LECTURE 8 - Mohorovičić’s inversion: the discovery of the crust-mantle boundary a.k.a. “Moho”

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"Andrija Mohorovičić" by a surrealist painter, Carlo Billich

***N.B. The material presented in these lectures is from the principal textbooks, other books on similar subject, the research and lectures of my colleagues from various universities around the world, my own research, and finally, numerous web sites. I am grateful for the material I used in this particular lecture to D. Skoko, M. Herak, and A. Mohorovičić himself, who was a founder of the Geophysical Institute of the University of Zagreb, at which I studied as an undergraduate student.***
Seismic “phases” and their nomenclature
Construction of travel time curves (hodochrones)
Observed and theoretical travel time curves

Kennett et al., 1991
1909 Earthquake and the Mohorovičić’s assumption

A. Mohorovičić (1910) - DISCOVERED CRUST-MANTLE BOUNDARY
HE CREATED AN EMPirical TRAVEL-TIME CURVE, BASED ON HIS OBSERVATIONS OF A CROATIAN 1909 EARTHQUAKE

\[ T = \frac{d\Delta}{\nu \cos i} \]

HE ASSUMED:

\[ N = \nu_0 \left( \frac{t_0}{t} \right)^k \]

WE DERIVED:

\[ 2\nu_0 \cos i \sqrt{R_0^2 - \nu_0^2 \cos^2 \alpha} \]

\[ \cos \alpha = \frac{1 - \sin^2 \alpha}{R_0^2 - \nu_0^2 \cos^2 \alpha} \]

\[ \frac{\Delta}{\nu \cos i} \frac{\nu}{\nu_0} \frac{k}{k-1} = \frac{\nu_0^2}{\nu^2} \left( \frac{2\nu_0 \cos i}{\nu \cos i} \right)^{k-1} \]

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Mohorovičić’s method
Mohorovičić’s method

\[
\Delta = 2 \left( \frac{\tan \iota}{\iota} \right) \int_{\tau_M}^{\tau_0} \frac{dt}{\sqrt{1 + \frac{2 \rho v_0 r_0^k}{\tau_M^2 - (\rho v_0 r_0^k)^2}}} 
\]

\[
\Delta = 2 \rho v_0 r_0^k \left[ \frac{2}{\pi (k+1)} \arccos \frac{\rho v_0 r_0^k}{\tau_0^2 (k+1)} \right] = 2 \left[ \frac{1}{1+k} \left( \arccos \frac{\rho v_0 r_0^k}{\tau_0^2 (k+1)} - \arccos \frac{\rho v_0 r_0^k}{\tau_M^2 (k+1)} \right) \right]
\]

\[
\Delta = 2 \left[ \frac{1}{k+1} \left( \arccos \frac{\rho v_0 r_0^k}{\tau_0^2 (k+1)} - \arccos \frac{\rho v_0 r_0^k}{\tau_M^2 (k+1)} \right) \right] = \frac{2}{k+1} \arccos \frac{\rho v_0}{\tau_0}
\]

\[
\Delta = \frac{2}{k+1} \arccos \frac{\rho v_0}{\tau_0}
\]
Mohorovičić’s method

\[ \Delta = \frac{2}{k+1} \arccos \frac{\nu_o}{v_o} = \frac{2}{k+1} \arccos (\sin i_o) = \frac{2}{k+1} \left( \frac{\pi}{2} - \arcsin (\sin i_o) \right) \]

\[ \Delta = \frac{2}{k+1} \left( \frac{\pi}{2} - i_o \right) = \frac{\pi}{k+1} - \frac{2i_o}{k+1} \Rightarrow \quad i_o = \frac{\pi}{2} - \frac{k+1}{2} \Delta \]

Substitution to \( T \)

\[ T = \frac{2}{1+k} \frac{\nu_o}{v_o} \cos i_o = \frac{2}{1+k} \frac{\nu_o}{v_o} \cos \left[ -\frac{k+1}{2} \Delta + \frac{\pi}{2} \right] = \frac{2\nu_o}{v_o(1+k)} \sin \left[ \frac{k+1}{2} \Delta \right] \]

This equation can be solved for two empirical values of \( \Delta \) and \( T \), for \( k \) and \( v_o \).
The depth of the discontinuity

Using the empirical values, Mohorovičić obtained

\[ \lambda = 3.6 \]
\[ v_0 = 5.56 \text{ km/s} \]

From
\[ v = v_0 \left( \frac{t_0}{t} \right)^k \] \[ \Rightarrow v_H = v_0 \left( \frac{t_0}{t_H} \right)^k \] \[ \Rightarrow \frac{v_H}{v_0 \sin \theta} = \frac{t_H}{t} \]

\[ \frac{t_H}{t} = \frac{v_0}{v_H \sin \theta} \]

For \( \Delta = 760 \text{ km} \) ( \( \overline{P} \) disappears)
\[ t_0 = \frac{\pi}{2} - \frac{k + 1}{2} \Delta = 74^\circ 16' \]
\[ t_H = 6318 \text{ km} \]

\[ H_{\text{Moho}} = t_0 - t_H = 6371 \text{ km} - 6318 \text{ km} = 53 \text{ km} \]

Voilà!

Andrija Mohorovičić 1910
The discontinuity in seismic wave speeds

Abrupt change in the composition and density of rocks results in a sharp change in seismic wave speeds.
The depth of Moho (crustal thickness)
Moho in popular culture

- The Mohorovičić Discontinuity is mentioned in one particular computer game, an RTS called *Total Annihilation*. Players can build a "Moho Mine" in order to mine metal at or close to the Mohorovičić Discontinuity. Due to the size of the structure, the public being unfamiliar with the Mohorovičić Discontinuity, and an expansion structure called the "Moho Metal-Maker", "Moho" is misinterpreted as meaning "big."
- The Mohorovičić Discontinuity is also mentioned in the novel *Abduction* by Robin Cook, in which a team of scientists are abducted by inhabitants of an underground civilization.
- In the cartoon *Inhumanoids* the monster, D-Compose's kingdom of Skellweb lies within the Moho.
- In *Star Control 2*, one of the "ramblings" of the odd Mycon race is referring to the Deep Children as "Dwellers in the Mohorovichic." This is a reference to the fact that a "Deep Child" will burrow deep into the surface of a planet to begin de-terraforming.
- *Deep Storm: A Novel* by Lincoln Child details an expedition where a team of scientists attempts to drill through the ocean floor to the Mohorovičić Discontinuity.
- In *The Mohole Mystery* by Hugh Walters lethal microbes and belligerent egg-shaped creatures inhabit the Mohorovičić Discontinuity when a manned rocket propelled capsule is sent down to investigate.
- In the *Mars trilogy* by Kim Stanley Robinson, the colonizers of Mars dig deep "moholes" to allow outgassing from the planet's interior as a means to increase the atmospheric pressure - thus contributing to the terraforming of the planet.
- In *Sid Meier's Alpha Centauri* game, the Mohorovičić Discontinuity is mentioned with the technology advance "Industrial Automation." Another technology advance, "Ecological Engineering," allows terraforming units to dig "Thermal Boreholes," which are equivalent to moholes and which, in the game, produce both energy and minerals.
**Reaching Moho**

- **Project Mohole** was an ambitious attempt to drill through the Earth's crust into the Mohorovičić discontinuity, and to provide an Earth science complement to the high profile Space Race. It was led by the American Miscellaneous Society with funding from the National Science Foundation. Phase One was executed in spring 1961. Off the coast of Guadalupe, Mexico, five holes were drilled, the deepest at 183 m below the sea floor in 3,500 m of water. This was unprecedented: not in the hole's depth but because of the depth of the ocean and because it was drilled from an untethered platform. The **Mohole** project failed due to poor management and cost overruns.

- The **Kola Superdeep Borehole** (KSDB) was the result of a scientific drilling project of the former USSR. The project attempted to drill as deep as possible into the Earth's crust. Drilling began on May 24, 1970 on the Kola Peninsula, using an "Uralmash-4E" and later an "Uralmash-15000" drilling device. A number of boreholes were drilled by branching from a central hole. The deepest, SG-3, was completed in 1989, creating a hole 12,262 metres (the deepest hole ever made by humans). However, due to higher than expected temperatures at this depth and location, 180º C instead of expected 100º C, drilling deeper was deemed infeasible and the drilling was stopped in 1992.

- **Chikyu Hakken** (地球発見), Japanese for "Earth Discovery", is a mission primarily led by the Japan Agency for Marine-Earth Science and Technology, or JAMSTEC. The half-billion dollar plus project aims to be the first to drill seven kilometers beneath the seabed and into the Earth's mantle.