 Weak Lateral Variation

For a stratified medium, with a 1-D velocity model depending on depth alone, we have no horizontal variation in seismic properties and the propagation path will automatically lie along the great circle between source and receiver. For a realistic medium we have to account for the presence of 3-D variations in seismic properties, and therefore need to know to what extent we can rely on the results of calculations for stratified media for interpretation.

For a smoothly varying medium where the seismic parameters depend weakly on the horizontal coordinates, Woodhouse (1974) has demonstrated that the propagation characteristics are governed by the vertical structure beneath each point on the propagation path. The path itself is governed by the variations in phase slowness and the incremental phase $\phi(\omega)$ along the path is simply the integral of the phase slowness:

$$\phi(\omega) = \omega \int \mathbf{p}_j \cdot d\mathbf{r},$$

where $\mathbf{p}_j$ is the horizontal slowness vector for the mode and $d\mathbf{r}$ is an element along the path. This style of phase representation is subject to the same limitations as the asymptotic approximation for the vector harmonics in the stratified case. We therefore require that, for a propagation range $r$, the combination $\omega \beta_1^{1-r} \gg 1$.

For structural models with such slow variations in seismic parameters, we can adapt the formulation for a stratified medium by envisaging the modal field to be launched in a uniform region with the source properties, to propagate, retaining the identity of the individual modes, and then be constituted into the observable displacements using the receiver structure.

The propagation component can then be further simplified if it is close to a great circle by using the averaged structure that produces the same incremental phase. Thus, for range $r$, we require

$$\phi(\omega) = \omega \int_{\text{path}} \mathbf{p}_j \cdot d\mathbf{r} = \omega p_j^{\text{av}} r,$$

where $p_j^{\text{av}}$ is the slowness for the averaged structure. Using the averaged structure to represent the main propagation characteristics, we can represent an approximation to the surface-wave part of the wavefield via a close analogue to the forms for a stratified medium (2.9). Thus, for Rayleigh waves,

$$\mathbf{u}_s(r, \phi, t) = \left( \frac{i e_z}{-e_z} \right) \sum_m e^{i m \phi} \int_{-\infty}^{\infty} d\omega$$

$$\times \sum_{j=0}^{N_m} \sqrt{\frac{2\omega p_j}{\pi r}} \frac{g_j}{4l_j} \mathbf{F}_m^{av}(p_j) \mathbf{A}_n(p_j)$$

$$\times \exp \left[ ig_j(p_j r - t) - i(2m + 1\pi) / 4 \right].$$

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Approximations for surface-wave propagation

A similar consideration arises during the course of a waveform inversion procedure. The optimum fit to the observed seismograms should be produced by updating the modal set used as the 1-D model for the propagation component is itself updated, since the partial derivatives for the seismograms depend on the modes. However, the incremental changes in model are generally small and so the modal eigenfunctions for the original trial structure can remain useful for several iterations. But, in regions with structural complexity, the propagation paths are such that there may be significant variations in the preferred average structure away from the reference structure. When a single set of modes has been employed, what is the likelihood that this might have an undue influence on the outcome of the inversion? The major effects can be examined by comparing the depth dependence of the modes of the original reference structure and those for the postulated average structure. When noticeable differences in mode shape occur, the estimated partial derivatives could well be in error and so the average structure itself could be suspect. We should note that the level of parameter differences between the averaged models for different traverses across a three-dimensionally varying region should be smaller than the actual variation in the 3-D structure.

3.1 Influence of crustal thickness

One of the most common circumstances in which it might be desirable to use different structural models for the source, receiver and propagation path is for regions where there is considerable variation in crustal thickness. Different paths from a common source location may then encounter rather different crustal structures, and this can be accommodated by using different 1-D models for the propagation component.

We will illustrate the way in which the crustal thickness affects the mode shapes by looking at the modal eigenfunctions for a sequence of models within which the lower crust is expanded in 10 km steps at the expense of the upper mantle. The model sequence is displayed in Fig. 1 with crustal thickness ranging from 65 to 25 km superimposed on the continental model penc of Dziewonski, Hales & Lapwood (1975) which has an original crustal thickness of 35 km.

The modal eigenfunctions for modes with phase velocity less than 7 km s\(^{-1}\) for this set of crustal and mantle structures are displayed as a function of depth in Fig. 2 for both Love waves and Rayleigh waves. Up to five modes are displayed at three periods which span the frequency range (0.014–0.06 Hz) associated with surface waves at regional ranges. For each selected period, the set of displacement eigenfunctions are displayed for each of the crustal models, with a slight offset between models to facilitate visual comparison. The eigenfunctions for each mode number are placed in order of diminishing crustal thickness (65, 55, 45, 35, 25) and are truncated at 400 km depth to focus attention on the shallower part of the behaviour. At the longer periods not all mode branches fall inside the phase-velocity window.

For the Love waves displayed in Fig. 2, we have only the tangential displacement eigenfunction (W\(_t\)). The sharp discontinuity in shear velocity and density at the...
Figure 1. P-wave velocity ($\alpha$) and S-wave velocity ($\beta$) models with varying crustal thickness used to generate the modal eigenfunctions shown in Fig. 2.

Figure 2. Displacement eigenfunctions for the first five modes of Love waves and Rayleigh waves as a function of depth for the sequence of models shown in Fig. 1. The modes are ordered by decreasing crustal thickness for each mode number. For Love waves, the transverse horizontal component is displayed. For Rayleigh waves, the vertical component ($U$) is shown as a solid line and the horizontal component ($V$) as a dashed line.
crust–mantle boundary is marked by a change in the slope of the eigenfunctions at this depth for all periods. These discontinuities in slope are most noticeable for the shorter periods (frequency greater than 0.04 Hz) and the fundamental mode. The change in modal shape between adjacent models, with a shift in crustal thickness of 10 km, also becomes more pronounced as the frequency increases. Quite strong changes across the suite of models can be observed for the first and second higher modes for the two shortest periods.

There is a similar pattern of behaviour for the Rayleigh waves in Fig. 2, for which we have to consider both the vertical-component eigenfunctions (\(U_r\)) and the radial-component eigenfunctions (\(V_r\)) for each mode. The influence of the crust–mantle boundary is most pronounced for the vertical component of the fundamental mode, and is more significant than for the fundamental Love mode at each frequency. The difference arises because the fundamental Rayleigh mode tends to have more energy concentrated in the crustal layer.

For the Rayleigh modes, the changes between the modal shapes for different crustal thicknesses are again most evident for the two shortest periods. Comparison with adjacent traces suggests that it should be possible to accommodate differences of crustal thickness of 10 km between contiguous structures (i.e. source to propagation path, propagation path to receiver) for frequencies up to 0.03 Hz for the higher modes of both Love and Rayleigh waves. For the fundamental modes, the approximate representation works less well, but it should still be possible to allow for a 10 km change in crustal thickness for frequencies up to 0.015 Hz.

### 3.2 Influence of lithospheric structure

We can investigate the extent to which changes in the averaged structure along the propagation path should affect the simple approximation of using a single set of reference modes in the course of the linearized waveform inversion (Nolet, van Trier & Hulsman 1986) by constructing a set of models in which the lithospheric properties are modified without changing the rest of the model. The largest variations in seismic velocity are to be expected within the lithosphere and we can provide an upper bound on the influence of structure by making a bulk shift in the velocity through the whole lithosphere.

The linearized inversion scheme constructs the model perturbations required to improve the match between observed and computed seismograms by making use of the partial derivatives of the seismograms. These partial derivatives depend on the modal eigenfunctions throughout the model. If the set of reference modes departs too far from the modes for the current structure there is a potential for error, both from an inaccurate estimate of the update parameters and in the comparison of seismograms to determine the misfit for a particular model. We note from the approximation (3.3) that the dominant influence of the propagation path comes from the phase dispersion, which can be recomputed readily for each new model with much less effort than for the modal eigenfunctions for a number of modes at many frequencies.

We have again modified the peme model. On this occasion the crustal thickness has been kept constant at 35 km, but the velocities in the lithosphere down to 120 km have been varied. Five models have been considered with perturbations of \(-5, -3, 0, 3\) and 5 per cent, and a set of modal eigenfunctions have been calculated in each case for frequencies in the range 0.01–0.06 Hz. The resulting velocity models are shown in Fig. 3. When the velocity is lowered, the low-velocity zone in the original model can be eliminated, whilst a very strong contrast between the lid and the low-velocity zone is generated when the lithospheric velocity is raised. Nevertheless, these changes in velocity are somewhat smaller than when mantle is replaced by crust.

Despite the significant changes in velocity, the modal shapes for the fundamental modes and first few higher modes of both Love and Rayleigh waves show a very high correlation across the different models to frequencies of at least 0.03 Hz. However, as the frequency increases, differences between the different cases begin to emerge and rapidly become pronounced. In Fig. 4 we display the mode shapes for Love and Rayleigh modes at two periods. In each panel we display the eigenfunctions in order of increasing fundamental frequency for each mode number. The longer period (34.12 s) marks the point at which differences between mode shapes can be discerned but the major features are preserved across the full set of models. At the shorter period (16.67 s), however, there are noticeable differences in the modal shapes because the shear wavelength is comparable to the thickness of the lithospheric lid and so the different parts of the structure exert their own influences on the modal field. For the models with enhanced lid velocities there is a shift of horizontal component energy into the pronounced low-velocity zone between 120 and 220 km depth.

Hence, even when quite large perturbations of up to 5 per cent in velocity have been introduced, the modal shape of the reference structure will represent a good approximation to the true modes for frequencies up to 0.035 Hz.

### 3.3 Frequency restrictions

We have just considered the influence of two classes of heterogeneous structure via the rather simple procedure of examining the shapes of the modal eigenfunctions for 1-D structures. In each case we were able to show that the simple approximation (3.3) for the representation of the wavefield for modelling, or inversion, could be used for frequencies up to 0.03 Hz, at least, for the higher mode portion of the surface wavefield. For variable crustal thickness, the contribution of the fundamental mode waves can only be expressed in terms of simple 1-D structures for frequencies less than 0.15 Hz. Lithospheric heterogeneity has a much smaller effect because of the limited depth penetration of the fundamental modes at higher frequencies.

Kennett & Nolet (1990) have taken a very different approach to examining the influence of heterogeneity in the crust and mantle on the surface wavefield. They have employed the coupled-mode propagation procedure of Kennett (1984) to undertake direct computations of the interactions of surface waves with two-dimensionally varying structures for propagation paths up to 3000 km. For realistic
Figure 3. P-wave velocity ($\alpha$), S-wave velocity ($\beta$) and density ($\rho$) models with varying lithospheric properties used to generate the modal eigenfunctions shown in Fig. 4.

Figure 4. Love-wave and Rayleigh-wave eigenfunctions, for a suite of lithospheric models for two periods illustrating the greater differences in mode shapes at shorter periods. The eigenfunctions are ordered by increasing lithospheric velocity for each mode number.
models, based on the results of Nolet (1990) with the inclusion of medium-scale heterogeneity (Kennett & Bowman 1990), the lower order modes could be regarded as propagating independently up to 0.02 Hz. This would correspond to a frequency limit for the use of a single set of reference modes, and a consequent 1-D model, for a 2-D structure with heterogeneity of ±5 per cent on horizontal scales of 400 km or more and ±1 per cent on horizontal scales of 200 km or less. For the higher order modes representing body-wave arrivals, the upper limit in frequency for the direct use of the reference modes is about 0.05 Hz. As noted by Kennett & Nolet (1990), in a three-dimensionally varying medium it might be possible to raise the frequency limits slightly because the chance of multiple interactions is reduced due to the angular dependence of scattering processes.

These estimates of frequency limitations based on intensive calculations for heterogeneous media are concordant with the results deduced by simpler means above. The classes of heterogeneity induced by changes in crustal thickness and a bulk shift in lithospheric velocity are more extreme than distributed heterogeneity over 3000 km, and in consequence the suggested upper frequency limits are slightly lower.

For propagation paths from 1000 to 4000 km, the use of asymptotic representation of the surface waves on which (3.3) is based (ωβr, r ≫ 1) requires that the frequencies employed should not be much less than 0.01 Hz.

The recommended frequency limits for the application of the approximation (3.3) for the surface wavefield will give apparently satisfactory synthetic seismograms.

4 PROPAGATION ACROSS STRUCTURAL BOUNDARIES

The approximations discussed in the previous section work most effectively when applied to situations where there are modest variations in seismic properties within structures of a similar class. However, in some cases we observe surface waves that have crossed major structural boundaries, such as the transition from an oceanic to a continental regime. Such waves carry with them valuable information about the structures through which they have passed. It is therefore desirable to develop calculation schemes for the surface-wave part of the wavefield that can include the influence of such structural boundaries.

If we consider a source in a nearly stratified region (e.g. an oceanic region), we can produce a good representation of the surface wavefield in that region by using approximations such as (3.3) to include some allowance for near-source or near-receiver heterogeneity. However, once the receiver moves outside this zone into a different structure, we have to include an allowance for the transition between the source and receiver structures. When we consider long-period waves (<0.03 Hz), the shear wavelengths will generally be much longer than the width of the transition (commonly 30 to 120 km wide, see e.g. Keen & Hyndman 1979), so that a simplified model of an abrupt transition can be used to include the major effects of the structural change.

Within the oceanic zone surrounding the source, the wavefield should not be strongly affected by the presence of the structural transition. Thus we can project the wavefield up to the boundary using the seismic properties of this zone. The superposition of different modal contributions can be used to build up the profile of seismic displacements and tractions at the boundary, as a function of depth.

If the source and receiver lie well away from the boundary, we can use the ray formulation of Levshin (1985) to consider propagation to the boundary, transmission with refraction due to the contrast in structures, and then propagation in the receiver structure to the recording location. In this high-frequency ray approximation, the continuity conditions for displacement and traction have only to be applied at the point where the ray crosses the boundary. Gregersen & Alsop (1974) have introduced a convenient approximation for long-period propagation in which any diffraction processes or other local effects at the boundary are ignored. The displacement and traction field calculated for the source structure are re-expanded in terms of the modes of the receiver structure and these contributions are then propagated to the receiver.

We consider a source at a distance r<up>1</up> from the boundary, and then construct the vertical and radial components of the displacement profile as a function of depth in the frequency domain, as a sum of Rayleigh modes,

\[
\mathbf{u}_s(r, \phi, z, \omega) = \left( \begin{array}{c} i e_x \\ -e_y \end{array} \right) \sum_m e^{i m \phi} \times \sum_{j=0}^{N/2} \sqrt{\frac{2 \omega \rho_j}{4 \pi r^2}} \frac{F_{Rj}(\rho_j)}{4 \pi r^2} \left( \begin{array}{c} g_{xj} \\ 0 \end{array} \right) \exp \left[ i \omega \rho_j r - i(2m + 1)\pi/4 \right],
\]

(4.1)

with a comparable expression for the tangential component of displacement in terms of Love modes. Similar representations can be written for the traction components in terms of the requisite eigenfunctions. These displacement and traction profiles are then to be expanded into the modes of the receiver structure by using the orthogonality properties of the modal set (including both Love and Rayleigh polarizations). The expansion coefficients of the wavefield on the vertical interface in terms of the receiver modes are

\[
C_{ij} = i \int_0^r \frac{dz}{\rho_j} \left[ \mathbf{w}_j(-p_j, z, \omega) \right]^T \mathbf{t} \left[ \mathbf{w}_j(-p_j, z, \omega) \right],
\]

(4.2)

where \( \mathbf{t} \) is the traction acting on the vertical boundary. Gregersen (1978) has shown the need for including both Love and Rayleigh waves to describe the effects for waves at oblique incidence on a ocean–continent transition. Vaccari et al. (1989) have shown that for models with uniform layers an analytic integration can improve the efficiency of the computation of modal coefficients. However, for more general structural models, numerical integration cannot be avoided.

Once the modal coefficients (4.2) have been evaluated, we can construct the displacement at the receiver at a distance
where $J$ represents the geometrical spreading along the refracted ray path (Levshin 1985).

The two-stage computation procedure involves propagation in both source and receiver structures with transfer coefficients evaluated via the continuity of displacement and traction at the point of refraction. Vaccari et al. (1989) have shown that, for the lower frequency part of the surface wavefield, there is a high efficiency of transmission between a continental model with a 60 km crustal thickness. Strong transfer between modes occurs across the boundary, for normally incident waves, for frequencies above 0.03 Hz, which agrees well with the results of the simplified treatment in the previous section.

For an ocean–continent transition, the contrast between modal shapes is largely confined to the surficial water layer in the ocean for frequencies less than 0.02 Hz for the fundamental and first few higher modes. For 0.04 Hz waves, however, the influence of the crustal thickness in each zone is much more pronounced and differences in modal character can extent to 50 km and deeper. Thus the simple interface modal becomes inadequate at a similar frequency to that at which the mismatch in modal characters between the end-members becomes pronounced.

The vertical contact model will give a good representation of the phase and amplitude interactions for lower frequency waves. For the fundamental mode, the phase velocities for both oceanic and continental models are similar at low frequencies so that errors in the path-averaged approximation can be confined below 0.25 per cent up to 0.02 Hz: the phase velocities diverge at higher frequencies and the errors in the path average then increase rapidly. The behaviour of the higher mode dispersion for Rayleigh waves is rather different for the oceanic and continental models; for example at 0.04 Hz the oceanic fundamental mode has a higher phase velocity than that for the continental mode, but the higher modes have lower phase velocities. In consequence, it is somewhat difficult to find a compromise model with an appropriate character to represent mixed oceanic–continental paths at higher frequencies. Furthermore, the contrasts in velocity can give rise to noticeable refraction effects which negate the assumption of great-circle propagation.

Nevertheless, by working with sufficient paths for lower frequency waves it should be possible to use the coupled displacement formulae (4.1), (4.3) as a basis for non-linear waveform inversion for the structures on either side of the structural boundary. The first stage would be a simple path-averaged treatment, which would then be refined using the more accurate formulae.

REFERENCES


