In the tutorial today:

Adams Williamson equation and the procedure for the determination of density as a function of Earth’s radius
Homework #2
PHY3070
II semester 2007 Week 4-5 (Hrvoje Tkalčić)

Due Wednesday, 8 August by 10:10am, in class

1. (15 points) Using the parametric form of the travel-time and epicentral distance equations for earthquakes (derived in the lectures), obtain the following relation:

$$ t = pA + 2 \int \frac{\sqrt{r^2 - p^2}}{r} dr $$

2. (20 points) Assume that the earthquake shown in Figure 16.58 of the C. M. R. Fowler book took place in the upper crust ($\alpha=6$ km/s and $\beta=3.5$ km/s).

a) Use the $P$ and $S$ arrival times to calculate the distance from the focus to the seismometer.

b) What estimates can you make about the height of the focus above the reflecting horizon and the depth of the horizon beneath the surface?

3. (25 points) Using the same syntax and set-up of the problem for the LIQUID-SOLID interface problem we considered in the tutorial, determine the third equation for transmission coefficients from the third boundary condition that the normal stresses are continuous. (We derived the first two equations).

Prove that the resulting equations can be written in the following form:

$$ \begin{bmatrix} \frac{\pi}{\sqrt{2}} & 0 & 2\alpha & \alpha \tan \theta & -\alpha \tan \theta \\ \frac{\pi}{\sqrt{2}} & 0 & 2\alpha & \alpha \tan \theta & -\alpha \tan \theta \\ \pi & 0 & 2\alpha & \alpha \tan \theta & -\alpha \tan \theta \\ \pi & 0 & 2\alpha & \alpha \tan \theta & -\alpha \tan \theta \\ \pi & 0 & 2\alpha & \alpha \tan \theta & -\alpha \tan \theta \end{bmatrix} \begin{bmatrix} \alpha \tan \theta \\ \alpha \tan \theta \\ \alpha \tan \theta \\ \alpha \tan \theta \\ \alpha \tan \theta \end{bmatrix} $$

where $\rho_1$ and $\rho_2$ are the densities of the liquid and solid layer, respectively.

4. (40 points) Using the same syntax (as in the previous problem), calculate the equations of the amplitude ratios for the SOLID-LIQUID interface assuming the incidence of a $P$ wave from solid to liquid from above (assume the $P$ wave velocity and density at the interface).

Optional - Bonus 30 points: Based on your equations derived in 3) or 4) and a computer program, plot the displacement amplitude ratios $U/A$ (reflected $P$ over incident $P$), $R/A$ (reflected $SV$ over incident $P$) and $T/A$ (reflected $P$ over incident $P$) as a function of the angle of incidence $\vartheta$, where $\vartheta = 90^\circ - \gamma$ (where we defined $\gamma = 90^\circ - \vartheta$ as the angle of emergence). Use at least 100 points for $\vartheta$ to plot the curve. Assume the following values for the elastic parameters when plotting the transmission and reflection coefficients as functions of the angle of incidence $\vartheta$: rock $\rho=2.785$ g/cm$^3$, $\alpha=9.5$ km/s, $\mu=46.0$ km/s, liquid $\rho=0.95$ g/cm$^3$, $\alpha=12.0$ km/s, $\mu=6.3$ km/s. What can you say about the critical angles?
Dr. Sambridge from RSES gave a presentation on the geophysical inverse theory and the principles of seismic tomography...
\[ \frac{\mathrm{d}p}{\mathrm{d}t} = -f(t)g(t) \frac{\mathrm{d}t}{t} \] (\textit{a})

\[ \frac{\mathrm{d}p}{\mathrm{d}t} \text{ the gradient of the hydrostatic pressure} \]

\[ g(t) = G \cdot \frac{m}{r^2} ; \ G = 6.67 \times 10^{-8} \text{ g cm}^3 \text{s}^{-2} \]

incompressibility (bulk modulus):

\[ K = -\frac{\mathrm{d}p}{\mathrm{d}V} ; \ \Theta \text{ is dilatation of volume} \]

\[ K = -\frac{\mathrm{d}p}{\mathrm{d}V} \] (\textit{a})

\[ \lambda = \sqrt[3]{\frac{\lambda^2 - 2\mu}{\rho}} \Rightarrow \lambda^2 = \lambda + 2\mu \]

\[ \mu = \sqrt[6]{\frac{\mu}{\rho}} \Rightarrow \mu^2 = \mu \]

\[ K = \lambda + \frac{2}{3} \mu , \text{ where } \lambda \text{ and } \mu \text{ are Lame's constants, and consequently} \]
From (**) ⇒ \[ k = \frac{x^2}{s} - \frac{2}{3} \beta^2 s + \frac{2}{3} \beta^2 \phi = \phi \left( \frac{x^2}{s} - \frac{4}{3} \beta^2 \right) \] (***)

From (*) ⇒ \[ dp = -k \frac{dv}{V} \]

\[ V = \frac{m}{s}; \quad dV = -\frac{m}{s^2} dp \]

Therefore \[ dp = -k \cdot \frac{m}{s^2} \frac{ds}{s} = k \frac{dp}{s} \] (***)

From (**) and (***) combined with (***)

\[ dp = -s \frac{m}{r^2} dt = k \frac{d\phi}{s} \]

\[ \frac{d\phi}{dt} = -s^2 \frac{m}{r^2} \frac{1}{k} = -s^2 \frac{Gm}{r^2} \frac{1}{s^2 \left( \frac{x^2}{s} - \frac{4}{3} \beta^2 \right)} = -\frac{Gm}{s^2 \left( \frac{x^2}{s} - \frac{4}{3} \beta^2 \right)} \]

\[ \frac{d\phi}{dt} = -\frac{Gm}{s^2 \left( \frac{x^2}{s} - \frac{4}{3} \beta^2 \right)} \]

Adams & Williamson equation (1923)

A-W equation is then used to determine density as a function of radius.