Surface Wave Tomography for the Australian Region Using a 3-Stage Approach II: 3-D Shear Wave Speed Models

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Abstract. A set of 3-D shear wave speed models for the Australian region is constructed as the final step in a three-stage inversion of surface wave information. The first step is the construction of 1-D path-specific models by waveform inversion. These models are treated as summaries of multi-mode dispersion which is used to build phase-speed maps for the fundamental and first few higher modes as a function of frequency. In the construction of the phase-speed maps a variety of effects including off-great-circle propagation and the influence zone surrounding the surface wave paths at finite frequency can be treated in a single formulation. The different styles of phase-speed maps based on different levels of approximation provide the basis for the construction of the 3-D shear wave speed distribution, which is assembled from local inversion of multi-mode dispersion for the structure beneath a cell. The three-stage approach provides a means of checking the models derived from the conventional two-stage inversion, in which 1-D path-average models directly constrain the 3-D structure, as well as providing a means of incorporating less restrictive approximations. The final 3-D model which includes the effects of ray tracing and finite frequency shows significant improvement in the regions of high gradients in shear velocity, such as near tectonic boundaries, especially in the eastern Australia. Despite the natural smoothing imposed by considering the influence zone around the surface wave paths the final models require very rapid change in shear wave properties in the neighborhood of the edge of the craton.
1. Introduction

Most work on 3-D shear wave speeds from global surface wave studies has worked through the intermediary of phase-speed dispersion maps for the fundamental mode [e.g., Nataf et al., 1986; Montagner and Tanimoto, 1990, 1991; Trampert and Woodhouse, 1995, 1996; Laske and Masters, 1996; Zhang and Lay, 1996; Ekström et al., 1997] and, to a lesser extent, higher modes [Stutzmann and Montagner, 1993; van Heijst and Woodhouse, 1997, 1999]. In contrast, regional surface wave tomography has been dominated by a two-stage approach in which path-specific 1-D models are derived by nonlinear inversion of the waveforms of surface waves and the path information is combined to construct a 3-D shear wave speed model [e.g., Zielhuis and Nolet, 1994; van der Lee and Nolet, 1997; Simons et al., 1999; Debayle and Kennett, 2000a,b]. The 3-D models are treated as path-average constraints in the 3-D distribution and a linear inversion is carried out to find a compatible 3-D wavespeed model. Regularization of the inversion is achieved either by some form of damping in cellular models or via a Gaussian smoothing profile when a continuous parameterization is employed [e.g., Montagner, 1986].

An alternative approach to regional surface wave tomography is to adopt a three-stage approach [Kennett and Yoshizawa, 2002] in which the 1-D models are regarded as a summary of path specific dispersion from which phase speed maps for the fundamental and first few higher modes can be constructed as a function of frequency. As discussed in Paper I [Yoshizawa and Kennett, 2003], this approach allows a systematic treatment of off-great-circle propagation and the inclusion of the sensitivity of the dispersion to the zone around the surface wave path, at finite frequency.
In this second paper we exploit the different sets of phase speed maps as a function of frequency for the Australian region constructed in Paper I. These phase speed maps explore the consequences of different levels of approximation for nature of surface wave propagation (i.e., with or without ray bending and finite frequency effects). We therefore generate five sets of 3-D shear wave speed models for the Australian region which reflect the different influences in the underlying multi-mode phase speed maps.

In the third step of the three-stage inversion approach local multi-mode dispersion curves are extracted in particular cells from the phase speed maps, and are inverted then for local shear wave speed profiles, which are assembled into the final 3-D shear wave speed model. This step includes a significant dependence on local crustal properties which therefore need to be known well.

The sets of 1-D models of Debayle and Kennett [2003] were used to estimate the multi-mode phase speeds in the first step of the three-stage inversion (Paper I). It is, therefore, straightforward to obtain a 3-D model using a conventional two-stage approach (e.g., partitioned waveform approach such as Nolet [1990] and Debayle and Kennett, [2000a]). This two-stage model provides a useful comparator for the models produced from the three-stage approach, particularly with regard to the influence of finite frequency at the deviations of propagation paths from the great-circle between source and receiver.

The five different sets of multi-mode phase speed models in Paper I will be used in this paper. The model descriptions used throughout this paper are summarized below:

Initial models

GC0 : models based on geometrical ray theory with great-circle approximation

GCiz : models incorporating effects of finite frequency with great-circle approximation
Updated models

Ray-GC0: models updated from GC0 incorporating effects of ray bending only

Riz-GC0: models updated from GC0 incorporating effects of both ray bending and finite frequency

Riz-GCiz: models updated from GCiz incorporating effects of both ray bending and finite frequency

Readers may refer to Figure 6 in Paper I for more detailed relations among these models.

2. Multi-Mode Phase Speed Maps and 3-D Shear Wave Speed Structures

Local multi-mode dispersion data carry valuable information of a local shear wave speed structure, and thus a set of multi-mode phase speed maps can be used to exploit a final 3-D wave speed model.

The relation of multi-mode phase dispersion and local one-dimensional structure can be represented by a linearized relation as [e.g., Takeuchi and Saito, 1972; Dahlen and Tromp, 1998],

\[
\frac{\delta c(\omega)}{c} = \int_0^R \left\{ K_\rho(\omega, z) \frac{\delta \rho(z)}{\rho} + K_\alpha(\omega, z) \frac{\delta \alpha(z)}{\alpha} + K_\beta(\omega, z) \frac{\delta \beta(z)}{\beta} \right\} dz, (1)
\]

where \( \delta \rho, \delta \alpha \) and \( \delta \beta \) are the perturbations of density, P wave speed and shear wave speed, respectively, \( \delta c \) is perturbation of the phase speed and \( R \) is the radius of the Earth. The sensitivity kernels represents partial derivatives of phase speed with respect to each model parameters, i.e., \( K_\rho = (\rho/c) \frac{\partial c}{\partial \rho} \), \( K_\alpha = (\alpha/c) \frac{\partial c}{\partial \alpha} \) and \( K_\beta = (\beta/c) \frac{\partial c}{\partial \beta} \).

Examples of the sensitivity kernels \( K_\rho, K_\alpha \) and \( K_\beta \) for the first three Rayleigh modes at periods of 50 s and 100 s are plotted in Figure 1. In all cases, sensitivity of shear wave speed is dominant and other two components have rather smaller sensitivity and have only
small influence on the phase speed variation. The sensitivity of the fundamental mode are usually confined only in the shallower part of the mantle, whereas that of the higher modes reach much deeper part of the mantle. As the period of surface waves increases, the maximum depth of the sensitivity also increases and the absolute amplitude is somewhat reduced.

In general, the effects of density and P wave speed on a Rayleigh wave phase speed perturbation is not very significant compared to the influence of shear wave speed, especially, in the intermediate frequency range used in this study. Thus we fix the density and P wave speed structures and only a perturbation of shear wave speed is considered in the inversion for a 1-D model.

We should emphasize that the 1-D models in stage 1 are path-specific and are employed to estimate multi-mode dispersion, whereas in the third stage we are interested in an ensemble of local 1-D wave speed models to assemble the 3-D model. The local 1-D models are derived from a nonlinear inversion of local dispersion for a number of modes. The results are sensitive to the local shallow structure and therefore we need good representations for the crustal structure across the region.

Throughout this study, we use the crustal structure of the 3SMAC model [Nataf and Ricard, 1996], which defines local wave speed structures at 2° × 2° cells with some local corrections. The use of a global crustal model such as 3SMAC is convenient, but even with the corrections it may be an inadequate representation of the true structure in the Australian region. A systematic development of a detailed model of crustal structure is being made, but the 3SMAC model is retained here for comparison with previous studies.
since the emphasis is on the influence of different approximations in carrying out the inversion for 3-D structure.

3. Non-linear Inversion for Local Shear Wave Speed Models

3.1. Method of Inversion

The third stage of the three-stage process is to invert for a set of local shear wave speed model using local multi-mode dispersion information assembled from a set of phase speed maps derived in the second stage (Paper I).

The iterative least-squares inversion scheme of Tarantola and Valette [1982] is used to invert multi-mode dispersion data for a local 1-D shear wave speed model. This nonlinear inversion procedure has been widely adopted in a number of surface wave studies [e.g., Montagner and Jobert, 1981; Nataf et al., 1986; Cara and Lévêque, 1987; Nishimura and Forsyth, 1989]. The detail of the method is explained in Appendix A, and here we explain only the practical application of the method, especially focusing on the choice of a priori smoothing parameters.

With this iterative method, the smoothness of the model is controlled by a priori information, which is used in the linearized inversion as a model covariance (A5). The standard deviation $\sigma$ in (A5) constrains the amplitude of variations of model parameters, and the correlation length $L$ controls the smoothness of the model variations with depth, and so determines how rapidly the model can vary as a function of depth.

Previous studies have shown that the shear wave speeds in the upper mantle have a peak to peak variation around 0.5 km/s over a 100 km interval in depth [e.g., Nishimura and Forsyth, 1989]. We prefer models which vary smoothly with depth but allow some degree of rapid variation so that we can treat quick changes with large wave speed perturbation.
Therefore, we choose the value of correlation length $L \approx 20$ km, and the $\sigma \approx 0.1$ km/s over the depth range below the Moho. For the shallower structure above the Moho, the correlation length is chosen to be $L \approx 5$ km so that more rapid variation in the shallower structure is allowed.

As the reference models used to initiate the inversion, we use PREM [Dziewonski and Anderson, 1981] for the oceanic region and PREMC, whose upper mantle structure is modified to represent the continental structure, for the continental region. Both the PREM and PREMC models are adapted to have smooth variation across the 220 km, 400 km, and 670 km boundaries. Prior to the inversion, the crustal structure of the reference model is corrected by using the 3SMAC model [Nataf and Ricard, 1996].

3.2. Local Shear Wave Speed Models

The local phase dispersion curves are assembled from the multi-mode dispersion maps at $2^\circ$ grid points along both longitude and latitude, and then inverted for local shear wave speed structures.

The number of iterations required for the satisfactory convergence of the inversion varies with the choice of reference model and a priori information. With an appropriate choice of the reference model to start the inversion in (A2), the model can converge in the first few iterations, since the nonlinearity between the phase dispersion and shear wave speed is quite moderate. However, typical 1-D shear wave speed models in the Australian region often lie quite far from PREM, with significant velocity perturbations in the upper 250 km.

We therefore repeat the whole process of inversion (A2) using updated reference models which are derived from the previous inversions. This allows us a good recovery of large
velocity perturbations which sometimes reach about ±10% from PREM. In most cases, the shear wave speed models converges at satisfactory levels within the first 5 to 10 iterations.

We display some examples of the local 1-D shear wave speed profiles along a latitudinal line (24°S) and a longitudinal line (130°E) for the locations shown in Figure 2.

The 1-D shear wave speed models for Riz-GCiz along the 24°S latitude across the middle of the Australian continent are shown in Figure 3. We can see significant variations in the shear wave speed structure across the continent, especially above 300 km. In the western region, there are prominent high velocity anomalies in the upper 300 km corresponding to the Proterozoic and Archaean cratonic region in the western Australia. Whereas, a conspicuous set of low velocity anomalies are seen from 145° to the east representing slow velocities in the Phanerozoic region of the eastern Australia and the Coral Sea.

The other set of 1-D shear wave speed models along a N-S line at 130° E longitude is displayed in Figure 4. We can again see the high velocity anomalies near the center of the Australian continent in the upper 300 km. Along this longitude, there is a continent-ocean boundary around 32°S latitude, and we can identify the slower velocities to the south of the boundary and the faster anomalies to the north suggesting the rapid changes in the shear wave speed at the boundary. The slower anomalies around 15°S correspond to a patch of slower wave speeds near the northern edge of Australia which can be seen in the phase speed maps (Figure 2).

The resolution kernels for a typical 1-D shear wave speed profile are shown in Figure 5. It is apparent that the shallower parts of the 1-D structure from 100 to 250 km are comparatively well constrained, whereas the deeper parts of the upper mantle are not well resolved even though we include information of up to the third higher mode. This is
mainly due to the smaller absolute sensitivity of the higher modes to the shear wave speed structure compared to that of the fundamental mode. Still we can see some sensitivity around 400 km depth where it is almost impossible to resolve structure with just the fundamental mode at the maximum period of 150 s used in this study.

4. 3-D Models in the Australian Region

We obtain the final 3-D models by repeating the inversions for the local shear wave speed profile across the whole region. The minimum scale length of the lateral heterogeneity that can be resolved by our phase speed inversion is longer than a few hundred kilometers, corresponding to the minimum wavelength. We therefore assemble the local phase dispersion curves for the set of modes for each $2^\circ \times 2^\circ$ cell and invert them for the local shear wave speed structure. We have obtained five types of 3-D shear wave speed models from the corresponding sets of phase speed models as explained in the end of section 1.

In this section, we mainly focus on the comparison of these 3-D models, especially how the models can be improved by considering the influence zone of surface wave paths. A 3-D model derived from a 2-stage approach is also displayed for the comparison of the models derived from different inversion techniques.

4.1. Comparison of 3-D Models I: Two-Stage and Three-Stage Approaches

Although the three-stage approach for surface wave tomography is able to provide us with a number of benefits for improving 3-D models, the process of obtaining a final 3-D model is rather indirect compared to the conventional two-stage approach. Therefore, a comparison of the 3-D models derived from the two-stage and three-stage inversions is
helpful for assessing how the differences in the inversion processes affect the final 3-D models.

To compare the models derived from different types of inversion scheme, we first obtain 3-D models using a form of two-stage inversion scheme with the direct use of the path-specific 1-D profiles of Debayle and Kennett [2003] that are used to estimate multi-mode phase speeds in this study. Following arguments in the appendix to Debayle and Kennett [2000a], a linear relation for a path-average shear wave speed perturbation at a particular depth \( z \) can be given as,

\[
\frac{\langle \delta \beta \rangle_{\text{obs}}}{\beta} \Bigg|_{z} = \frac{1}{\Delta} \int_{g.c.} \delta \beta(s) \Bigg|_{z} ds.
\]  

(2)

This linear relation can be solved in the same way as the phase speed inversion explained in section 3 of Paper I, using the LSQR algorithm [Paige and Saunders, 1982] for a damped least-squares problem.

The 3-D model derived from the two-stage inversion scheme shows similar behavior in the trade-off between the misfit and model norm, and thus we choose an appropriate damping parameter which shows trade-off behavior similar to Figure 7 in Paper I. Variance reductions achieved through the direct use of the path-average 1-D models are more than 70 % for the models above 200 km, whereas, for the models below 250 km, the variance reduction are achieved around 40 %. However, the model explains the data quite well with respect to the estimated errors.

The 3-D models derived from the two-stage and three-stage approach are displayed in Figure 6. The three-stage model has been derived from a set of phase speed models \( \text{GC0} \), whereas the two-stage models are directly retrieved from the path-specific 1-D profiles.
Despite the intrinsic differences in the inversion processes, the final 3-D models are extremely well correlated. The geographical correlation coefficients of these velocity structures exceed 0.95 at all depths. Resolution maps for the two-stage models are similar to those shown in Figure 13 (a,c,e) in Paper I depending on the depth. Although the process of recovering these models are different, the underlying assumptions are the same. That is, all the surface wave paths are assumed to lie along the corresponding great-circle and no finite-frequency effects are considered. The similarity of these models indicate that the roundabout route to reach the final 3-D model in the three-stage approach (i.e., the use of the intermediary of the phase speed maps) does not bring in any noticeable error in the final models for the intermediate frequency range used in this study. This means that we can also expect to gain from the application of the three-stage approach when we bring in additional aspects of surface wave propagation such as frequency dependent off-great-circle propagation and the influence zone around the paths.

It should be noted that the appearance of the 3-D models is slightly different from that of Debayle and Kennett [2000a], because of differences of the data set. However the 3-D models in Figure 6 are quite similar to those of Debayle and Kennett [2003] whose dataset is utilized in this paper, despite the different inversion technique.

4.2. Comparison of 3-D Models II: Three-Stage Models

Using the five sets of phase speed models in Paper I, we obtain five types of corresponding 3-D shear wave speed models. In this section, we carefully examine the differences between these models.

4.2.1. Differential maps at 120 km depth. The shear wave speed models at 120 km depth for the different types of 3-D models are shown in Figure 7. The major features
of these models are quite similar to those of the corresponding phase speed maps in Paper I, sharing the similar patterns of the fast and slow wave speed anomalies.

In order to reveal the differences between these models, differential velocity perturbation $\Delta \beta / \beta_0$ between models are estimated by,

$$\frac{\Delta \beta_{ab}(x)}{\beta_0} = \frac{\delta \beta_a(x)}{\beta_0} - \frac{\delta \beta_b(x)}{\beta_0} = \left\{ \frac{\beta_a(x) - \beta_0}{\beta_0} - \{ \beta_b(x) - \beta_0 \} \right\}$$

$$= \frac{\beta_a(x) - \beta_b(x)}{\beta_0},$$

(3)

where $a$ and $b$ denote two arbitrary heterogeneous maps, and $\beta_a(x)$ and $\beta_b(x)$ are shear wave speeds at a geographical location $x$ in these models.

Examples of differential maps with several combinations of five models are shown in Figure 8. Figure 8 (a), (b) and (c) show differential maps between updated models (including ray tracing) and the corresponding starting models, whereas (d) shows a differential map between updated models which include the effects of both ray bending and the influence zone.

It is apparent that the differences between $Riz-GC0$ and $GC0$ (Figure 8 (b)) are the most conspicuous among all the differential maps; while there is almost no differences (less than $\pm 0.5\%$) between updated models $Riz-GCiz$ and $Riz-GC0$ (Figure 8 (d)) even though these two models are updated from different reference heterogeneous models. This fact suggests that the process of global iteration (i.e., the update process for the models) using the influence zone gives quite consistent results even when we use different initial models.

Since both models $Riz-GCiz$ and $Riz-GC0$ are derived considering ray tracing and finite-frequency effects of surface waves in the updating processes, these two can be considered as
the best models derived with our three-stage inversion scheme. Therefore, the significant
differences between $Riz-GC0$ and $GC0$ in Figure 8 (b) suggest that the great-circle model
$GC0$, which is only based on the geometrical ray theory, has a number of erroneous
features in its velocity distribution that are revealed as relatively large differences in
velocity perturbation ($\pm 1 \sim 2\%$).

Even when we consider the inclusion of just off-great-circle propagation, such errors in
the models may not be sufficiently reduced, as can be seen from the comparison of Figure
8 (a) and (b). The models based simply on the geometrical ray theory shown in Figure 7
(a) and (b) have rather similar patterns of heterogeneities and their amplitude. Thus the
model $Ray-GC0$, which is updated just by ray tracing, shows subtle differences from $GC0$
in most region (Figure 8 (a)), except for the western margin of the Australian continent
where the ray coverage is poor. This suggests that the updating process considering only
ray path deviation is not, by itself, enough to improve the tomography models. In other
words, the assumption of the surface wave propagation along the great-circle works quite
well at the period longer than 40 seconds used in this study. This can also be expected
from the extensive results of two-point ray shooting experiments of Yoshizawa [2002],
which show that the actual ray paths with the minimum travel times in the sense of
Fermat’s principle, are not very far away from the corresponding great-circles in many
cases.

On the other hand, the models derived including the influence zone around the surface
wave paths (Figure 7 (c), (d) and (e)), share some similar features of somewhat smoothed
heterogeneities, regardless of the inclusion of effects from the off-great-circle propagation.
Figure 8 (c) is a differential map between the best updated model $Riz-GCiz$ and a finite-
frequency great-circle model $GCiz$. The differences are less than $\pm 1\%$ and are barely noticeable, suggesting that the finite-frequency model $GCiz$ is rather similar to the model $Riz-GCiz$ even without ray tracing. Therefore, it would be fair to say that the inclusion of the finite-frequency effects based on the influence zone will be more important contribution to the improvement of tomographic models than the ray tracing for the intermediate frequency range adopted in this study.

There is an interesting feature in Figure 8 (b). Major differences between $Riz-GC0$ and $GC0$ are somehow confined in the continent and little clear differences are seen in the oceanic region to the east of the Australia. Such large velocity differences in the continent are mainly found in particular regions where the velocity gradients are relatively large, such as near the western and eastern margins of the continent and around craton in the central Australia.

**4.2.2. Effects of the influence zone and structural implications.** The major features in Figure 8 are common in other slices of 3-D models at different depths, that is, we can see noticeable differences between models $Riz-GC0$ and $GC0$. In this section, we make further comparisons of these two models (Figure 9) focusing particularly on how the influence zone affects the models, and investigate the structural implications inferred from the best models $Riz-GC0$ and $Riz-GCiz$.

In Figure 9, we can see that some small patch-like features in the model $GC0$ are smoothed out in the $Riz-GC0$. This is especially apparent near the faster velocity anomalies beneath the Proterozoic blocks in the center of the Australian Continent, and also in the slower velocity regions in the Coral Sea and Tasman Sea to the east, off-shore of Australia. The smoothing effects in the finite-frequency model $Riz-GC0$ become clearer
in the deeper parts of the mantle (≥ 150 km). This can be attributed to the fact that the structures at these depths are constrained mainly by the higher modes and long-period fundamental modes whose influence zones are wider than those for the short-period fundamental mode.

The effect of the smoothing caused by the finite-width of the rays is also preserved in the cross sections shown in Figure 10, especially in the region where there are noticeable smoothing in the heterogeneity in the map projections in Figure 9, i.e., beneath central Australia and the oceanic region to the east of Australia.

Differential cross sections between these two models are shown in Figure 11. The most striking feature is that the differences in the east of Australia (around 148°E and 20° ∼ 30°S) exist down to 400 km depth, whereas other noticeable differences of velocity distribution in cratonic region in the central and western Australia are confined in the top 200 km.

In both models in Figure 9, we can clearly see higher velocity anomalies down to depths of 200 to 250 km beneath the middle and western parts of Australia, corresponding to the continental lithosphere of the Australian Continent. The depth of the root of such continental lithosphere can be estimated from the largest velocity gradient in the wave speed profiles. In a region just beneath the Proterozoic blocks in the central Australia (around 20°S and 132°), the continental lithosphere seem to reach 300 km. This is quite consistent with the results of Simons et al. [1999] who have also suggested that the higher wave speed anomaly in this region is likely to persist to depth over 300 km. Such feature is clearer in a full set of map slices of the model Riz-GCiz (Figure 12), which shares an
almost identical wave speed distribution with \textit{Riz-GC0}. We can track the faster wave speed anomaly in the center of the continent down to 300 km depth.

In Figure 10 (c), we can also identify a relatively faster wave speed anomaly in the north of the Australian Continent (around 130°E and 0° $\sim$ 10° S), which corresponds to the subduction beneath the Indonesia. Also we can see fast velocity anomalies beneath New Zealand corresponding to the subducting plate (Figure 10 (d)), although, unlike travel time tomography of body waves, surface wave tomography do not have very good resolution to recover the detailed nature of the downgoing plate.

One of the major difference from the models of \textit{Simons et al.} [1999] is that our models do not show extreme high wave speed anomalies in the eastern Australia beneath 250 km around 140° $\sim$ 150°E and 20° $\sim$ 25°S. At this depth, the wave speed perturbation from the PREM become very small beneath the Australian Continent and the maximum perturbation do not exceed $\pm$2% in our models. The cause of the differences can be attributed to the differences in the data analysis, that is, the waveform fitting procedures are essentially different. Our data set of path-specific 1-D models of \textit{Debayle and Kennett} [2003] have been derived from fitting cross-correlograms as secondary observables as we have explained in Paper I, whereas the \textit{Simons et al.} [1999] have used a procedure of fitting the multi-mode waveforms directly for path-specific 1-D models [e.g., \textit{Nolet}, 1990], which is more sensitive to the choice of initial models to start the inversion [\textit{Hiyoshi}, 2001].

Higher wave speed anomalies in the western Australia just beneath the NWAO station, corresponding to the root of the Archaean craton seem to get fainter at a depth of 250 km of \textit{Riz-GC0} (Figure 9 (h); Figure 10 (d) at 30°S). Earlier studies [\textit{Simons et al.}, 1999; \textit{Debayle and Kennett}, 2000a] have shown high wave speed anomalies at this depth, similar
to the model GC0. We may say the weakened higher wave speed anomaly in this region in the model Riz-GC0 can be caused by the inclusion of the effects of ray-path bending and the influence zone about the paths.

However, with the presently available path coverage, this region has limited resolution because of the sparse path coverage. Therefore we need to be careful about discussing the structure in the western blocks of the Australian Continent. Even so, the ray tracing experiments shown in Yoshizawa [2002] suggest that conspicuous ray path deviations from the great-circle are likely to appear in the paths from the Tonga-Kermadec region to the NWAO station. Therefore, the use of ray tracing in the estimation of the phase speed maps should play a role to suppress some undesirable effects caused by large ray path deviations at the NWAO station.

5. Discussion

In this paper, we have completed the development of the three-stage inversion scheme of Kennett and Yoshizawa [2002] by obtaining final 3-D shear wave speed models based on the sets of multi-mode phase speed maps as a function of frequency developed in Paper I.

Although the three-stage inversion scheme is a rather indirect approach for recovering the 3-D shear wave speed models compared to the conventional two-stage approach, our experiments with inversion for 3-D models based on both two-stage and three-stage approaches using the identical data sets suggest that the use of phase speed maps as intermediaries does not cause significant errors in the final 3-D model.

The advantage of the three-stage approach is that various types of information can be combined in a common framework to form a final 3-D model. The polarization anomaly due to ray path bending and the influence zone [Yoshizawa and Kennett, 2002b] that takes
account of finite frequency effects of surface waves can be efficiently brought together by working with multi-mode phase speed maps at each frequency.

In practical applications, the three-stage inversion scheme requires the computation of a number of phase speed maps to better constrain the final 3-D models, followed by inversions for local shear wave speed models. Therefore, in total, the three-stage approach requires more computation than for the two-stage process, even though our approach is still efficient enough to treat the complicated phenomena of off-great-circle propagation and finite frequency effects.

The final 3-D shear wave speed models, which are obtained using the influence zone, retain naturally smoothed characters in corresponding multi-mode phase speed maps that are derived from the incorporation of finite frequency effects. Besides, the models which are updated from a different initial models show extremely similar patterns of shear wave speeds variation, which implies that independence of updated models from the starting models.

The three-stage approach enable us to include slightly higher frequencies of surface waves since the effects of ray bending caused by moderate lateral heterogeneity can be treated through the updating process of phase speed maps. Such effects is particularly important for the fundamental mode, which samples the shallower layers of the Earth with stronger heterogeneity.

In order to go to higher frequency ranges than those used in this paper, a careful treatment of the effects of the mode-branch coupling as well as of crustal structure will be required to avoid undesirable effects on the tomography models [Kennett and Nolet, 1990; Kennett, 1995]. The treatment of mode-branch coupling in a full 3-D structure is
still too complex for the practical use, because different directions of propagations for all the scattered waves in a 3-D structure must be considered [Kennett, 1998].

The three-stage inversion scheme has considerable potential for further development. Future studies will address the recovery of azimuthal anisotropy, the use of Love and Rayleigh waves and the simultaneous use of group and phase speed information.

Anisotropy in the mantle can be treated in a framework of the three-stage approach utilizing anisotropic ray tracing [Tanimoto, 1987; Larson et al., 1998]. With the use of finite frequency kernels, we can expect that the azimuthal anisotropy will also be affected by the smoothing effects of finite frequency in a similar way to the smoothing of velocity structure.

We have only worked with Rayleigh wave observations to constrain the isotropic shear wave speed models, for the preliminary application of the three-stage approach. Although higher-mode phase speed measurements for Love waves tend to be more ambiguous compared to Rayleigh waves due mainly to the significant overlap of the different modes and a low signal-to-noise ratio in horizontal components, path-specific 1-D models derived from appropriate waveform fitting for Love waves can also give satisfactory results of multi-mode phase speeds for the first few modes, as we have shown in Yoshizawa and Kennett [2002a]. Independent treatment of Rayleigh and Love waves will allow us to exploit the possible maximum path coverage for both types of waves, which will be essential to obtain high resolution 3-D $SV$ and $SH$ velocity models and the three-dimensional distribution of polarization anisotropy [cf., Debayle and Kennett, 2000b].

The use of phase speed depends on the knowledge of the source mechanisms, which is crucial for the accuracy of the measured phase speed. On the contrary, group speed
measurements do not require source mechanism information and give an effective way of producing stable and reliable measurements of fundamental mode dispersion [e.g., Levshin et al., 1992; Ritzwoller and Levshin, 1998]. Group speed information has different sensitivities to the depth from the phase speed, Therefore, there are advantages in working with both the phase and group speed information simultaneously to constrain the final 3-D shear wave speed models.

For the fundamental-mode with appropriate path lengths (longer than 50°), we can also make observations of the arrival angle anomalies of Love and Rayleigh waves using three-component seismograms [Laske and Masters, 1996; Yoshizawa et al., 1999]. Such polarization information, which has more sensitivity to the smaller-scale heterogeneity than phase information, can also be incorporated within the framework of the three-stage approach. Since polarization anomaly is sensitive to a velocity gradient, it can also be a significant constraint on the size of velocity perturbations.

Long-wave length structure, which dominates most of global models, cannot be derived simply from regional scale studies. Therefore there are benefits in using phase speed maps at global scales as starting models for the regional tomographic inversion of phase speeds, which will enable us to combine tomography models at different scales.

The three-stage approach for surface wave tomography gives us significant benefits in retrieving accurate 3-D shear wave speed models. The concept of the influence zone about the propagation path allows us an efficient treatment of the finite-frequency effects for the recovery of multi-mode phase speed maps with a reasonable approximation of physical properties of surface wave propagation, and is of significant help for improving surface wave tomography.
Appendix A: Formulation of Generalized Nonlinear Inversion for 1-D models

In this appendix, the inversion for the shear wave speed structure is formulated using the least-squares generalized inversion scheme of Tarantola and Valette [1982]. Since the nonlinear inversion procedure of Tarantola and Valette [1982] has been widely used in surface wave studies [e.g., Montagner and Jobert, 1981; Nataf et al., 1986; Cara and Lévéque, 1987; Nishimura and Forsyth, 1989], and has already been explained in many papers, we only briefly explain the method of inversion.

We consider a problem where the observed data \( d \) is represented as a function of model parameters \( p \) as follows,

\[
d = g(p),
\]

where \( d \), in this case, consists of a set of local multi-mode phase speed perturbation \( \delta c(\omega) \) as function of \( \omega \). The model parameter vector \( p \) consists of a 1-D profile of the local shear wave speed perturbation \( \delta \beta(z) \) as a function of depth \( z \). The phase speed perturbation depends also on the P wave speed and density, but in the intermediate period range, which is of interest in this study, there are only slight effects from these parameters. Therefore, we concentrate on the shear wave speed perturbation in this study.

With an appropriate reference model \( p_0 \), a model parameter vector at the \((k + 1)\)th iteration for an underdetermined problem can be extracted as [Tarantola and Valette, 1982],

\[
p_{k+1} = p_0 + C_{pp} G_k^T \left( C_{dd} + G_k C_{pp} G_k^T \right)^{-1} \left[ d - g(p_k) + G_k (p_k - p_0) \right],
\]

where \( p_k \) is the model vector at the \( k \)th iteration, \( C_{dd} \) is the \textit{a priori} data covariance matrix, and \( C_{pp} \) is the \textit{a priori} model covariance matrix. \( G_k \) is the kernel matrix which
consists of the partial derivatives of the data with respect to the model parameters with components, for the $i$th data and $j$th model parameter,

$$G_{ij} = \frac{\partial d_i}{\partial p_j}.$$  \hspace{1cm} (A3)

In this application, (A3) represents the partial derivatives of phase speeds with respect to shear wave speeds.

The model resolution matrix for the algorithm of Tarantola and Valette [1982] is given by [Montagner and Jobert, 1981],

$$R = C_{pp} G_k^T \left( C_{dd} + G_k C_{pp} G_k^T \right)^{-1} G_k.$$  \hspace{1cm} (A4)

The nature of the resolution matrix $R$ is the same as that discussed in the previous section for the inversions at the second-stage. By investigating the rows of $R$, we can check how well the model parameters has been resolved.

We assume that the $a$ priori model covariance matrix $C_{pp}$ can be represented by a Gaussian distribution as follows,

$$C_{pp}(r_i, r_j) = \sigma(r_i)\sigma(r_j) \exp \left\{ -\frac{1}{2} \frac{(r_i - r_j)^2}{L^2} \right\},$$  \hspace{1cm} (A5)

where $r_i$ is the depth of the $i$th model parameter, $\sigma(r_i)$ is the standard deviation for the $i$th model parameter and $L$ is the average correlation length between the model parameters $r_i$ and $r_j$.

The standard deviation $\sigma$ constrains the amplitude of variations of model parameters, and the correlation length $L$ controls the smoothness of the model variations with depth, and so determines how rapidly the model can vary as a function of depth. The choice of these parameters depends simply on one’s preference. The $a$ priori parameters used in this study is explained in detail in Section 3.
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References


**Figure Captions**

**Figure 1.** Sensitivity kernels $K_\rho, K_\alpha, K_\beta$ for the fundamental and first two higher modes of Rayleigh waves at (a) 50 s and (b) 100 s.

**Figure 2.** The locations of the local shear wave speed models in the east-west section in Figure 3 are shown as purple dots and those in the north-south section in Figure 4 as red triangles, superimposed on the Rayleigh wave phase speed map for the fundamental mode at 100 s.

**Figure 3.** Local shear wave speed models along latitude 24°S with varying longitude. The locations of these models are indicated as purple dots in Figure 2.

**Figure 4.** Local shear wave speed models along longitude 130°E with varying latitude. The locations of these models are indicated as red triangles in Figure 2.

**Figure 5.** Resolution kernels for shear wave speed structure beneath the location (24°S, 140°E).

**Figure 6.** 3-D shear wave speed models in the Australian region at depths from 100 to 250 km with 50 km increment. The model in the left column is derived from the two-stage approach and that in the right column from is the GCO three-stage model derived from the great-circle approximation. The reference wave speeds are 4.41 km/s at 100 km, 4.43 km/s at 150 km, 4.51 km/s at 200 km, 4.61 km/s at 250 km.

**Figure 7.** Five types of shear wave speed models at 120 km derived from the corresponding phase speed maps in Figure 9 of Paper I based on the three-stage approach.
Figure 8. Differential maps at 120 km between shear wave speed models in Figure 7; (a) *Ray-GC0* and *GC0*, (b) *Riz-GC0* and *GC0*, (c) *Riz-GCiz* and *GCiz* and (d) *Riz-GCiz* and *Riz-GC0*.

Figure 9. 3-D shear wave speed models derived from the great-circle approximation based on the geometrical ray theory (*GC0* in the left column) and those updated from *GC0* including the effects of off-great-circle propagation and the influence zone (*Riz-GC0* in the right column) at the depth from 100 to 250 km with 50 km increment. Reference wave speeds are the same as Figure 6.

Figure 10. Cross sections of 3-D shear wave speed models in Figure 9. (a) N-S cross sections through varying longitudes for the model *GC0*. (b) E-W cross sections through various latitudes for the model *GC0*. (c) Same as (a) but for the model *Riz-GC0*. (d) Same as (b) but for the model *Riz-GC0*.

Figure 11. Differential cross sections between shear wave speed models *Riz-GC0* and *GC0* corresponding to Figure 10.

Figure 12. Shear wavespeed model *Riz-GCiz* in the upper mantle. Reference velocities are 4.41 km/s at 100 km, 4.43 km/s at 150 km, 4.51 km/s at 200 km, 4.61 km/s at 250 km, 4.70 km/s at 300 km and 4.75 km/s at 350 km.
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