Effect of 2-D topography on the 3-D seismic wavefield using a 2.5-D discrete wavenumber-boundary integral equation method

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SUMMARY
A full treatment of topographic effects on the seismic wavefield requires a 3-D treatment of the topography and a 3-D calculation for the wavefield. However, such full 3-D calculations are still very expensive to perform. An economical approach, which does not require the same level of computational resources as full 3-D modelling, is to examine the 3-D response of a model in which the heterogeneity pattern is 2-D (the so-called 2.5-D problem). Such 2.5-D methods can calculate 3-D wavefields without huge computer memory requirements, since they require storage nearly equal to that of the corresponding 2-D calculations.

In this paper, we consider wave propagation from a point source in the presence of 2-D irregular topography, and develop a computational method for such 2.5-D wave-propagation problems. This approach is an extension to the 2.5-D case of the discrete wavenumber-boundary integral equation method introduced by Bouchon (1985) and Gaffet & Bouchon (1989) to study 2-D topographic problems. One of the most significant advantages of the 2.5-D calculations is that calculations are performed for a point source and so it is possible for us to take into account the 3-D radiation pattern from the source. We demonstrate that this discrete wavenumber-boundary integral equation procedure, coupled with a Green’s function decomposition into P- and S-wave contributions, provides a flexible and effective means of evaluating the wavefield.

Key words: P waves, S waves, topography, wave propagation.

1 INTRODUCTION
The influence of topography in the immediate neighbourhood of the seismic source is of considerable interest in two disparate classes of problem. The way in which the ground motion at the surface is modified by topography is of major interest in seismic hazard assessment, and in this case interest is focused on the near-field environment of the source. In addition, the influence of topography on far-field radiation can be significant in teleseismic studies, especially for discrimination between shallow underground nuclear explosions and earthquakes, since nuclear device emplacement by horizontal adit is feasible in mountainous regions.

A full treatment of the influence of such topography on the wavefield requires a 3-D treatment of the topography and a 3-D calculation for the wavefield. Recent advances in high-performance computers have brought such (3, 3)-D calculations just within reach. We have used here a notation (w, h) to indicate the spatial dimensionality of the wavefield (w) and the heterogeneity of a medium (h). Finite-difference calculations have been performed for such large-scale (3, 3)-D media by Frankel & Leith (1992), Frankel & Vidale (1992) and Yomogida & Etgen (1993). However, such full (3, 3)-D calculations are still very expensive to perform because of the large memory requirements. Nevertheless, in order to provide a quantitative analysis of real seismic records from complex regions we need to be able to calculate the 3-D wavefields.

An economical approach which does not require the same level of computational resources as (3, 3)-D modelling is to examine the 3-D response of a model in which the heterogeneity pattern is 2-D. Such (3, 2)-D wave modelling (the so-called 2.5-D problem) can provide useful results for many problems of practical interest. Luco, Wong & De Barros (1990) proposed a formulation for a 2.5-D indirect boundary method in order to obtain the 3-D response of an infinitely long canyon, in a layered half-space, for plane elastic waves impinging at an arbitrary angle with respect to the axis of the canyon. Randall (1991) developed a 2.5-D velocity-stress finite-difference tech-
3-D Problem

\[ \tilde{u}^{IN} \quad \tilde{u}^{SC} \]

2.5-D Problem

\[ \tilde{u}^{IN} \quad \tilde{u}^{SC} \]

**Figure 1.** Configuration of 2.5-D topographic problems. The shape of the free surface \( S \) is constant in the \( y \)-direction. The solid star denotes a point source which is located in the \( x-z \) plane \( (y = 0) \).

**Figure 2.** Schematic illustration of the \( k_y \) coupling for a \((3,3)\)-D problem and the absence of such coupling between \( k_y \) components for the \((3,2)\) i.e. the 2.5-D case.

In this paper, we consider the problem of the interaction of the seismic wavefield with 2-D irregular topography for excitation by a point source, which may be either an explosion or a double couple. The numerical treatment is based on an extension to the \((3,2)\)-D case of the discrete wavenumber-boundary integral equation method introduced by Bouchon (1985) and Gaffet & Bouchon (1989) to study the \((2,2)\)-D topographic problems. Thus we employ a 2.5-D discrete wavenumber-boundary integral equation method.

**Figure 3.** Three explosive source configurations for which the wavefields are compared in Figs 4, 5 and 6. The medium parameters are \( \alpha = 5.2 \text{ km s}^{-1}, \beta = 3.0 \text{ km s}^{-1} \) and \( \rho = 2.8 \text{ g cm}^{-2} \). (a) Flat surface model. Source position \( x = y = 0, z = 1 \text{ km} \). (b) Mountain model. Source position \( x = y = 0, z = 1 \text{ km} \). (c) Mountain model. Source position \( x = 0.8 \text{ km}, y = 0, z = 1 \text{ km} \).
2 METHOD

2.1 A boundary integral equation for the 2.5-D topographic problems

Consider a homogeneous isotropic medium with an irregular surface (free surface) $S$ subject to a harmonic excitation by an incident wave radiated from a point source. We take a Cartesian coordinate system $[x, y, z]$ with the $z$-axis taken positive downwards. We assume the shape of free surface is invariant in the $y$-direction, and is represented by the relation

$$S: z = s(x, y), \quad -\infty < x, y < +\infty .$$

The cross-section of the surface $S$ in the $x-z$ plane is denoted by $C$, and is described by

$$C: z = c(x) = s(x, 0), \quad -\infty < x < +\infty .$$

We set the $y$-coordinate of the source position at $y = 0$ without loss of generality. Fig. 1 shows the schematic configuration of the relation of the source and the topography.

![Figure 1](image1)

Figure 1. The response of a flat free surface due to an explosion source (Fig. 3a). (a) Snapshots of horizontal components of the scattered $P$-wave part $u_{x}^{S}$ at the free surface; (b) snapshots of horizontal components of the scattered $S$-wave part $u_{y}^{S}$ at the free surface; (c) record section of vertical-component seismograms $u_{y}^{F}$ at $x = 1$ km and variable $y$ position; (d) record section of vertical-component seismograms $u_{y}^{F}$ at $x = 1$ km and variable $y$ position. Positive amplitude means upward motion. The peak amplitude is shown on the right of each trace. It is in cm if the unit of the moment tensor is $10^{20}$ dyn cm, density is in g cm$^{-3}$, and medium velocity is in km s$^{-1}$.
dence $\exp(+i\omega t)$ will be assumed throughout, but will be suppressed in the equations.

The total displacement field in the half-space can be represented as

$$u(x) = u^I(x) + u^{SC}(x),$$

where the superscripts 'IN' and 'SC' denote the incident and scattered wavefields, respectively. Following an indirect boundary integral equation approach (see e.g. Bouchon 1985), the scattered fields are represented by distributed body forces along the diffracting surface (Huygens' principle). The scattered displacement field in the half-space can then be written as

$$u^{SC}(x) = \int_S dS(x') G(x; x') f(x'),$$

where $G$ is a full-space displacement Green's function tensor whose entry $G_{ij}(x; x')$ corresponds to the $i$th component of the displacement vector at $x$ due to a unit harmonic force in the $j$th direction at $x'$ (see e.g. Aki & Richards 1980) and $f(x')$ is an unknown body-force vector at $x'$. On performing a Fourier transform of (2.3) and (2.4) with respect to $y$ we obtain the full wavefield and the scattered contribution in the form.

Figure 4. (Continued.)
where we have used the notation
\[ \tilde{g}(x, k_y, z) = \int_{-\infty}^{1/\omega_y} dy g(x, y, z) \exp(-ik_y y), \]
and we have also employed the coordinate-shift property of the full-space Green's function for homogeneous media:
\[ G(x, y, z; x', y', z') = G(x, y - y', z; x', 0, z'). \]

The traction field in the half-space can be expressed in the \((x, k_y, z)\) domain, in a similar way to the displacement field, as
\[ \tilde{t}(x, k_y, z) = \tilde{t}^{\text{IN}}(x, k_y, z) + \tilde{t}^{\text{SC}}(x, k_y, z), \]
where \(\tilde{t}^{\text{SC}}(x, k_y, z) = \int d\mathbf{C}(x', \mathbf{z}') \tilde{h}(x, k_y, z; x', 0, \mathbf{z}') \tilde{t}(x', k_y, \mathbf{z}'). \]

The free-surface boundary condition in the mixed coordinate–wavenumber \((x, k_y, z)\) domain is given by
\[ \tilde{t}(x, k_y, z) = 0, \quad (x, z) \in C. \]

On inserting (2.8), we obtain a boundary integral equation in the \((x, k_y, z)\) domain:
\[ \int_{C} d\mathbf{C}(x', \mathbf{z}') \tilde{t}(x, k_y, z; x', 0, \mathbf{z}') = -{\tilde{t}}^{\text{IN}}(x, k_y, z), \]

\((x, z) \in C. \quad (2.10)\)

It should be noticed that, for a fixed value of the wavenumber \(k_y\), this integral equation depends on only two space coordinates, i.e. \(x\) and \(z\). We call eq. (2.10) the 2.5-D boundary integral equation. If this integral equation can be solved, the displacement in the \((x, k_y, z)\) domain can be obtained from (2.5). The displacement in the space domain can then be derived by an inverse Fourier transform over the \(y\) coordinate. For each value of \(k_y\), eqs (2.5), (2.8) and (2.10) can be solved as independent 2-D equations, because in this configuration with 2-D topography there is no coupling between waves of different \(k_y\). This property arises from the \(y\)-shift-invariance (i.e. space-invariance in the \(y\)-direction) of the 2.5-D system, i.e. the relation (2.7). In a full 3-D calculation there would be coupling between different \(k_y\) components (Fig. 2).

2.2 Discrete wavenumber solution

In general, eq. (2.10) cannot be solved analytically, so a solution must be sought by numerical procedures. The kernel of the integral equation (2.10) has a singularity at \((x, z) = (x', z')\), and so we must exploit numerical methods that can
avoid this singular behaviour. Furthermore, the infinite extent of the free surface is not tractable in a numerical procedure, and so we must truncate in order to deal with a finite region for computational purposes. One of the simplest approaches to the numerical evaluation of the integral equation (2.10) is a discrete wavenumber representation (Bouchon 1985; Gaffet & Bouchon 1989), which introduces a horizontal spatial periodicity in the surface shape and source configuration and employs a truncated Fourier series in place of the actual Green’s function.

The numerical solution of eq. (2.10) requires a discretization of the boundary C. To this end, body forces are applied at equal spacing $\Delta x$ along C. The integrals in eqs (2.5), (2.8) and (2.10) are then to be replaced by discrete summations to give

\[
\tilde{u}^P(x, k_x, z) = \Delta x \sum_{m = -\infty}^{\infty} \tilde{G}(x, k_x, z; x_m, 0, z_m) \tilde{f}(x_m, k_x, z_m),
\]

(2.11)

\[
\tilde{u}^S(x, k_x, z) = \Delta x \sum_{m = -\infty}^{\infty} \tilde{H}(x, k_x, z; x_m, 0, z_m) \tilde{f}(x_m, k_x, z_m),
\]

(2.12)

Figure 5. The response due to an explosion source beneath a mountain (Fig. 3b). (a) Snapshots of horizontal components of the scattered $P$-wave part $u^P_{sc}$ at the free surface; (b) snapshots of horizontal components of the scattered $S$-wave part $u^S_{sc}$ at the free surface; (c) record section of vertical-component seismograms $u^V$ at $x = 1$ km and variable $y$ position; (d) record section of vertical-component seismograms $u^V$ at $x = 1$ km and variable $y$ position.
The displacement and traction Green's functions can be represented as a superposition of plane waves, i.e., as a wavenumber integral. We assume the source medium configuration to be periodic along the x-axis with repetition length $L_x$, so that the radiation from each body force can be computed using the discrete wavenumber representation (Bouchon 1979; Campillo & Bouchon 1985). If one period $L_x$ of the boundary $C$ is represented by an array of $2M + 1$ points $[L_x = (2M + 1)\Delta x]$, eqs (2.11) to (2.13) lead to the representation

\begin{equation}
\Delta x \sum_{m=-\infty}^{\infty} \tilde{H}(x, k_y, z; x_m', 0, z_m') \tilde{G}(x_m', k_y, z_m') = -\tilde{f}^m(x, k_y, z), \quad (x, z), (x_m', z_m') \in C, \quad (2.13)
\end{equation}

\begin{equation}
\Delta x \sum_{m=-\infty}^{\infty} \tilde{G}(x, k_y, z; x_m', 0, z_m') \tilde{H}(x_m', k_y, z_m') = -\tilde{f}^m(x, k_y, z), \quad (x, z), (x_m', z_m') \in C, \quad (2.14)
\end{equation}

\begin{equation}
\Delta x \sum_{m=-M}^{M} \tilde{H}(x, k_y, z; x_m', 0, z_m') \tilde{G}(x_m', k_y, z_m') = -\tilde{f}^m(x, k_y, z), \quad (x, z), (x_m', z_m') \in C, \quad (2.15)
\end{equation}

where $\tilde{G}$ and $\tilde{H}$ are the discrete wavenumber representations of displacement and traction Green's function tensors. The
expressions for $\hat{G}$ and $\hat{H}$ are displayed in the Appendix, which employ symmetry or antisymmetry properties with respect to the $x$-coordinate to include only the components for non-negative horizontal wavenumber $k_x$. We have represented the elastic wavefield diffracted by the surface as a force distribution applied at $2M + 1$ points regularly spaced at intervals of $\Delta x$ in the $x$-direction. This sampling implies a periodicity with repetition width $2\pi/\Delta x$ in the $x$ wavenumber domain, so that the infinite summations (infinite series) over the $x$-component of the horizontal wavenumber $k_x$ ($k_x = 2m\pi/L_x$; $m$: non-negative integer) in the expressions for $\hat{G}$ and $\hat{H}$ are reduced to finite summations over $k_x$ in the range $[0, 2\pi M/L_x]$, as demonstrated by Bouchon (1985) and Campillo & Bouchon (1985). Then we choose $(x_m, z_m)$ (the body force points; $m = -M, \ldots, M$) as the sampling points at which we perform matching to solve eq. (2.16) by the collocation method (using the finite summations over $k_x$).

In actual calculations, we must also discretize the $y$-component of the horizontal wavenumber $k_y$. This discretization can be achieved by assuming the source configuration to be periodic along the $y$-axis as well as along the $x$-axis (Bouchon 1979). A repetition length $L_y$ along the $y$-axis gives $k_y$ ($k_y = 2\pi n/L_y$, $n$: integer). The total displacement can then be represented as

$$u(x, y, z) = \frac{1}{L_y} \sum_{n=-N}^{N} \hat{u}(x, k_y, z) \exp(-ik_yn), \quad (2.17)$$

where the $y$ wavenumber summation is truncated at $n = \pm N$.

When the force system describing the source has either symmetry or antisymmetry in the $y$-coordinate, the number of $k_y$ calculations can be reduced by a factor of nearly 2. When the source is symmetric with respect to $y$, such as for the single forces $f_x$ and $f_y$ and the moment tensor components $M_{xx}$, $M_{yy}$, $M_{yx}$, $M_{zx}$, and $M_{zy}$, (2.17) can be simplified to

$$u_x(x, y, z) = \frac{-i2}{L_y} \sum_{n=1}^{N} \hat{u}_x(x, k_y, z) \cos(k_yn),$$

$$u_y(x, y, z) = \frac{-i2}{L_y} \sum_{n=1}^{N} \hat{u}_y(x, k_y, z) \sin(k_yn),$$

$$u_z(x, y, z) = \frac{2}{L_y} \sum_{n=0}^{N} \hat{u}_z(x, k_y, z) \cos(k_yn). \quad (2.18)$$

The $x$- and $z$-components of displacement field $u_x$, $u_z$ are then even functions with respect to $y$, while the $y$-component $u_y$ is an odd function with respect to $y$.

When the source is antisymmetric with respect to $y$, such as for a single force $f_x$ and the moment tensor components $M_{xy}$, $M_{yx}$, $M_{zy}$, and $M_{zx}$, (2.17) can be reduced to

$$u_x(x, y, z) = \frac{-i2}{L_y} \sum_{n=1}^{N} \hat{u}_x(x, k_y, z) \cos(k_yn),$$

$$u_y(x, y, z) = \frac{2}{L_y} \sum_{n=0}^{N} \hat{u}_y(x, k_y, z) \sin(k_yn),$$

$$u_z(x, y, z) = \frac{-i2}{L_y} \sum_{n=1}^{N} \hat{u}_z(x, k_y, z) \cos(k_yn). \quad (2.19)$$
Since any force system of sources can be decomposed into symmetric and antisymmetric components with respect to $y$ and represented as a linear combination of them, the formulae (2.18) and (2.19) can also be used in the case of a source that does not have simple symmetry and antisymmetry in the $y$ coordinate.

### 2.3 Decomposition of scattered wavefield into $P$- and $S$-waves

An advantage of the method described in the previous subsection is that it leads to a convenient decomposition of the scattered wavefield into $P$- and $S$-wave parts. In the mixed coordinate–wavenumber domain $(x, k_x, z)$, the scattered wavefield can be represented as

$$
\tilde{u}^{sc}(x, k_x, z) = \tilde{u}^{P, sc}(x, k_x, z) + \tilde{u}^{S, sc}(x, k_x, z),
$$

where $\tilde{G}_P$ and $\tilde{G}_S$ are the $P$- and $S$-wave parts of the Green's function tensor $\tilde{G}$ (see Appendix). In order to recover the $P$- and $S$-wave contributions in the space domain, we can make

$$
\tilde{u}^{P, sc}(x, k_x, z) = \Delta x \sum_{m=-M}^{M} \tilde{G}_{P,5}(x, k_x, z; x_m, 0, z_m) \tilde{f}(x_m, k_x, z_m),
$$

(2.20)

where $\tilde{G}_P$ and $\tilde{G}_S$ are the $P$- and $S$-wave parts of the Green's function tensor $\tilde{G}$ (see Appendix). In order to recover the $P$- and $S$-wave contributions in the space domain, we can make

![Figure 6](https://example.com/figure6.png)

**Figure 6.** The response due to an explosion source lying beneath the flanks of a mountain (Fig. 3c). (a) Snapshots of horizontal components of the scattered $P$-wave part $u^{P, sc}$ at the free surface; (b) snapshots of horizontal components of the scattered $S$-wave part $u^{S, sc}$ at the free surface; (c) record section of vertical-component seismograms $u^{P, sc}$ at $x = 1.8$ km (1 km offset from the source position) and variable $y$ position; (d) record section of vertical-component seismograms $u^{S, sc}$ at $x = 1.8$ km and variable $y$ position.
use of the formulae (2.17), (2.18) or (2.19) to invert the spatial transforms.

3 APPLICATIONS

We will illustrate the (3,2)-D wavefield calculations for a number of different source configurations in models with both elevated and depressed topography at the free surface of a uniform half-space. We will use the same functional form for the cross-section $c(x)$ of the free surface (Sills 1978; Gaffet & Bouchon 1989) for all the numerical examples of topography in this section:

$$c(x) = h[1.0 - (x^2/w^2)] \exp[-3(x^2/w^2)],$$  \hfill (3.1)

where $w$ denotes the half-width of the valley or mountain and $|h|$ is the depth of the valley or the height of the mountain ($h > 0$ for a valley, $h < 0$ for a mountain). The properties of the elastic half-space are taken as $\alpha = 5.2 \text{ km s}^{-1}$, $\beta = 3.0 \text{ km s}^{-1}$ and $\rho = 2.8 \text{ g cm}^{-3}$.

We will also employ a source-time function in the form of a zero-phase Ricker wavelet with a central frequency of 2 Hz:

$$s(t) = 0.5\sqrt{\pi}[(2\pi t)^2 - 0.5] \exp[-(2\pi t)^2].$$  \hfill (3.2)

The temporal moment tensor for the source, $M_{ij}(t)$, is then represented as

$$M_{ij}(t) = M_{ij}s(t),$$  \hfill (3.3)

where $M_{ij}$ is the constant tensor, henceforth referred to as 'moment tensor'. In all calculations in this section, we use the repetition length $L_s = L_t = 20 \text{ km}$ and the truncation of the wavenumber at components $M = N = 50$. 

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3.1 Explosion source

We will illustrate the power of the 2.5-D computational procedure by comparing three different source configurations: first the very simple case of a plane free surface and then two models of a mountain with different source positions. The cross-sections of the three cases in the plane \( y = 0 \) are shown in Fig. 3.

Rather than display the whole wavefield we will concentrate on the \( P \) and \( S \) contributions to the scattered field and will compare snapshots of the displacement components at the free surface as well as record sections parallel to the axis of the topography at 1 km x-offsets from the source position, which lies on the flank of the topography.

In the case of a plane free-surface, the scattered field represents the difference between the wavefield expected for a whole-space and that for a half-space. In consequence, the horizontal-displacement snapshots (Figs 4a, b) show a radial symmetry about the source position in the centre of the display. At shorter elapsed times, when \( P \) waves are more prominent, we can see the \( P \) wavefield associated with the reflection process move out across the free surface followed by larger scattered \( S \) waves. The locus of Rayleigh-wave energy can be recognized by the coincidence of \( P \) and \( S \) components in the field induced by the presence of the free surface. The vertical-component record sections for a fixed x-coordinate, 1 km offset from the source position (Figs 4c, d), show that the maximum contribution to the scattered \( P \)-wave component comes in a 45° cone about the source, but weaker effects persist to wider angles, where the scattered \( S \) wave is somewhat stronger. Following the \( P \)-wave disturbance with some delay is the Rayleigh wave, which is more readily seen in the scattered \( S \) wavefield.

Once topography is introduced, however, the patterns of the scattered waves become much more complex. The snapshots of the horizontal components of the scattered fields with the source immediately below the mountain range (Figs 5a, b) indicate dominant patterns perpendicular to the mountain ridge, with a dearth of energy on the crest. The patterns are very different from those without topography and indicate the profound modification to the wavefield introduced even by simple 2-D topography. The record sections of the vertical components of the scattered \( P \) and \( S \) contributions (Fig 5c, d) show a comparable modification to the snapshots. There has been an amplification of the scattered \( P \) waves within a 60° cone about the source \((|y| < 1.5)\), even for the illustrated profile which is parallel to the axis of the ridge. In the same distance range, the change in waveform for the scattered \( S \) waves is even more noticeable, and the amplitudes for small \( y \) overshadow the Rayleigh-wave contributions, which become slightly more prominent farther from the source. The apparent source of the Rayleigh waves lies at the locus of the change of slope in the topography, at \( x = 1.5 \).

With the offset explosion source illustrated in Fig. 6, the
complex influence of the mountain topography becomes more apparent because the previous symmetry in the horizontal component snapshots about $x = 0$ is removed in Figs 6(a) and (b). The patterns of the scattered field have drastically changed across the ridge axis. There is still a symmetry in $y$ in the vertical-component sections, but the details of the scattered wave patterns in Figs 6(c) and (d) with the offset source are substantially modified from Figs 5(c) and (d), where the source lies immediately below the axis of the mountain ridge. The amplitudes of the seismic disturbance in Figs 6(c) and (d) are much larger for small $y$ than those in Figs 5(c) and (d), and they decrease more rapidly with distance from the axis ($y = 0$).

3.2 Double-couple source

One of the most significant advantages of the 2.5-D calculation scheme is that calculations are performed for a point source and so it is possible for us to take into account the 3-D radiation pattern from the source.

We will illustrate such calculations for a valley structure using a strike-slip fault model at a depth of 1 km, with a strike parallel to the axis of the structure (i.e. a moment tensor $M_{xy} = M_{yx} = 1$, all other components zero). Because of the symmetry in the radiation pattern, the calculations will also be appropriate for a fault with a strike along the $x$-axis perpendicular to the topography. We will compare two sets of calculations with different horizontal source positions. In the first case, Fig. 7, the source lies directly beneath the valley; whilst in the second case, Fig. 8, the source is shifted to the flanks of the valley. The source geometries are illustrated in Figs 7(a) and 8(a). Snapshots of the horizontal component of the wavefield at the free surface are displayed for two different times after source initiation. The first snapshot, shown in Figs 7(b) and 8(b), is at 0.1 s, when the wavefield is dominated by $P$ waves. The second snapshot, shown in Figs 7(c) and 8(c), is at 0.5 s, when the $S$ wavefield is well developed. In each model, the characteristic four-lobed radiation pattern from the source can be clearly seen. Symmetry is preserved for the central source configuration (Figs 7(b) and 8(b), but the influence of the flank topography has imposed significant distortion on the displacement pattern for both $P$ and $S$ waves in Figs 8(b) and (c).
4 DISCUSSION

We have demonstrated the versatility of a (3, 2)-D wave propagation scheme for examining the 3-D character of the seismic wavefield excited by a point source in a model with 2-D topography. The discrete wavenumber-boundary integral equation (DWBIE) procedure, coupled with a Green's function decomposition into $P$- and $S$-wave contributions, provides a flexible and effective means of evaluating the seismic wavefield which can readily be used to construct a movie for various aspects of the propagation. Many of the elusive features introduced by topography are best recognized in such a dynamic presentation, rather than in the simple snapshots used here.

We have illustrated the 2.5-D DWBIE approach with simple analytic models of topography, but for this technique there is no restriction to small or simple topography. In our treatment above we have concentrated on the near-field aspects of the seismic wavefield. The procedure can, however, be readily adapted to consider far-field effects induced by near-source topography. The wavefield can be evaluated on a reference surface some way below the source and then transferred to teleseismic distances using other propagation techniques.

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**APPENDIX A: EXPLICIT EXPRESSIONS FOR GREEN'S TENSOR COMPONENTS**

The expressions for $\hat{G}$ and $\hat{H}$ in (2.14) and (2.15) are shown here.

$$\hat{G} = \begin{bmatrix} \hat{G}_{xx} & \hat{G}_{xy} & \hat{G}_{xz} \\ \hat{G}_{yx} & \hat{G}_{yy} & \hat{G}_{yz} \\ \hat{G}_{zx} & \hat{G}_{zy} & \hat{G}_{zz} \end{bmatrix}, \quad \hat{H} = \begin{bmatrix} \hat{H}_{xx} & \hat{H}_{xy} & \hat{H}_{xz} \\ \hat{H}_{yx} & \hat{H}_{yy} & \hat{H}_{yz} \\ \hat{H}_{zx} & \hat{H}_{zy} & \hat{H}_{zz} \end{bmatrix}. \tag{A1}$$

Each entry of $\hat{G}$ and $\hat{H}$ is the following discrete wavenumber representation for the corresponding component of the Green's function tensor in the mixed coordinate–wavenumber domain $(x, k_y, z)$:

$$\hat{G}_{pq}(x, k_y, z; x'_m, z'_m) = \sum_{l=0}^{\infty} \hat{G}^q_{pr}(k_{sl}, k_y, z; z'_m)CS_{lm}(x); \tag{A2}$$

$$\hat{H}_{pq}(x, k_y, z; x'_m, 0, z'_m) = \sum_{l=0}^{\infty} \left[ T^p_{ps}(k_{sl}, k_y, z; z'_m)n_s(x, z) + T^q_{ps}(k_{sl}, k_y, z; z'_m)n_s(x, z) \right]; \tag{A3}$$

where $[n_s(x, z), 0, n_t(x, z)]^T$ (T: transpose) is the normal vector of the free surface, pointing to the medium, and

$$\hat{G}^q_{pr}(k_{sl}, k_y, z; z'_m) = \left( \psi^p_{sl} \right) \frac{1}{2\mu k_y^2} \left( \begin{array}{c} \phi^q_{ps} \\ \psi^q_{ps} \end{array} \right) \left( k_{sl}, k_y, z, z'_m \right),$$

$$T^p_{ps}(k_{sl}, k_y, z; z'_m) = \frac{\psi^p_{ps}(k_{sl}, k_y, z; z'_m)}{\psi^q_{ps}(k_{sl}, k_y, z; z'_m)}CS_{lm}(x),$$

$$\hat{H}_{pq}(x, k_y, z; x'_m, 0, z'_m) = \sum_{l=0}^{\infty} \left[ T^p_{ps}(k_{sl}, k_y, z; z'_m)n_s(x, z) + T^q_{ps}(k_{sl}, k_y, z; z'_m)n_s(x, z) \right];$$


D is a layer matrix whose entries \( d_{ij}(i = 1, \ldots, 8; j = 1, \ldots, 4) \) are

\[
\begin{align*}
    d_{11} &= -d_{24}, \quad d_{12} = 0, \quad d_{13} = i \text{sgn}(z - z_m)\nu_\beta, \quad d_{14} = -d_{33}, \\
    d_{21} &= -d_{32}, \quad d_{22} = -d_{13}, \quad d_{23} = 0, \quad d_{24} = ik_{\text{st}}, \\
    d_{31} &= -i \text{sgn}(z - z_m)\nu_\alpha, \quad d_{32} = ik_\gamma, \quad d_{33} = -d_{24}, \quad d_{34} = 0, \\
    d_{41} &= \mu(2k_{\alpha}^2 - k_{\beta}^2 - 2k_{\gamma}^2), \quad d_{42} = 0, \quad d_{43} = 2d_{74}, \quad d_{44} = -2d_{82}, \\
    d_{51} &= \mu[2(k_{\alpha}^2 + k_{\gamma}^2) - k_{\beta}^2], \quad d_{52} = 2d_{63}, \quad d_{53} = -2d_{74}, \quad d_{54} = 0, \\
    d_{61} &= -2d_{82}, \quad d_{62} = -d_{54}, \quad d_{63} = \mu \text{sgn}(z - z_m)k_\gamma\nu_\beta, \quad d_{64} = \mu(k_{\alpha}^2 - k_{\gamma}^2), \\
    d_{71} &= -2\mu \text{sgn}(z - z_m)k_\gamma\nu_\alpha, \quad d_{72} = \mu(k_{\alpha}^2 + 2k_{\gamma}^2 - k_{\beta}^2), \quad d_{73} = -d_{82}, \quad d_{74} = \mu \text{sgn}(z - z_m)\nu_\gamma k_{\text{st}}, \\
    d_{81} &= -2\mu \text{sgn}(z - z_m)\nu_\gamma k_{\text{st}}, \quad d_{82} = \mu k_{\gamma} k_\alpha, \quad d_{83} = \mu(k_{\beta}^2 - 2k_{\alpha}^2 - k_{\gamma}^2), \quad d_{84} = -d_{63},
\end{align*}
\]

with

\[
    k_\alpha = \omega/c, \quad \nu_\gamma = (k_{\alpha}^2 - k_{\beta}^2 - k_{\gamma}^2)^{-1/2}; \quad \Im[
u_\gamma] \leq 0 \quad (\gamma = \alpha, \beta),
\]

\[E = \text{diag}\{\exp(-i \nu_\beta |z - z_m|), \exp(-i \nu_\alpha |z - z_m|), \exp(-i \nu_\gamma |z - z_m|), \exp(-i \nu_\gamma |z - z_m|)\}, \]

and the entries of source potential vectors are

\[
\begin{align*}
    \phi^\alpha &= k_{\alpha}/\nu_\alpha, \quad \psi^\alpha_1 = 0, \quad \psi^\alpha_2 = -\text{sgn}(z - z_m)\nu_\beta, \quad \psi^\alpha_3 = k_\gamma/\nu_\beta, \\
    \phi^\beta &= k_{\beta}/\nu_\beta, \quad \psi^\beta_1 = \text{sgn}(z - z_m), \quad \psi^\beta_2 = 0, \quad \psi^\beta_3 = -k_{\alpha}/\nu_\beta, \\
    \phi^\gamma &= \text{sgn}(z - z_m), \quad \psi^\gamma_1 = -k_\gamma/\nu_\gamma, \quad \psi^\gamma_2 = k_\alpha/\nu_\gamma, \quad \psi^\gamma_3 = 0.
\end{align*}
\]

In the expressions (A2) and (A3) we have employed x-symmetry or antisymmetry of each Green’s function tensor element to reduce the range of wavenumber summation by half (i.e. non-negative \( k_{\alpha} \) only).

The Green’s function tensor shown above can be decomposed into \( P \)- and \( S \)-wave parts. The \( P \)-wave part is derived by resetting \( \psi^\alpha_1, \psi^\beta_2 \) and \( \psi^\beta_3(q = x, y, z) \) in (A8) to zero, while the \( S \)-wave part is obtained by resetting \( \phi^\gamma \) to zero instead.

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