Quantifying Uncertainty in Reservoir Performance Prediction

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Summary

This paper will describe a technique for generating an ensemble of history matching models using the Neighbourhood Approximation algorithm.

Using the ensemble of models generated, we demonstrate how uncertainty in reservoir performance prediction can be quantified by sampling from the posterior distribution. This involves using the neighbourhood approximation algorithm in a Bayesian framework.

We validate the technique on the SPE 10th Comparison Solution Project dataset. Fine grid oil rate and average reservoir pressure for 300 days are used in lieu of field data. We generated multiple coarse grid reservoir models and assessed the misfit in oil rate and average field pressure. By running multiple Markov Chain walks on the misfit surface, we are able to quantify the posterior probability of the models in the input ensemble and predict the range of possible fine grid profiles out to 2000 days.

The Neighbourhood Approximation algorithm

The Neighbourhood Approximation (NA) algorithm is a stochastic sampling algorithm, originally developed for earthquake seismology. It works by adaptively sampling parameter space using geometrical properties of Voronoi cells to bias the sampling to regions of good fit to data. By its nature, the algorithm exploits information in all previous models to selectively sample parameter space. For a full description of the algorithm, see [4]. Two tuning parameters, n_s and n_r, control the performance of the algorithm. n_s determines the number of models generated at each iteration and n_r specifies the number of cells to resample at each iteration.

We have adapted the algorithm to generating multiple history matching models. For our earlier application of the algorithm in petroleum engineering, see [2] and [6].

Sampling from the posterior distribution

The aim at the appraisal stage is to infer information from a finite set of models generated to quantifying the uncertainty in forecasting.

Given model m in the ensemble, our prior knowledge about this model can be quantified through a prior probability density function $p(m)$. By comparing the model output with the observed data, we can calculate the likelihood that the observed data is explainable by the model for which the likelihood holds. Using this likelihood, we can update our prior knowledge about the models in a Bayesian framework.

By Bayes theorem, the updated posterior probability density for model m is given by Equation (1). $p(O|m)$ is the likelihood, i.e., the probability that the observation O can be

$$p(m \mid O) = \frac{p(O \mid m)\, p(m)}{\int p(O \mid m)\, p(m)\, dm} \tag{1}$$

obtained, given the model *m*. It quantifies the model as well as the data errors. In calculating *p(m|O)*, one of the challenge lies in evaluating the integral expression, i.e., the normalising term. This task is usually non-trivial, even for problems of very low dimension. However, Markov Chain Monte Carlo (MCMC) provides a method for generating a sequence of realisations that are samples from the posterior distribution without the need to calculate this term. The NA-Bayesian algorithm uses a MCMC approach to generate new models whose distribution approximates that of the input ensemble. For details of the NA-Bayesian algorithm, see [5].

Application- model and problem description

The fine grid is the SPE 10th Comparison Solution Project model [3]. It is a geostatistical model of about 1.2million cells. The model dimensions are [1200x2200x170] cubic feet and the top 70ft represent the Tarbert formation, while the bottom 100ft is the Upper Ness. There are 4 producer wells in the corners of the model at constant BHP, and a central water injector at constant injection rate. There are only two phases namely, oil and water. The relative permeability curves are of Corey type.

The coarse grid is a single phase upscaling of the fine grid, and consists of [5x11x10] cells. The well positions and controls are identical to the fine grid model.

Firstly, we run the fine grid model to obtain fine grid average pressure and produced oil rate for 2000 days. We use data for 300 days as history data. The task is to use the coarse grid model and history match, by determining relative permeability curves. Finally, we forecast the fine grid behaviour out to 2000 days, and quantify the uncertainty in our forecast.

Parameterisation and misfit Quantification

We used oil and water relative permeability curves defined by [1]. We modelled each geological formation (Tarbert and Upper Ness) with a separate pair of curves. Hence we needed to determine 12 model parameters, i.e., 6 per formation.

In the least square sense, the measure of misfit, *M*, can be expressed as in equation (2).

$$M = \left\langle p_{obs} - p_{sim} \middle| C_p^{-1} \middle| p_{obs} - p_{sim} \right\rangle + \left\langle q_{obs} - q_{sim} \middle| C_q^{-1} \middle| q_{obs} - q_{sim} \right\rangle \qquad (2)$$

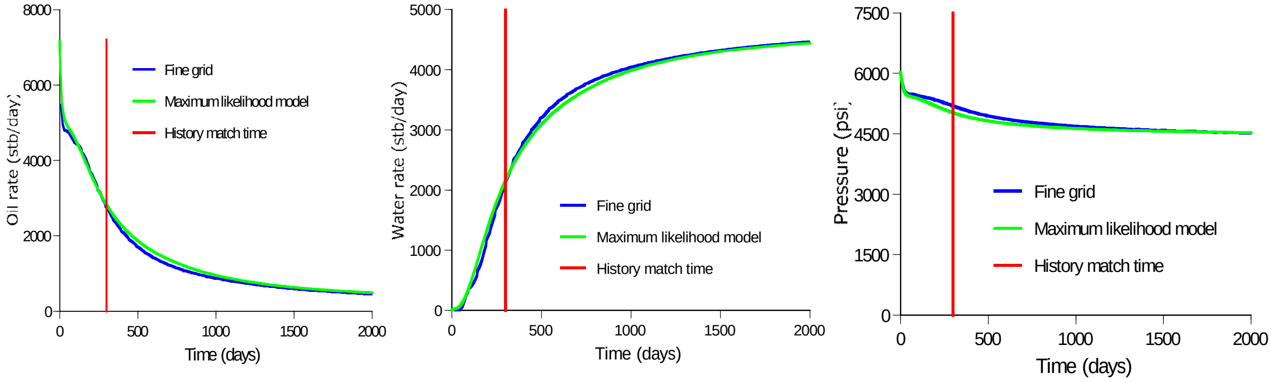
The subscripts, ‘obs’ and ‘sim’ refer to the observed and simulated data, respectively, while p and q are respectively the pressure and production rates data. Since the likelihood *p(O|m)* depends exponentially on the misfit value, correctly quantifying the model misfit values is imperative for the evaluation of the posterior probability *p(m|O)*. Quantifying the model misfit requires accounting for the model as well as the data errors. This can be done by introducing a covariance matrix, *C*, which captures these errors.

We have to take into account time correlation in errors, which we do using an exponential model for the covariance structure and a correlation time of 100 days. We note however that this is one of several possible error models for the covariance structure. The application of other error models will be addressed elsewhere.

Results and discussions

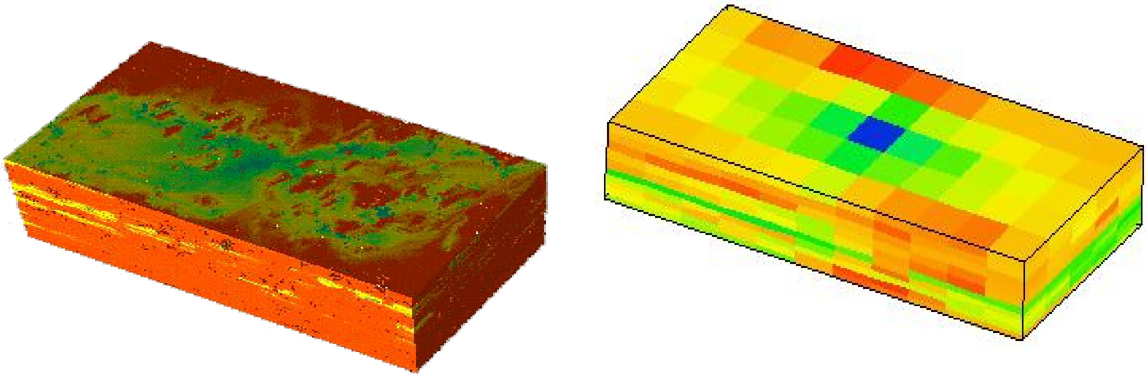
Using the NA-algorithm, we generated 312 models, sampling a 12 dimensional parameter space. Figure 2 shows the maximum likelihood model prediction out to 2000 days. These figures show that the maximum likelihood model is highly accurate in predicting the fine grid behaviour. Given a fine scale description and a simulation grid, an upscaling algorithm assigns suitable values of petrophysical properties, e.g., porosity, permeability, to cells on the coarse simulation grid. We have employed the simplest form of upscaling namely, single-phase upscaling, where the aim is to preserve the gross features of the flow on the simulation grid. One of the challenges with upscaling is being able to capture fine scale features with the coarse (upscaled) grid. In our particular case, we have upscaled from 1.2million cells to a coarse grid of 550 cells. With this degree of coarseness, capturing fine scale features is a challenge. Figure 2 displays the water saturation at 800 days for the fine and coarse grids. We observe that the coarse grid captures, to a high degree, much of the fine grid features in, and around the center of the fine grid. The high permeability streaks (layers) in the fine grid also feature prominently in the coarse grid.

To quantify the uncertainty in predicting the fine grid behaviour we employed the NA-Bayesian algorithm on the ensemble of models generated. We run a long chain of the MCMC algorithm and performed a Bayes update of the probabilities. This approach calculates the relative probability of each model in the ensemble, based on the frequency of visits to each model during the random walk. We next calculated the mean and standard deviation of the profiles. Using these parameters, we determined the P_{10} and P_{90} cut-off for each of the predicted profiles. Figure 3 shows plots of the maximum likelihood prediction, the average, and the P_{10} — P_{90} cut-off profiles for the oil and water rates, and the field average pressure. We observe that the P_{10} — P_{90} profiles envelope the maximum likelihood prediction as well as the fine grid data.



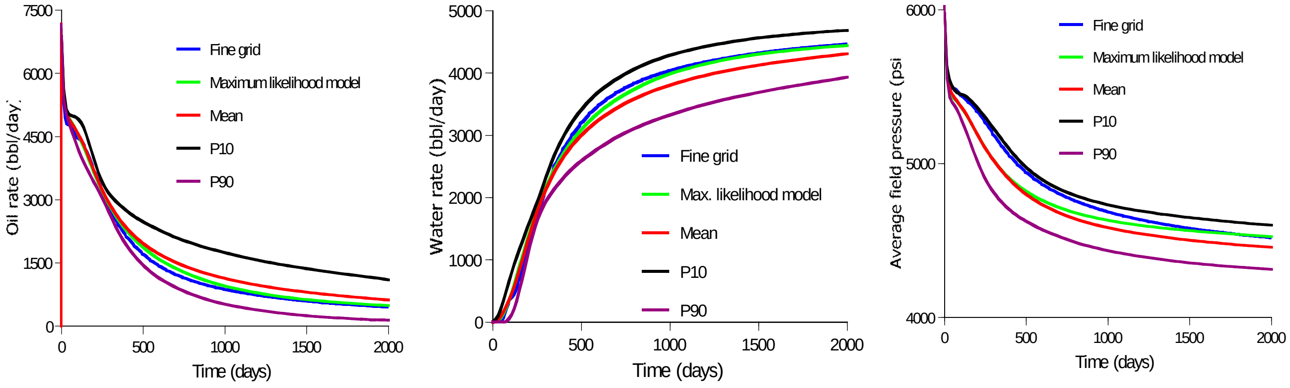
a. Oil rate **b**. Water rate **c**. Pressure

Figure 1. Maximum likelihood predictions



a. Fine grid **b.** Coarse grid

Figure 2. Water Saturation at 800 days



a. Oil rate **b**. Water rate **c**. Pressure

Figure 3. Quantifying uncertainty

Conclusions

We have demonstrated the use of the Neighbourhood Approximation algorithm to generate multiple history matching models, and investigated using synthetic data from the 10th SPE Comparative Solution Project. Our results show that the maximum likelihood model is highly accurate in predicting the fine grid behaviour.

Using the Neighbourhood Approximation algorithm in a Bayesian framework, we have quantified the uncertainty in predicting the fine grid behaviour. The fine grid profiles lie within the uncertainty bounds predicted by the algorithm.

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