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Seismic Source Characterization using a Neighbourhood Algorithm

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Abstract. The full characterization of a seismic source requires the specification of the hypocentre location and the source mechanism. The non-linear inversion is accomplished in two stages using the Neighbourhood Algorithm (NA), a direct search method in parameter space which is able to preferentially sample those regions with least data misfit; there are just two control parameters and no differentiation is employed. The first step is hypocentre location using a 4-D search space and a focussed search strategy. The second step is waveform inversion of the early part of P and S wavetrains to refine source depth and extract the source mechanism. With a moment tensor representation, this second stage explores an 8-D parameter space. For both the hypocentre and source mechanism inversion the NA method provides rapid and effective results with information from just a few stations, as illustrated with an event in southern Xinjiang.

1. Introduction

The characterization of a seismic source in terms of location and source mechanism provides information which may be used for a wide variety of purposes, but is particularly relevant in the context of monitoring the Comprehensive Nuclear-Test-Ban Treaty. Events deeper than 15 km are very unlikely to be man-made, and if waveform observations are compatible with a source without any explosive (isotropic) component a further discriminant is available.

We extend methods for waveform inversion to determine source mechanism (e.g., *Dziewonski et al.*, [1981], *Sipkin*, [1994], *Kawakatsu*, [1995]) to smaller events as in the work of *Goldstein and Dodge* [1999], and link this directly to source location by using a fully non-linear inversion procedure which does not require the calculation of any derivatives. The Neighbourhood Algorithm (NA) [*Sambridge*, 1999a,b] is a new method of exploring a multi-dimensional parameter space to find regions with acceptable data fit, whose properties are controlled by just two parameters. For the initial step of hypocentre location, we use the NA scheme in the 4-D space of *latitude*, *longitude*, *depth* and *origin time* in a form which resembles an irregular contracting grid. Whereas in the second stage when we match the waveforms of the onsets of P and S phases, we use the NA approach with a broader exploration of the 8-D space formed from the six independent components of the source moment tensor [*Aki and Richards*, 1980], the source depth and a parameter defining the source time function. In each step we construct data misfit functions by comparison of the observations with direct calculations, using arrival times of phases in the first stage and synthetic seismograms calculated used generalized ray theory in the waveform inversion. The overall inversion can be carried out rapidly and good results can be achieved with a small number of observations, as illustrated by application to an event in southern Xinjiang, China.

2. The Neighbourhood Algorithm

The Neighbourhood Algorithm (NA) is a derivative-free method of searching a parameter space based on the properties of some measure of data fit [*Sambridge*, 1999a,b]. The NA exploits the self adaptive properties of Voronoi cells, the nearest neighbour regions in multi-dimensional space subject to a suitable distance norm [*Voronoi*, 1908]. The procedure can be summarized as:

1. A set of n_s initial models are generated pseudo-randomly, and a misfit measure calculated for each model. The space is then divided into a set of Voronoi cells characterized by the misfit at the point within the cell.
2. The n_r models with least misfit are determined and a uniform random walk is performed inside the Voronoi cells to generate n_s new models and estimates of misfit. The Voronoi cells for the whole ensemble are then constructed, so that all prior information is exploited.
3. The second step is repeated until some pre-defined termination criterion is met e.g level of misfit, or number of iterations.

The parameters n_s, n_r ($n_s \geq n_r$) control the behaviour of the coverage of parameter space. For small (n_s, n_r) the NA scheme represents a focussed search, whilst larger (n_s, n_r) gives a more exploratory character. Only the rank of the misfit estimates are used, so that any style of user-defined measure can be employed. The NA approach is rapid because the Voronoi cells are not calculated explicitly, only the projections on the axes are required (see *Sambridge* [1999a]). The NA procedure is able to cope with misfit surfaces containing multiple minima [*Sambridge*, 1998] and ultimately exploit the best data fit.

3. Application to source characterization

We consider a situation in which we have a limited number of seismograms for an event, for which we make N_t arrival time picks for P and some S phases. For a source at

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origin time t_s and spatial location $\mathrm{x}_s = (x_s, y_s, z_s)$, we construct an arrival time estimate for the ith phase as

$$t_{ci}(\mathrm{x}_s, t_s) = t_s + t_{ri}(\mathrm{x}_s), \tag{1}$$

where we construct the travel-time using ray tracing in a suitable Earth model. We have used the *ak135* model of *Kennett et al.* [1995] with the IASPEI software based on the tau-spline method of *Buland and Chapman* [1983]. We compare the arrival time estimates with the observations$\{t_i\}$ using a robust L_1 measure,

$$C_t = \frac{1}{N_t} \sum_{i=1}^{N_t} |t_i - t_{ci}|/\sigma_i, \tag{2}$$

where σ_i is the estimate of the error in the ith arrival time observation.

The search of the 4-D hypocentral space (t_s, x_s) is then carried out using the NA scheme to find the region where C_t is least. The use of the L_1 measure minimizes the influence of outliers with a limited data set (for further details see *Sambridge and Kennett*, [2000]).

Once we have an estimate of the source location we extract the epicentral distances and azimuths to the stations and move to comparison of waveform segments at the onset of the P wavetrain and, where possible, the S wavetrain. We use a moment tensor representation of the seismic source (with 6 independent components) and generalized ray theory (cf. *Langston and Helmberger*, [1975]) to calculate synthetic seismograms for the primary phase and its surface reflections (e.g. P, pP, sP). The time separation of the P and pP, sP phases varies rapidly and non-linearly with depth, but for fixed depth the seismograms are linearly dependent on the moment tensor components. Once we allow the depth to be variable we therefore have a non-linear inversion problem for the source parameters. We use a simple source time function for small events, a trapezoid with three time segments in the ratio 1:3:1, controlled by the rise time.

For this stage of the inversion we fix the origin time and epicentre for the event and so need to explore an 8-dimensional parameter space. We wish to penalize differences from the observed waveforms to extract the strongest possible constraints and, from a variety of tests, we favour an L_3 norm. The measure of fit between the N_s observed and synthetic seismogram segments u_j and u_j^c, is thus

$$C_s = \frac{1}{\langle S \rangle N_s} \sum_{j=1}^{N_s} S_j \int_{t_1}^{t_2} \mathrm{d}t \, |u_j(t) - u_j^c(t)|^3, \tag{3}$$

where S_j is the signal to noise ratio at the jth station, and $\langle S \rangle = N_s^{-1} \sum_j S_j$ averaged across all the stations. The NA procedure is now used in exploratory mode to find the region in source-parameter space for which C_s is minimized. If the estimate of source depth from the waveform fitting differs significantly from the hypocentral estimate, we undertake a new NA inversion for the hypocentre with a constrained depth and then repeat the waveform inversion

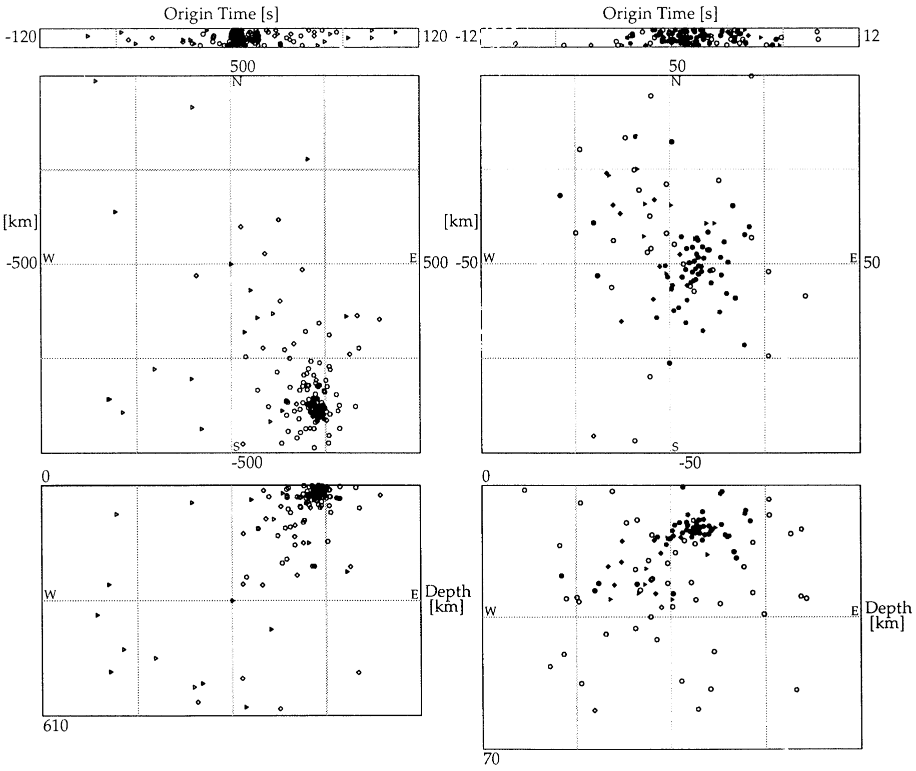

Figure 1. Representation of the progress of the NA technique towards convergence on the location estimate for the Xinjiang event via projections onto the different spatial planes. The group of models generated at each iteration are indicated by the same style of polygons. The number of sides increases as the iterations proceed, but for clarity solid symbols are introduced for the last few iterations. (a) full domain, the centre is the reference point about which the parameter bounds were specified. (b) 10 fold zoom about the region of acceptable models.

4. Application to southern Xinjiang event

We illustrate the NA approach to source characterization by application to an m_b 5.6 event in southern Xinjiang (1997 iv 6 04:36:35, ISC location 39.54°N, 77.00°N, 31 km, 17km *pP-P*). This event was well recorded around the globe and moment tensor estimates are available for comparison from NEIC and the Harvard group [*Dziewonski et al.*, 1999].

We have used a limited number of broad-band waveforms extracted from the IRIS data Management Centre archive and employ our own time picks. The event occurs in a continental region and the *ak135* model [*Kennett et al.*, 1995] provides a suitable reference for both the location and waveform inversion.

The location step employs the NA procedure with n_s=9, n_r=2 searching a broad domain ±5° in latitude and longitude, ±600 km in depth and ±120 s in origin time centred at 43°N, 74°E, 10 km deep and 30 s onset time. The search is roughly equivalent to a contracting irregular grid. In 20 iterations (i.e. 189 sets of traveltime calculations) we achieve a tight constraint on the hypocentre with a best fit at 39.537°N, 76.852°E, 10.6 km deep, 34.00 s. The NA inversion rapidly exploits the character of the misfit to home in on a region of acceptable fit. The progress of the NA scheme is illustrated in Figure 1 where we show both the full search domain and a ten-fold zoom around the area of least misfit.

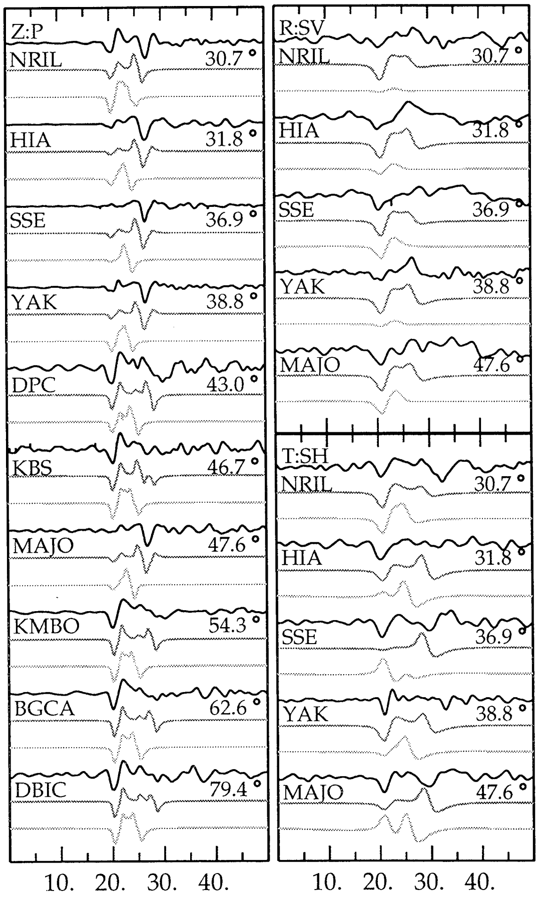

Figure 2. Comparison between seismograms for the southern Xinjiang event: observed (black traces), synthetics for NA solution (grey traces) and for NEIC solution (light grey traces). The vertical component of *P*, and the radial and transverse components of *S* are displayed. Each set of traces is annotated with the station name and epicentral distance.

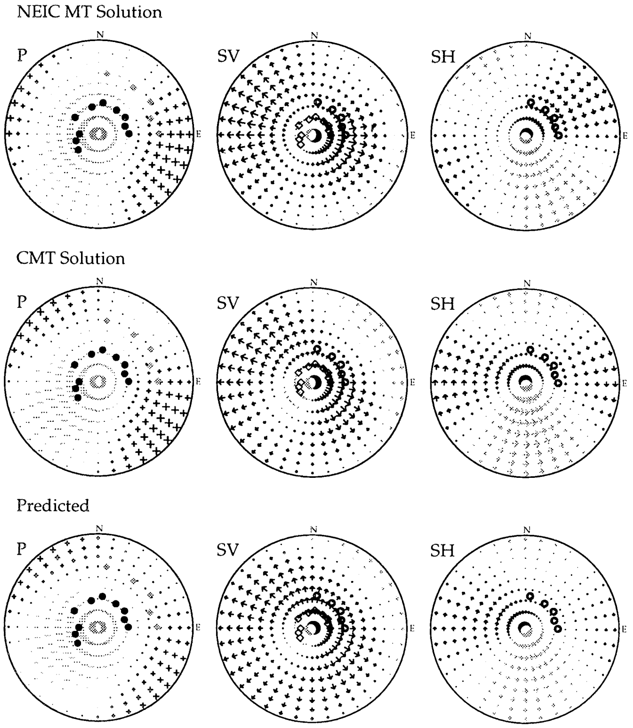

Figure 3. Source radiation patterns for *P*, *SV* and *SH* waves for the southern Xinjiang event, NEIC moment tensor solution (top), Harvard CMT solution (middle), and source mechanism from the NA inversion (bottom). The size and configuration of the symbols indicate the sense of motion (for *P* waves, pluses indicate compression and minuses dilatation). The azimuth and take-off angle of each phase is represented by the following: solid circles for *P*, solid triangles for *pP*, solid (grey) diamonds for *pS*, open circles for *S*, open triangles for *sS*, and open diamonds for *sP*.

We use the epicentre from the location step in the waveform inversion and allow the source depth to vary between the surface and 35 km depth. Ten stations are used in the waveform inversion, but only five of the stations have suitable *SV* and *SH* waveforms. The waveforms are bandpass filtered in the range 0.01 Hz to 0.7 Hz before inversion.

A joint *P* and *S* waveform inversion results in a significant isotropic (implosive) estimate, with a normalized moment tensor trace of -1.1. This isotropic estimate is not really surprising given the sampling distribution of the source radiation pattern (see Figure 3). However, constraining the isotropic component to zero results in a poor fit to the *SH* wave seismograms. We therefore perform inversion with a small, but not significant, isotropic component of -0.1. An excellent fit is achieved for the *P* wave seismograms, and a reasonable fit is achieved for the *S* wave seismograms (Figure 2). Note that the tangential component at station SSE is not fit by any of the solutions. The recovered source depth is 15 km, which is the same as the Harvard CMT estimate, but deeper than the NEIC moment tensor depth estimate of

8 km. A source depth of 15 km appears to fit the observed seismograms better than a depth of 8 km (Figure 2). We obtain good correspondence between the source mechanism estimate obtained from the NA inversion, and both the NEIC and Harvard moment tensor solutions (Figure 3).

The final hypocentral estimate from just ten stations lies within 14 km of the ISC location based on 604 arrival times and close to the *pP-P* depth estimate of 17km.

5. Discussion

We have been able to demonstrate the systematic use of Neighbourhood Algorithm inversion for source characterization with no need for differentiation. We have shown here the construction of suitable solutions, but the information acquired in the course of the inversion can be used to provide estimates of *a posteriori* probability functions [*Sambridge*, 1999b]. This would not be necessary in the case of 4-D hypocentre location where a simple grid search would be practical. However, for the 8-dimensional joint depth/mechanism inversion, the NA can be used for 'resolution' analysis, provided the probability distributions of all noise processes are known. The NA procedure has advantages over grid search methods for location [*Kennett*, 1992] and source mechanism assessment [*Pearce*, 1977, 1980], because it is adaptive to the character of data fit and can be applied in higher dimensional spaces where regular searches are infeasible. With a limited amount of waveform data we are able to achieve an effective characterization of a source and have illustrated the value of using *SV* observations, where available, to improve constraints on the isotropic component of a moment tensor representation.

References

Aki, K. and P.G. Richards, *Quantitative Seismology: Theory and Methods*, W. H. Freeman, San Francisco, 1980.

Buland, R. and C.H. Chapman, The computation of seismic travel times, *Bull. seism. Soc. Am.*, **73**, 1271–1302, 1983.

Dziewonski, A., T.A. Chou and J.H. Woodhouse, Determination of earthquake source parameters from waveform data for studies of global and regional seismicity, *J. Geophys. Res.*, **86**, 2825–2852, 1981.

Dziewonski, A.M., G. Ekström and N.N. Maternovskaya, Centroid-moment tensor solutions for April-June, 1997, *Phys. Earth Planet. Int.* **112**, 1–9, 1999.

Goldstein, P. and D. Dodge, Fast and accurate depth and source mechanism estimation using P-waveform modeling: a tool for special event analysis, event screening, and regional calibration, *Geophys. Res. Lett.*, **26**, 2569–2572, 1999.

Kawakatsu, H., Automated near-realtime CMT inversion, *Geophys. Res. Lett.*, **22**, 2569–2572, 1995.

Kennett, B.L.N., Locating oceanic earthquakes - the influence of regional models and location criteria, *Geophys. J. Int.*, 108, 945–854, 1992.

Kennett, B.L.N., E.R. Engdahl and R. Buland, Constraints on seismic velocities in the Earth from travel times, *Geophys. J. Int.*, 122, 108–124, 1995

Langston, C.A. and D.V. Helmberger, A procedure for modelling shallow dislocation sources, *Geophys. J. R. astr. Soc.*, **42**, 117–130, 1975.

Pearce, R.G., Fault plane solutions using relative amplitudes of P and pP, *Geophys. J. R. astr. Soc.*, **50**, 381–394, 1977.

Pearce, R.G., Fault plane solutions using relative amplitudes of P and surface reflections: further studies, *Geophys. J. R. astr. Soc.*, **60**, 459–487, 1980.

Sambridge M.S., 1998. Exploring multidimensional landscapes without a map, *Inverse Problems*, 14, 427–440.

Sambridge, M.S., Geophysical inversion with a neighbourhood algorithm -I. Searching a parameter space, *Geophys. J. Int.*, 138, 479–494, 1999a.

Sambridge, M.S., Geophysical inversion with a neighbourhood algorithm -II. Appraising the ensemble, *Geophys. J. Int.*, 138, 727–746, 1999b

Sambridge, M.S. and B.L.N. Kennett, Seismic event location: nonlinear inversion using a neighbourhood algorithm, *Pageoph.*, in press, 2000.

Sipkin, S., Rapid determination of global moment-tensor solutions, *Geophys. Res. Lett.*, **21**, 1667–1670, 1994.

Voronoi, M. G., Nouvelles applications des paramètres continus à la théorie des formes quadratiques, *J. Reine Angew. Math.*, 134, 198–287, 1908.

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