Supporting Information for “Receiver structure from teleseisms: auto- and cross-correlation”

Weijia Sun¹ and B.L.N. Kennett²

¹ Key Laboratory of Earth and Planetary Physics, Institute of Geology and Geophysics, Chinese Academy of Sciences, Beijing 100029, China.

² Research School of Earth Sciences, Australian National University, Canberra, ACT 2601, Australia.

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**S1. Moveout corrections in the slowness domain**

The trajectories of reflected arrivals in the time-slowness domain are approximately elliptical and when we stack the contributions from events at different epicentral distances we need to correct to a common time base, typically that for vertical incidence (zero slowness).

Consider a single interface at a depth $h_0$ with velocity $v_0$ above. The trajectory of the reflection in $\tau - p$ space [Kennett, 2001, pp 56] for vertical reflection time $\tau_0 = 2h_0/v_0$ is,

$$\tau^2 + 4h_0^2p^2 = \tau_0^2.$$  \hspace{1cm} (S1)

Rearranging the equation

$$\tau^2 + v_0^2\tau_0^2p^2 = \tau_0^2, \quad \text{so} \quad \tau^2 = \tau_0^2(1 - v_0^2p^2),$$  \hspace{1cm} (S2)

and thus

$$\tau = \tau_0(1 - v_0^2p^2)^{1/2} \approx \tau_0(1 - \frac{1}{2}v_0^2p^2 + ...).$$ \hspace{1cm} (S3)

The moveout correction for slowness $p$ is thus $-\frac{1}{2}v_0^2p^2\tau_0$. Although the negative correction for increasing slowness may appear to be counter-intuitive, since the corresponding corrections in the $t-x$ domain are positive, the slowness moveout behaviour can be clearly seen in Figure S1 for a complex velocity profile.

With the aid of the moveout correction, the traces for teleseismic arrivals with a specific slowness can be converted to be equivalent to slowness $p = 0$ indicating vertical incident below stations. The correction involves a slight stretch of the traces in time, which becomes more pronounced for later times. Rather than correct directly to $p = 0$, one can first choose a reference slowness $p_r$, e.g., $6 \text{ s/deg}$ for $P$ with slowness between $4.4$ and $8.8 \text{ s/deg}$, then correct the slowness of each phase to the reference slowness using equation (S3). The corrected traces are then stacked. An extra moveout correction is finally used to adjust the stack trace for the reference slowness $p_r$ to zero.
The effect of incident slowness on auto-correlograms

The application of Claerbout’s (1968) idea requires that waves propagate nearly vertical beneath seismic stations assuming the discontinuities are flat. Where discontinuities show topography, the method prefers the contributions perpendicular to the interface, e.g., Moho.

We illustrate the behaviour of the reflection response with slowness with synthetic calculations for a 1-D lithospheric model taken from the recent study by Kennett and Furumura [2016] of multi-scale heterogeneity across Australia. The multi-scale model employed by Kennett and Furumura [2016] includes variations in the lithospheric mantle with horizontal correlation length of several kilometers and vertical correlation length around 0.5 km. These fine scale features are needed to explain the long durations and large amplitudes of $P$ and $S$ coda recorded by stations in northern Australia from events in Australia and from the Indonesian subduction zone.

The finely sampled 1-D model is taken from a cratonic location (21°S, 119°E) in Western Australia, and shows complex behaviour similar to that seen in the observations reported in the main paper. In Figure S1 we show the reflection response including free surface effects for both incident $P$ and $S$ waves as a function of slowness across the full span appropriate to teleseismic arrivals. As in the observations the reflection response is constructed by auto-correlation of the transmitted waves, incident at 290 km depth. The strong peak near zero time is reduced by tapering.

In Figure S1, the bands of slowness associated with the main seismic phases are indicated by colour blocks behind the traces. Since the traces are represented in terms of reflection transit time $\tau$ the duration of the $S$ train is longer. The reflection responses are built up by complex interference effects between the arrivals from the impedance variations and their internal multiples. The longer delays for
S waves give somewhat more extended pulses and the apparent frequency is further reduced by the inclusion of attenuation in the calculations.

Across the full suite of teleseismic slowness there is little moveout for crustal reflections, so that Moho reflections can be expected to stack well. However, for reflections from greater depth there is more change in reflection timing as a function of slowness.

For incident $P$ waves, even for waves returned from 200 km depth (approximately 50 s two-way time) the change in moveout with slowness is slow but not entirely negligible. For very distant events, $PKP$ phases show very little moveout and so can be expected to stack coherently without any corrections; this corresponds to the stationary phase condition employed by Ruigrok and Wapenaar [2012] for a study in Tibet. For Australia, although there is a good distribution of events out to $120^\circ$, there is much less available seismicity at larger distances and hence teleseismic $P$ waves become important. Stacking is still possible provided that appropriate moveout corrections are made for each slowness value, as in the treatment in Section S1.

Even without moveout corrections, teleseismic $P$ waves are effective in rendering reflections from the Moho and later reflectivity to about 100 km depth (30 s). But, better results are obtained throughout the mantle lithosphere when moveout corrections are applied to the autocorrelograms for individual events based on their arrival slowness before stacking. The build up in the size of the moveout corrections with increasing depth means that it may become difficult to extract reflections from discontinuities deeper than 300 km even when using distant $PKP$.

For $S$ waves, the best results for $S$ reflectivity are to be obtained with $SKS$, provided there are events of sufficient size to overcome the noise. Far distant $S$ can also give good results, but the moveout corrections are quite significant and need to be applied. A complication for closer events for $S$ is that
their slownesses couple to $P$ waves in the crust, so that the apparent reflectivity includes $S$-$P$ coupling. The effects are clearly seen in Figure S1 by the complex band of arrivals for slowness greater than 12 s/deg for $S$ waves. The coupling phenomena is the same as gives rise to the shear-coupled PL phase following the onset of teleseismic $S$.

**S3. Frequency band and interpretation**

The recovered $P$ auto-correlograms (AC) have broader frequency band of 0.5–4 Hz than for receiver functions where the band employed is generally rather lower, at best 0.03–1 Hz, and often with a much lower upper limit. Higher frequencies can sample finer structures in the Earth.

We illustrate the issues in Figure S2 using the same 1-D model as above with the fine sampling interval of 0.1 km in depth. To investigate the effects from the frequency band used, we compare the $P$ reflectivity and $Sp$ receiver functions (RF) with different frequency bands in Figure S2. For the higher upper frequency limit, the $P$ auto-correlograms (0.5–3 Hz) show similar frequency characteristics to the $Sp$ receiver function (RF1) with comparable frequency band of 0.05–2 Hz.

The used 1-D model is extracted from a multi-scale heterogeneity model [Kennett and Furumura, 2016], which was designed to fit a range of different types of seismological observations. This realistic model shows complex velocity variations with a significant Moho, but no prominent reflectors below Moho. Reflections can be often tracked to wavespeed variations with changes of frequency characteristics for the higher frequency results (AC, RF1, RF2). Whereas the receiver functions with the lowest frequency bands (RF4, RF5) seem to show several discontinuities, but these are actually the result of interference of subtle arrivals from rapid wavespeed variations of the 1-D model. In this case, the low frequency components are less helpful to understand the complex structures of the Earth. Where
sufficient higher frequencies can be extracted discontinuities in realistic media can be interpreted by an abrupt change in frequency characteristics, rather than just a single prominent pulse.

References


Figure S1. Simulation of reflection response including free-surface reflections constructed using auto-correlation of the transmission response for incident $P$ and $S$ waves across the full suite of teleseismic slowness in the frequency band from 0.5–3 Hz. The slowness regimes associated with the major seismic phase regimes are indicated by blocks of colour. The synthetics are calculated for a 1-D model extracted from a multi-scale heterogeneity model for Australia [Kennett and Furumura, 2016] at the cratonic location $21^\circ$S, $119^\circ$E. The $P$ and $S$ wavespeeds are plotted as a function of vertical reflection time $\tau$ to the left of each suite of reflection traces.
Figure S2. Comparison of $P$ reflectivity (AC) and $Sp$ receiver functions (RF1-5) for the 1-D model at the cratonic location 21°S, 119°E. The $P$ reflectivity is extracted from transmission response with frequency band of 0.5–3 Hz. The $Sp$ receiver functions are shown in different frequency band: RF1: 0.05–2.0 Hz, RF2: 0.05–1.0 Hz, RF3: 0.05–0.5 Hz, RF4: 0.05–0.25 Hz, RF5: 0.05–0.125 Hz. The used 1-D model is shown at the left with red lines.