Practical Probabilistic Seismic Risk Analysis: A Demonstration of Capability

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INTRODUCTION

Smith (2005) presents a compelling argument encouraging seismologists to move beyond probabilistic seismic hazard analysis and embrace seismic risk analysis. According to Smith (2005) the results of a seismic risk analysis may take several forms (*e.g.,* average annualized loss or exceedance probability curves) whereby all forms can be derived from "probabilistic estimates of losses for specific portfolios of assets." This paper demonstrates how Australian government agencies already are achieving Smith's proposal by using probabilistic seismic risk analysis (PSRA) to model earthquake risk in Australian cities. The paper provides a brief description of the process for conducting a PSRA and demonstrates typical results for two Australian cities, Newcastle and Perth, located on the eastern and western seaboards, respectively. This work has been conducted at Geoscience Australia, an Australian federal government agency, within a risk modeling group that focuses on the assessment of risk for a range of natural hazards. Experts within the group include seismologists, geophysicists, mathematicians, structural engineers, software engineers, economists, and social scientists. The work has benefited greatly from collaboration with members of the global re-insurance sector, something that also was encouraged by Smith (2005).

The methodology outlined in this manuscript has been coded into a comprehensive tool for PSRA and probabilistic seismic hazard analysis (PSHA) known as the Geoscience Australia's Earthquake Risk Model (EQRM). A complete description of the approach can be found in Robinson *et al.* (2005), and detailed examples of the work can be found in Fulford *et al.* (2002) and Sinadinovski *et al.* (2005) for the Newcastle and Perth regions respectively.

METHOD: UNDERTAKING A PROBABILISTIC SEISMIC RISK ANALYSIS (PSRA)

This section compares the event-based approach for PSHA and PSRA with traditional approaches for PSHA and provides a brief description of how building damage can be calculated for an earthquake scenario (*i.e.,* a single earthquake) using the approach adopted by HAZUS (FEMA 1999). It also demonstrates that the generation and use of a synthetic earthquake catalog can facilitate an approach to both PSHA and PSRA that allows a complete and consistent categorization of risk and hazard.

Traditional and Event-based Approaches to PSHA and PSRA

A typical formulation of a PSHA begins with

$$\frac{P[A > a^* \text{ in time } t]}{t} \approx \sum_i v_i \int\int G_{A|m,r}(a^*) f_M(m) f_R(r \mid m) dm dr \quad (1)$$

where a^* refers to a specified amplitude of interest (*e.g.*, PGA), v_i is the number of earthquakes per year in source zone i, t is time, m is magnitude, and r is distance (*e.g.,* McGuire and Arabasz 1990). The presence of the capitals A, M, and R suggests that the amplitude, magnitude, and distance are treated as random variables. The probability density function for earthquake magnitude $f_M(m)$ can be derived from any recurrence relationship. For example, the bounded Gutenberg-Richter recurrence relationship is widely used (*e.g.,* Kramer 1996 or McGuire and Arabasz 1990). It is a variant of the frequency relationship introduced by Gutenberg and Richter (1944). The probability density function for distance $f_R(r|m)$ is conditional on magnitude and depends on the unique geometry of source-to-site configurations. Cornell (1968) describes how $f_R(r|m)$ can be computed for annuli around a site of interest. The quantity $G_{A|m,r}(a^*)$ is the conditional exceedance probability of ground motion amplitude given by

$$G_{A|m,r}(a^*) = P(A \geq a^* \mid m,r) \quad (2)$$

and can be evaluated by assuming that the ground motion is log-normally distributed (*e.g.,* Campbell 2003) with log-mean $\mu_{\ln(A)}$ and log-standard deviation $\sigma_{\ln(A)}$ such that

$$G_{A|m,r}(a^*) = 1 - \Phi\left(\frac{\ln(a^*) - \mu_{\ln(A)}}{\sigma_{\ln(A)}}\right). \quad (3)$$

In equation 3, Φ is the standard Gaussian cumulative distribution and $\ln(a^*)$, $\mu_{\ln(A)}$, and $\sigma_{\ln(A)}$ typically are given by empirically derived attenuation formulae (*e.g.,* Toro *et al.* 1997; Atkinson and Boore 1997). A PSHA can be undertaken using attenuation formulae for bedrock (*e.g.,* Frankel *et al.* 2000) or attenuation formula that contain site-specific information (*e.g.,* Stirling *et al.* 2002). In either case an explicit form for equation 3 can be computed and hence the PSHA calculation is possible via equation 1. Cramer (2003) describes an approach for modifying ground motion attenuation formulae in such a way that the entire probability distribution for site effects can be incorporated in the PSHA.

In practice, computing the PSHA via the standard approach involves selecting a site of interest and evaluating equation 1 for a desired range of a*. The process can be repeated for a number of sites and the ($P[A > a^*$ in time $t]$, a^*) pairs sorted to produce hazard maps for the desired return periods. A consistent approach for PSRA requires a conditional exceedance probability for building damage that incorporates the probabilistic nature of ground motion attenuation, site effects, and building damage. As with PSHA, this process can be repeated for many buildings to produce ($P[A > a^*$ in time $t]$, a^*) pairs where a^* now represents the level of loss at each building site. Aggregating the ($P[A > a^*$ in time $t]$, a^*) pairs to produce risk estimates for all buildings in a portfolio is not trivial. An alternative approach is the event-based approach where loss values, a^*, are computed at all sites for specific events. The process can be repeated for a set of events and the probabilities $P[A > a^*$ in time $t]$ computed separately by considering the likelihood of each of the events (Robinson *et al.* 2005). Since the event-based approach computes losses on an event-by-event basis, it is straightforward to aggregate the losses for a single event across an entire city, portfolio of interest, or some other category such as building classification. Examples of different aggregations are presented in the section on case studies, below. The event-based approach for PSHA was tested as part of the "Joint validation of probabilistic seismic hazard analysis computer codes" conducted under the framework of the Pacific Earthquake Engineering Research Center (PEER) Lifelines Program. The event-based approach performed well compared with other codes, most of which were using a traditional approach to PSHA (Andres Mendez, personal communication 2006).

Computing Building Damage for an Earthquake Scenario

Techniques for modeling building damage associated with a specific earthquake can be broadly divided into engineering and empirical approaches. Both approaches divide buildings into classifications that attempt to capture the range in vulnerability associated with different building types. For example, an unreinforced masonry or double-brick building is more vulnerable to earthquake damage than a brick-veneer or wood-framed building. Empirical approaches to damage calculation use empirically derived relationships relating some measure of earthquake intensity (*e.g.,* PGA or MMI) and damage (*e.g.,* McCann *et al.* 1980). Such approaches have advantages: They are easy to use and will generally estimate damage accurately for the buildings and regions that were used to derive them. Empirical models become problematic when data are not available or of poor quality for the building types or regions of interest. This is not uncommon in many regions of the world or for many building types due to a lack of building damage data from observed earthquakes. Furthermore, it is also quite rare that sufficient data are available to create a probabilistic damage model that captures a range of damage states associated with a particular level of motion. Consequently, damage curves rarely are probabilistic in nature. Engineering approaches are based on engineering models of building performance during earthquakes (*e.g.,* Kircher *et al.* 1997; FEMA 1999). They are based on a combination of field observation, laboratory experimentation, and theory. The use of theory and laboratory observations allows the extension of engineering models to regions and building types where few data are available. Engineering models also can be probabilistic in nature and hence capture some of the uncertainty in modeling building damage.

Geoscience Australia has adopted an engineering approach based on the capacity spectrum method used in HAZUS (FEMA 1999) with a few minor modifications (Robinson *et al.* 2005). The approach models ground motion using a demand curve (damped response spectral acceleration versus damped response spectral displacement). A building's response to an input ground motion is captured by its capacity curve. The intersection of the capacity and iteratively modified demand curves gives an estimate of the building's peak response displacement (S_d^*) and acceleration (S_a^*). The S_d^* and S_a^* are used with the fragility curves to estimate probabilities that the building experiences different levels of damage. This process is repeated for three different components of the building: the displacement-sensitive structural system, the displacement-sensitive nonstructural components, and the acceleration-sensitive nonstructural components. A financial loss model is used to estimate damage in dollars or as a percentage of total building value. The process is repeated for all the buildings in a portfolio of interest to provide damage estimates to the portfolio for the scenario event. Note that a typical portfolio will contain a range of different building types that must be categorized and matched with appropriate capacity, fragility, and financial loss models. FEMA (1999) provides a categorization that has been further modified to suit Australian buildings (Robinson *et al.* 2005). A comparison of damage estimation using an empirical damage model and the capacity spectrum method is provided by Edwards *et al.* (2004) for the 28 December 1989 M 5.6 Newcastle, New South Wales, earthquake.

Using a Synthetic Earthquake Catalog to Undertake PSHA and PSRA

A synthetic earthquake catalog is similar to an historical earthquake database. The historical earthquake database represents observed earthquakes over some period of time and typically contains information such as earthquake magnitude, hypocenter, and date of occurrence. Similarly, a synthetic earthquake catalog represents a set of earthquakes using key parameters such as magnitude and location. Rather than an observation time, the time associated with a synthetic catalog represents the period over which the earthquakes are expected to occur. A synthetic catalog can be generated through Monte Carlo sampling of the probability density functions that govern the magnitude and geometry (*i.e.*, $f_M(m)$ and $f_R(r|m)$ respectively). Typically, $f_M(m)$ is such that the probability of having an earthquake in a specified period of time decays exponentially with magnitude. This means that it is necessary to sample many thousands of small earthquakes to ensure that any large earthquakes are present in the catalog. The larger earthquakes are required to ensure that the PSRA captures all probable events. In this paper the sampling of $f_M(m)$ is optimized by using a variation of Latin Hypercube Sampling to ensure that the entire range of earthquake magnitudes is represented. McKay *et al.* (2000) provide a general description of Latin Hypercube Sampling and Robinson *et al.* (2005) provide a detailed explanation of the approach adopted in this paper. The result is a synthetic earthquake catalog that has an equal representation of all magnitudes. Associated with each synthetic earthquake is the event activity that accounts for the likelihood of each of the synthetic events. The event activity is a function of the time interval considered (usually one year), the probability density function for earthquake magnitude $f_M(m)$, the total number of earthquakes expected in the time period (*e.g.*, the Gutenberg-Richter (GR) *a* value) and the number of synthetic earthquakes in the catalog. The sum of all event activities in the synthetic catalog reproduces the GR *a* value. A normalized histogram of event activities reproduces $f_M(m)$.

In summary, a PSHA can be conducted by:

1. generating a synthetic earthquake catalog by simulating earthquakes using an earthquake source (or occurrence) model;
2. estimating how the level of ground shaking propagates to sites of interest using the attenuation model;
3. including the local regolith and its effect on ground shaking by incorporating the site response model; and
4. estimating the hazard by calculating the maximum level of ground shaking expected to be exceeded in a given time period for all sites of interest.

Source, attenuation, and site response models are all treated as probability density functions. Specific values of the random variables are obtained by sampling the probability density functions using the techniques described by Robinson *et al.* (2005). The technique for obtaining the probability density function for the site response model is described by Dhu and Jones (2002) for the Newcastle region. An alternative technique is provided by Robinson *et al.* (2006).

A PSRA can be used to estimate the expected loss across a portfolio of buildings by:

1. repeating steps 1 to 3 above;
2. estimating the probability of structures being in different damage states using the capacity spectrum method (in this process the capacity curve is treated as a random variable by randomly selecting a vertical translation from a probability density function defined at the curve's yield point);
3. estimating the probable financial loss using the financial loss model with the weighted sum of loss for each damage state; and
4. aggregating the loss across the entire portfolio and estimating the probability of its exceedance.

CASE STUDIES

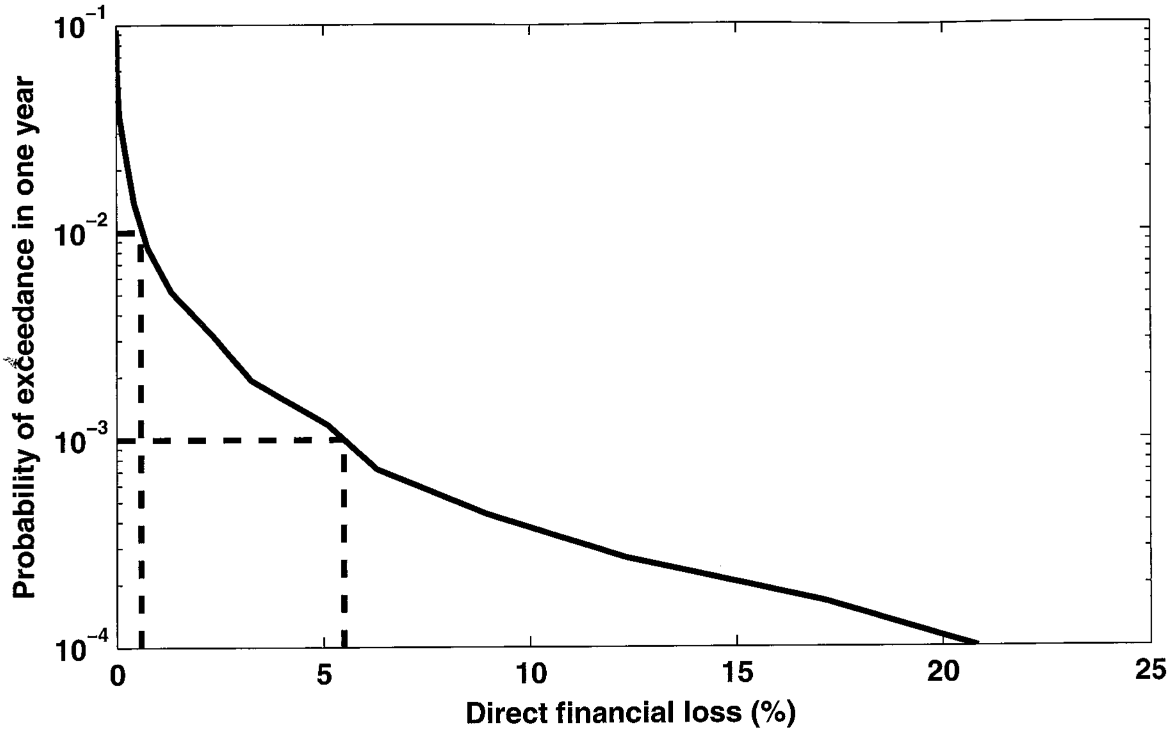
The capabilities of PSRA are demonstrated using case studies from two Australian cities: Newcastle and Perth. The modeled loss exceedance curve for Newcastle is shown in figure 1. It illustrates loss values that are expected to be exceeded in a one-year time frame with different levels of probability. The dashed lines represent probability levels of 0.01 (top) and 0.001 (bottom) per year that correspond to return periods of 100 and 1,000 years, respectively. The loss values associated with the 100- and 1,000-year return periods are 0.64% and 5.49%, respectively. An alternative but similar estimate of loss is the conditional expected loss for events in the ranges 0.0032 to 0.032 and 0.00032 to 0.0032 (terminology introduced by Smith 2005). For the Newcastle area these values are 0.615% and 5.2% for the 0.0032 to 0.032 and 0.00032 to 0.0032 ranges, respectively.

The annualized loss represents the estimated loss per year when all events from the synthetic catalog are considered. It is computed by numerically integrating the loss exceedance curve to compute the area enclosed between the curve and the two axes. Figure 2 illustrates the cumulative annualized loss as events of increasing return period are included. This curve is asymptotic to the true value of annualized loss, 0.034% per year.

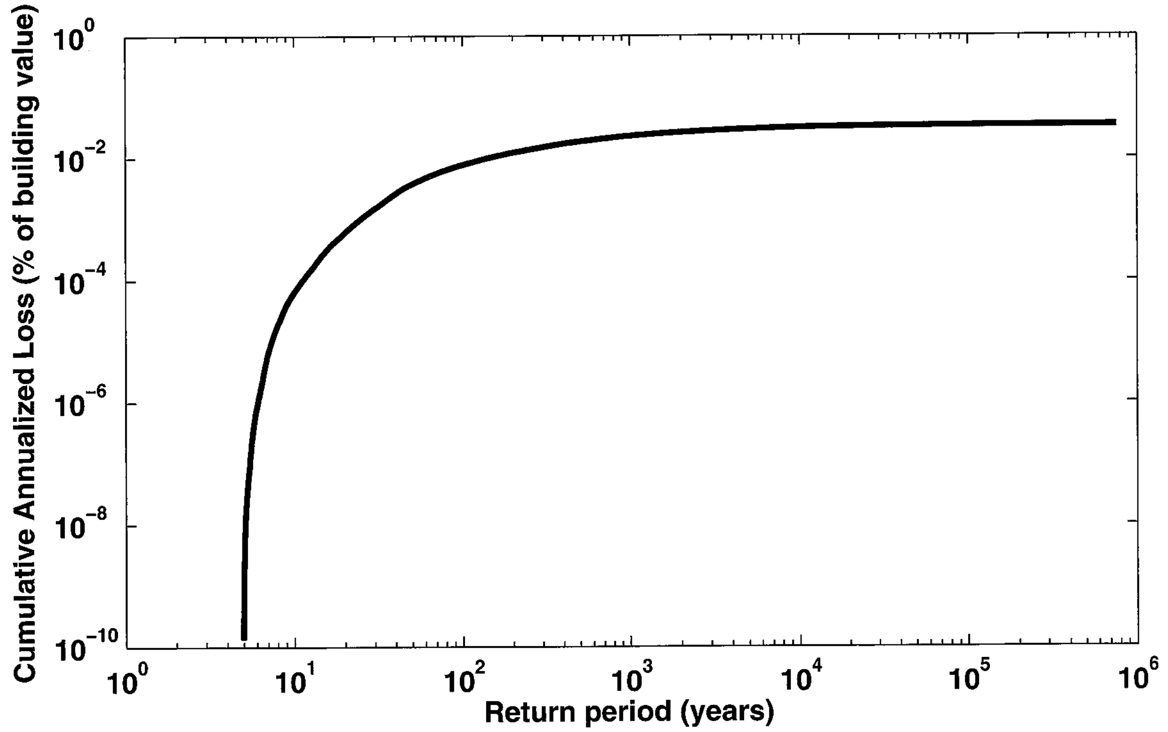
Figure 3 illustrates the annualized loss for Perth when it is disaggregated by distance and magnitude. In this figure each column represents a different distance (*e.g.*, 10–15 km) and magnitude range (*e.g.*, 5.5–6). The height of the column represents the contribution to annualized loss from all event-building combinations that fall in the designated distance and magnitude ranges. The modeled annualized loss for the region is 0.04% (Sinadinovski *et al.* 2005). Figure 4 illustrates the annualized loss for the four different primary building construction types as a percentage of the total value of building stock in each category. That is, it shows differences in the vulnerability between the building categories without considering the level of exposure in each category. The results demonstrate that modeled losses for unreinforced masonry (*e.g.*, double-skin brick) are the highest, with roughly twice the expected annualized loss of the timber frame buildings (*e.g.*, single-skin brick), which is in turn larger than concrete and steel-frame buildings.

DISCUSSION

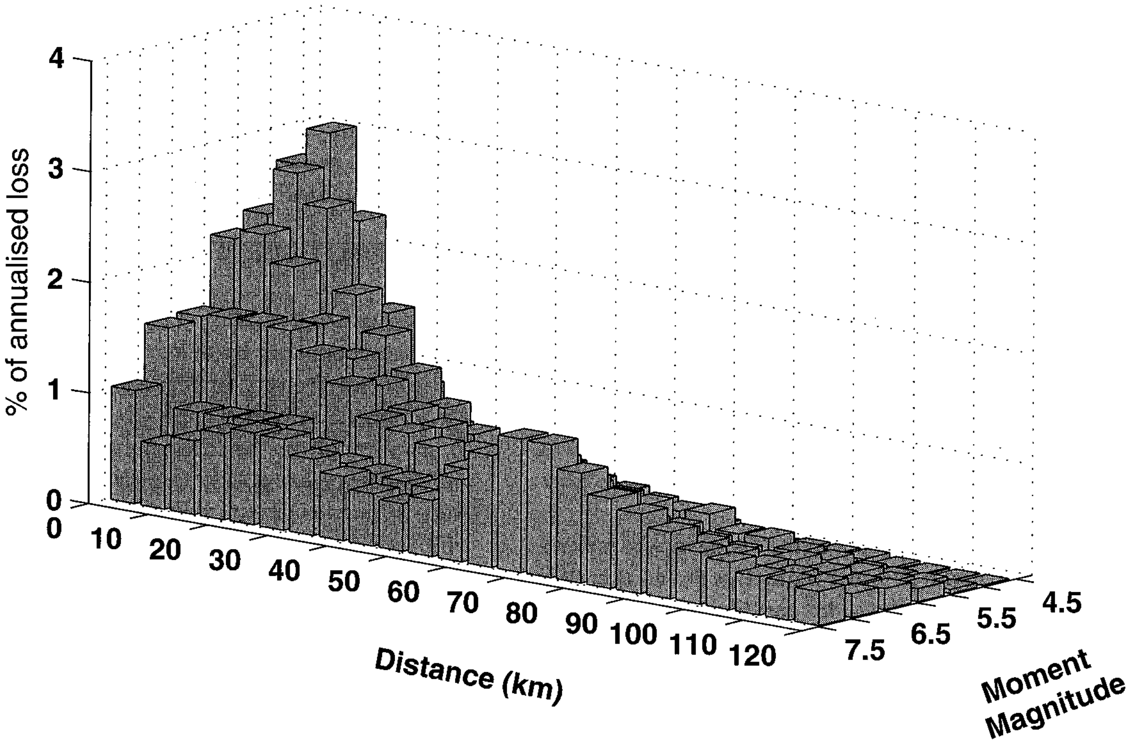
Smith (2005) proposes a discritization of the risk exceedance curve using conditional expected losses in three annual probability ranges: (1) 0.032 to 0.32, (2) 0.0032 to 0.032 and (3) 0.00032 to 0.0032 . We present conditional expected losses in Newcastle for the 0.0032 to 0.032 and 0.00032 to 0.0032 ranges (2 and 3) but not for the 0.032 to 0.32 range (1). The loss values of risk exceedance curves asymptotically approach zero as the event probability increases (figure 1). In regions such as Australia where earthquake recurrence is relatively low, it is common that the risk exceedance curve already is close to zero for probability levels corresponding to roughly 0.1 per year (*i.e.*, return periods of 10 years). In such cases there is little benefit in reporting the conditional expected loss for the 0.032 to 0.32



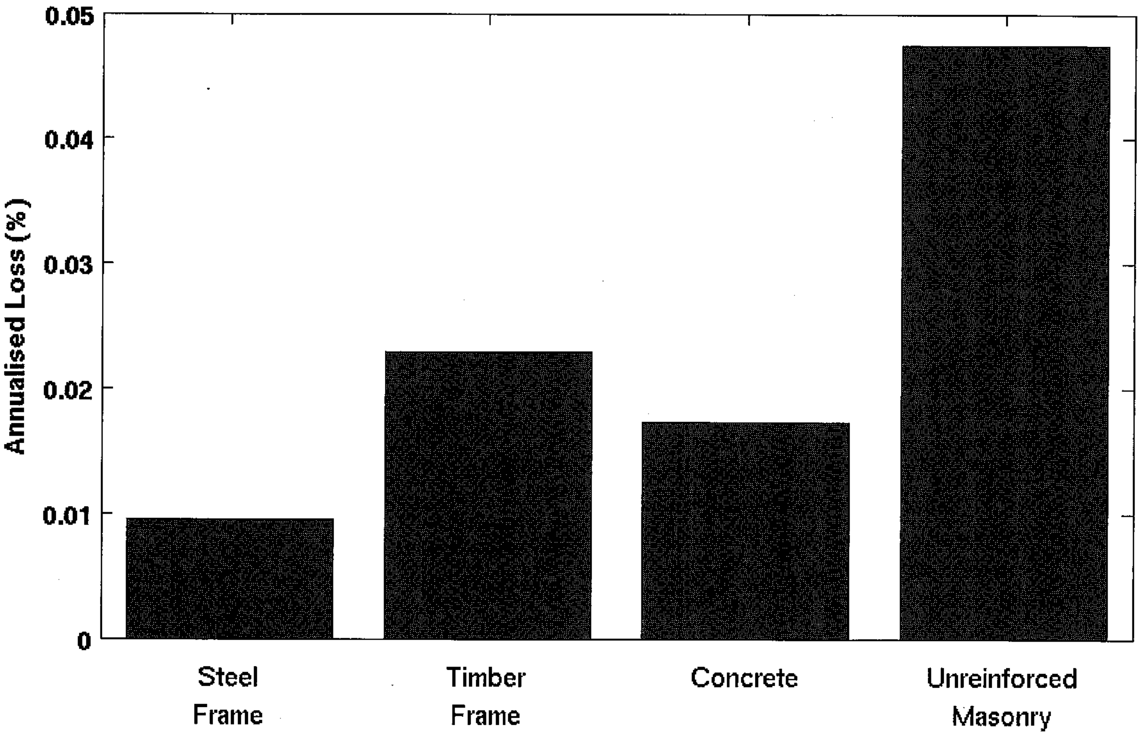
▲ **Figure 1.** Modeled loss exceedance curve for the city of Newcastle, Australia. The dashed lines represent the probability levels corresponding to return periods of 100 years (top) and 1,000 years (bottom).



▲ **Figure 2.** Modeled cumulative annualized loss as a function of return period in Newcastle, Australia.



▲ **Figure 3.** Modeled annualized loss for the city of Perth, Australia, disaggregated by distance and magnitude.



▲ **Figure 4.** Modeled annualized loss for the city of Perth, Australia, disaggregated by building construction type.

range. Furthermore, the conditional expected loss values for the 0.0032 to 0.032 and 0.00032 to 0.0032 ranges are similar to the estimated 100- and 1,000-year losses respectively (*i.e.,* 0.64% versus 0.615% for the former and 5.49% versus 5.2% for the latter). It is not clear that computing the conditional loss values for the Newcastle case provides any further information about the earthquake risk than the 100- and 1,000-year loss values.

The annualized loss represents the expected loss per year if all modeled events are considered. In Newcastle for example, this value is 0.034% per year. The flattening of the cumulative annualized loss curve in figure 2 indicates that the higher-impact events corresponding to larger return periods have little influence on the annualized loss. In fact, the curve indicates that events with return periods greater than 1,000 years contribute very little to the annualized loss. This flattening explains why the annualized loss value is a poor measure for catastrophe management.

The peak in the top left corner of the disaggregated annualized loss for Perth (figure 3) is the highest peak and therefore represents the region that provides the largest contribution to the annualized loss. It represents losses modeled for small events that are right underneath the city of Perth, that is, events with moment magnitude between 4.5 and 5.5 and earthquake source to building distances of less than 15 km. The expected loss for each of these events is small; however, the frequency of the events is higher relative to larger earthquakes. Hence, the peak in the top left corner represents many small events at close distances causing a high recurrence of low damage levels. A second peak occurs between 60 and 100 km for earthquakes with moment magnitudes between 7 and 7.5. The peak occurs in this distance range due to the presence of earthquake zones with higher seismicity to the east of Perth (Sinadinovski *et al.* 2005). Here the damage sustained by each individual earthquake is much higher than the smaller earthquakes contributing to the first peak; however the frequency of events is smaller. The combination of higher damage and lower frequency leads to a smaller peak in this case.

It is commonly known that some building types are more vulnerable than others during earthquakes. Figure 4 demonstrates how unreinforced masonry buildings are the most vulnerable of the building types found in Perth. This result is of particular interest to the Perth community because unreinforced masonry construction represents roughly 90% of the residential building stock. Consequently, the expected loss for the city of Perth would be higher than another region with equivalent hazard but lower percentage of unreinforced masonry structures. This simple example demonstrates the necessity of moving beyond a calculation of hazard and considering both the exposure and vulnerability of a region if the risk is to be understood.

CONCLUSION

Probabilistic seismic risk analysis (PSRA) provides probabilistic estimates of earthquake risk by considering earthquake hazard, building exposure, and vulnerability. The approach adopted by Geoscience Australia for PSRA is discussed and the technique coded into a robust piece of software known as Geoscience Australia's Earthquake Risk Model (EQRM). The capability of the EQRM is demonstrated using PSRA results from the cities of Newcastle and Perth in Australia. It is anticipated that the EQRM will be released as open source in 2006. Interested parties should contact the authors for further information.

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