Geophys. J. Int. (2004) **156,** 648–654 doi: 10.1111/j.1365-246X.2004.02123.x

RESEARCH NOTE

Constraints on earthquake epicentres independent of seismic velocity models

T. Nicholson,[1] Ó. Gudmundsson[2] and M. Sambridge[3]

[1]*Research School of Earth Sciences, Australian National University, Canberra ACT* 0200, *Australia. E-mail: todd@rses.anu.edu.au*
[2]*Danish Lithospheric Centre, Øster Voldgade* 20, 1350 *Copenhagen K, Denmark. E-mail: og@dlc.ku.dk*
[3]*Centre for Advanced Data Inference, Research School of Earth Sciences, Australian National University, Canberra ACT* 0200*, Australia. E-mail: malcolm@rses.anu.edu.au*

Accepted 2003 July 31. Received 2003 July 15; in original form 2002 May 31

SUMMARY

We investigate the constraints that may be placed on earthquake epicentres without assuming a model for seismic wave speed variation within the Earth. This allows location improvements achieved using 1-D or 3-D models to be put into perspective. A simple, arrival order misfit criterion is proposed that may be used in standard location schemes. The arrival order misfit criterion does not use a seismic velocity model but simply assumes that the traveltime curve for a particular phase is monotonic with distance. Greater robustness is achieved by including a contribution from every possible pairing of stations and the effect of timing inconsistencies reduced by smoothing. An expression is found that relates the smoothing parameter to the number of observations. A typical event is studied in detail to demonstrate the properties of the misfit function. A pathological case is shown that illustrates that, like other location methods, the arrival order misfit is susceptible to poor station distribution. 25 ground truth and 5000 other teleseismically observed events are relocated and the arrival order locations compared to those found using a least-squares approach and a 1-D earth model. The arrival order misfit is found to be surprisingly accurate when more than 50 observations are used and may be useful in obtaining a model independent epicentre estimate in regions of poorly known velocity structure or the starting point for another location scheme.

Key words: body waves, earthquake location, earthquakes, *P* waves, seismology, traveltime.

1 INTRODUCTION

Almost all earthquake location procedures rely on a model for the seismic velocity structure of the Earth. One-dimensional earth models, such as the JB tables (Jeffreys 1940), PREM (Dziewonski & Anderson 1981), IASP91 (Kennett & Engdahl 1991) and *ak135* (Kennett *et al.* 1995), are commonly used in teleseismic location, particularly for routine hypocentre determination. Recently 3-D models, such as S&P12/WM13 (Su & Dziewonski 1993), BDP98 (Boschi & Dziewonski 1999) and VWE97 (van der Hilst *et al.* 1997), have also been used to relocate small numbers of events. However, locations found using high-resolution block models are surprisingly poor compared to those of spherical harmonic models (Antolik *et al.* 2001). In a global context the uncertainty of earthquake location outside regions of dense station networks is still of the order of tens of kilometres.

It is self evident that inaccuracies in the earth model will map into event mislocation. Tradeoffs between structure and location are well known and have been studied by a number of authors (e.g. Crosson 1976). Pattern recognition techniques are often used informally in the location of mining explosions and may, in some cases, be used independent of an earth model (Nicholson *et al.* 2002). However, few studies have examined what level of constraint may be placed on hypocentre or epicentre parameters without using an assumed model of seismic wave speed variation. Here we address this question and by doing so we hope to determine a 'reference accuracy' for model independent location. Location schemes using an earth model should (hopefully) exceed this reference accuracy, and the degree to which they do will provide a measure of the effectiveness of the earth model. In the same way the relative improvements in location achieved with 3-D earth models over a 1-D models (e.g. Smith & Ekstrom 1996) may also be put into perspective. Anderson (1981) examined the degree of constraint placed on earthquake epicentres using only the order in which a phase was observed at stations in a local network. In this paper we adapt Anderson's original arrival order technique to epicentre determination using teleseismic observations. We modify the arrival order method to reduce mislocation caused by observational noise and traveltime perturbations caused by lateral heterogeneity. The modified arrival order technique is tested using a large number of globally distributed events

from the EHB catalogue (Engdahl *et al.* 1998) and a set of ground truth events that have previously been used to evaluate the accuracy of model based approaches (Smith & Ekstrom 1996; Antolik *et al.* 2001; Piromallo & Morelli 2001).

2 THE ARRIVAL ORDER METHOD

Anderson (1981) proposed the arrival-order method for the determination of earthquake epicentres in a local earthquake network. The basic notion is quite simple. If travel time, and thereby arrival time, is a monotonic function of distance, then arrival-time order is a valid proxy for distance order. For each pair of observations of the same phase it can therefore be assumed that the epicentre lies closer to the station that made the earlier observation (i.e. if the bisector between the two stations is drawn, then the epicentre lies on the side closest to the station that made the earlier observation). Combining such information from all pairs of observations allows the region that is consistent with all arrival order constraints to be identified.

In the presence of triplications in the traveltime curve care must be taken when applying the arrival order method. The traveltime curve may not be monotonic with distance if arrivals from different branches are compared. However, the arrival order method may still be used provided only arrivals from the same branch are compared and that traveltime is monotonic with distance on that branch. For retrograde branches this requires the earlier arrival actually to be at the more distant station. For simplicity, throughout the remainder of this paper we shall assume that only phases which have monotonically increasing traveltime curves are used.

A simple example of arrival order location, from teleseismic arrivals is illustrated in Fig. 1. Four stations are shown that reported P arrivals for an event near Fiji along with the bisectors of every possible pair of stations. Whereas the bisectors were simply lines in Anderson's local study, in global location the bisectors become great circles. In Fig. 1 the stations are labelled according to their arrival order. Therefore, the arrival order epicentre lies to the east of the bisector of stations 1 and 2, north west of the bisector of stations 3 and 4, and south west of the bisector of Stations 2 and 3. The area defined, which we call the optimal region, is labelled A in Fig. 1. The remaining three bisectors are not directly used to constrain the epicentre. If there is no region that satisfies all constraints then the one that satisfies the most constraints becomes the most likely region.

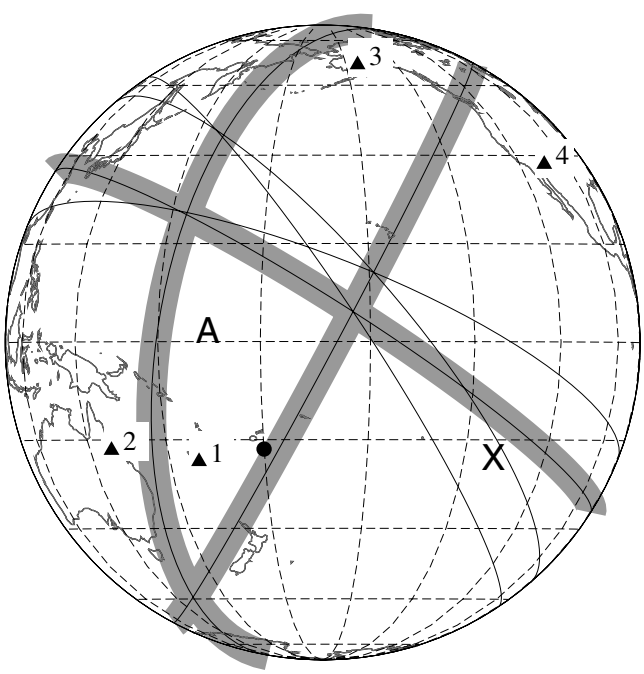

Figure 1. A simple example to demonstrate the arrival-order method for four stations (solid triangles). The stations are numbered in the order they observed the event. The six constraints are shown as solid lines and the three shaded constraints define a feasible region (marked with an A) for the event. The constraint X is redundant. The true position of the event is marked with a filled circle.

Sambridge & Kennett (1986) used the arrival-order method as part of a regional location algorithm for an area with a poorly constrained velocity structure in southeastern Australia. They realized that one can formulate the method using Heavyside functions across the corresponding bisector that have the value of one on the side of the earlier observation and zero on the side of the later observation. This effectively replaced the graphical approach of Anderson (1981) with the contouring of a sum of Heavyside functions. However, their approach still only directly uses a small number of the constraints to define the optimal region and may be susceptible to the effects of observational noise and lateral heterogeneity. Anderson's approach involves solving a set of linear inequality constraints, which may be inconsistent, while Sambridge & Kennett (1986) introduced a rank function, which may have discontinuous derivatives and is not suitable for standard location schemes using a local, derivative based optimization. In this paper we adapt, the arrival order method to provide a differentiable misfit measure suitable for use with standard optimization schemes. In the next section we present an arrival order misfit measure, which alleviates the effects of lateral heterogeneity and errors in picking and directly uses the information in the complete set of constraints.

3 THE ARRIVAL-ORDER-FITNESS MEASURE

The arrival-order method of Anderson (1981) provides an optimal region and directly uses only a small subset of the available inequality constraints. The size and shape of the optimal region depends critically on the small number of bisectors that form its boundaries. If the data were error free and not affected by lateral heterogeneity, there would be no additional information in the redundant inequality constraints (i.e. the unused bisectors) and their use would not improve the accuracy of the epicentre. However, in principle, all the bisectors provide some constraint on the epicentre, even though they are not used in the original arrival-order approach. It is reasonable to presume that if all bisectors could be used in some systematic way then we could increase the constraint on location. We therefore propose the introduction of a global measure of arrival order misfit based on every pair of constraints.

Anderson's original approach can be formulated in terms of a misfit function, $k(x)$:

$$k(x) = j_s(x) - j_{ns}(x), \tag{1}$$

where j_s is the number of constraints satisfied and j_{ns} is the number of constraints not satisfied. The optimal region is the region within which k is at a maximum. Eq. (1) can also be stated as

$$k(x) = \sum_{i=1}^{N} \frac{d_i(x)}{|d_i(x)|}, \tag{2}$$

where d_i is the distance, in km, from the point x to the nearest point on the ith bisector and N in the total number of bisectors. The distance d_i is taken as positive when x is on the same side of the bisector

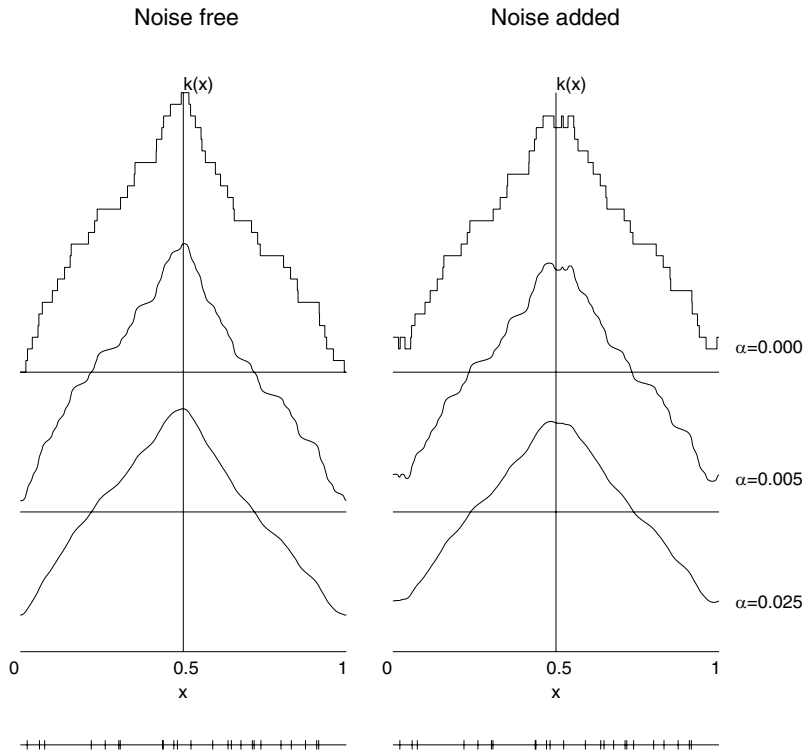

Figure 2. Smoothing of the arrival-order-misfit measure demonstrated with a one-dimensional example. Twenty four stations are distributed at random (uniform deviate) along the line $0 < x < 1$. An event is located at $x = 0.5$. The x-axis wraps around and thus the bisector consists of two points, making the arrival-order misfit is periodic. The curves show the misfit function at three different damping parameters ($\alpha = 0.000$, $\alpha = 0.005$ and $\alpha = 0.025$, respectively from top to bottom). The curves on the left show a case with no noise added, and those on the right the case with random Gaussian noise of standard deviation 0.1 added. The noise generates multiple extrema in the optimal region around the true location. Damping removes these multiple extrema.

as the station that made the earlier observation, and negative on the other side. Again the optimal region is the one for which $k(x)$ is a maximum.

This fitness measure is not ideal because the function $k(x)$ contains no information within the optimal region since it is constant. Furthermore, there is no smoothing over potentially inconsistent constraints in this approach. The arrival times contain an uncertainty and lateral velocity heterogeneity exists in the Earth giving rise to deviations from a monotonic traveltime curve. We can combine these effects in a time deviation, δt. For a given horizontal slowness, p, that time deviation corresponds to a distance deviation, $\delta x = \delta t/p$. We can either say that any inequality constraint smaller than δt in time difference cannot be trusted, or that we cannot locate the true bisector more precisely than to within the scale δx. We choose the later and smooth the function shown in eq. (2) to obtain

$$k(x) = \sum_{i=1}^{N} \frac{d_i(x)}{\alpha + |d_i(x)|}, \tag{3}$$

where α is a smoothing parameter. Note, that eq. (3) reduces to eq. (2) when $\alpha = 0$. Note also, that when d_i is small (i.e. x is close to the ith bisector) the contribution to k is small, reflecting the fact that a small picking error or lateral heterogeneity could cause the arrival orders of the two stations to be reversed. Conversely, when d_i is large the contribution to k is almost 1.

The effect of the smoothing is demonstrated by a 1-D example in Fig. 2, where we plot k as a function of x for three values of the smoothing parameter in both the noise free and noisy case. A random set of 24 stations is distributed along a line $0 < x < 1$. This line is assumed to be periodic and therefore wraps around on to itself. An event is placed at $x = 0.5$. The positions of the 24 stations are plotted on the line at the base of the figure. On the left we have used the distance between the stations directly to compute the function $k(x)$ for three damping parameters, $\alpha = 0.000$, 0.005 and 0.025. On the right hand side we have plotted $k(x)$ for the same smoothing parameter values, but added an uncertainty, δx (a Gaussian zero mean random variable with a standard deviation of 0.1) to the distance measurement for each station pair. This results in some inconsistent data, which cause multiple extrema in the optimal region. For sufficiently large values of the smoothing parameter, smoothing removes the multiple maxima and facilitates an unambiguous choice of location at a global maximum in $k(x)$. Note that every bisector constraint makes a contribution to k that varies as a function of x, so the constraints that Anderson (1981) did not use are no longer redundant. This improves the robustness of the approach against the effects of noise and lateral heterogeneity.

The arrival-order method has not been used previously for epicentre determination using teleseismic observations at globally distributed stations. In some cases the assumption that traveltime increases monotonically with distance can be inaccurate, such as, when stations lie in the distance range corresponding to triplications in the traveltime curve. To reduce the effects of triplications we have restricted our studies to the use of first arrivals of the phases P, PKP and PKiKP. The observations used here come from the EHB global data base (Engdahl *et al.* 1998). We have found that it is common for heterogeneity and picking errors to lead to inconsistencies in the constraints when the number of observations exceeds about 30 (435 bisectors).

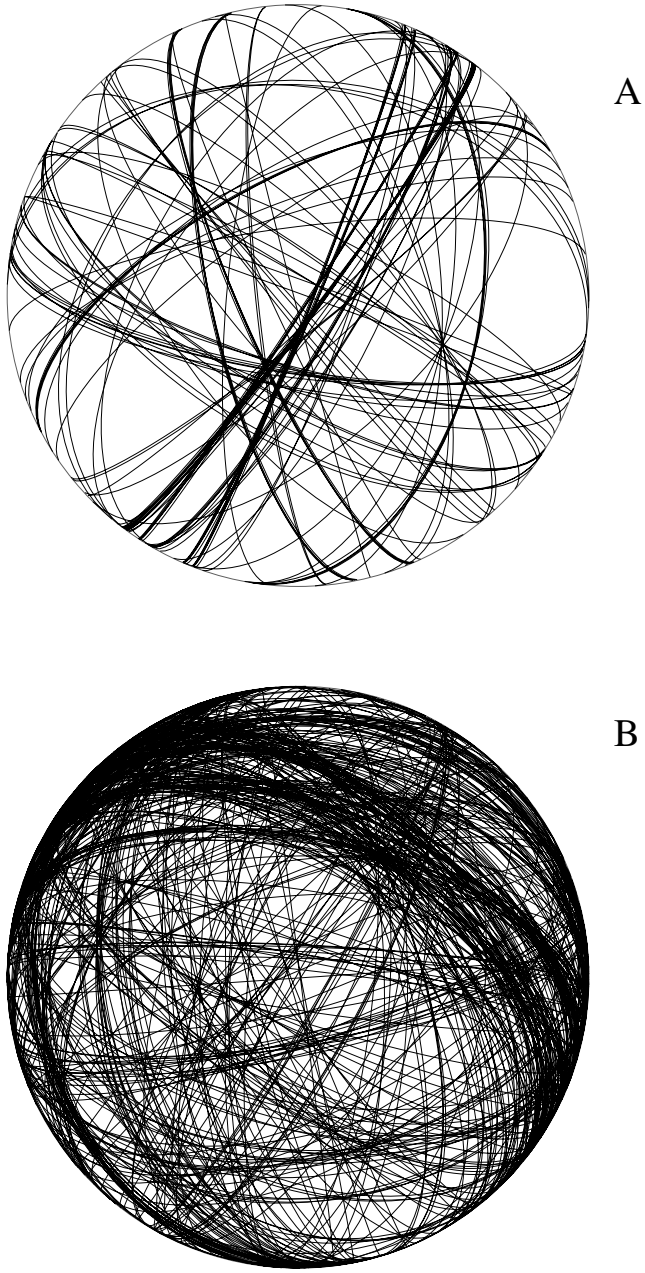

Figure 3. Examples of the bisectors for two events in the EHB catalogue. (a) An event observed at 16 stations (120 bisectors). (b) An event observed at 30 stations (435 bisectors). The number and coverage of the constraints improves quadratically with the number of observations.

4 CHOOSING THE SMOOTHING PARAMETER

The size of the optimal region is strongly dependent on the number of bisectors and hence the number of stations. The number of bisectors, m, is given by,

$$m = \frac{n(n-1)}{2}, \tag{4}$$

where n is the number of arrival-time observations. Therefore, the number of bisectors depends quadratically on the number of arrival-time observations. Fig. 3 shows the effect of this nonlinear dependence for two real events. Notice, that in Fig. 1, with four observations, each region defined by the bisectors is very large, but with 16 observations (Fig. 3a) the regions are much smaller and with 30 (Fig. 3b) they are smaller still. Note also that in Fig. 3 there is significant clustering of bisectors, caused by the non-uniform distribution of recording stations. As with most location procedures the poor distribution of stations can lead to a bias in the epicentre determination.

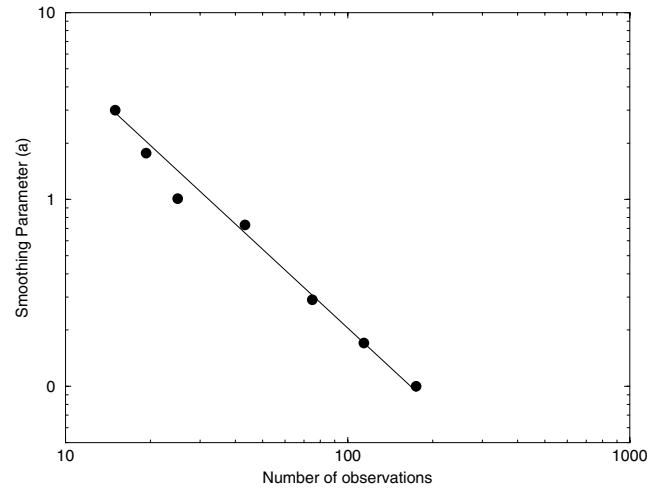

Figure 4. The optimal smoothing parameter as a function of the number of observations. Note the clear linear tend shown.

To estimate an appropriate smoothing parameter we selected 4 groups of events based on the number of observations reported (10–20, 20–50, 50–100, 100–250 observations), each containing 1000 events chosen at random from the EHB catalogue. These events were not used in the remainder of this study to avoid circularity. For each group, we defined an optimal smoothing parameter as that which minimizes the mean distance between the EHB location and the epicentre (Fig. 1). This objective roughly goes hand in hand with the objective to smooth over multiple maxima. This represents a compromise between smoothing real signal (i.e. if α is too large) and allowing local maxima when $k(x)$ becomes steplike (i.e. if α is too small). A large range of smoothing parameters gave satisfactory results in each case.

The results are shown in Fig. 4. A reasonable empirical fit to the relationship between the optimal value of the smoothing parameter, α (in km), and the number of arrivals is given by,

$$\alpha = \frac{230}{n^{1.5}}, \tag{5}$$

which is shown as the line in Fig. 4. However, there is an indication that the slope lessens with increasing number of observations. Beyond 25 observations the best slope through three points is -1.15. It is reasonable to expect that the smoothing parameter required to smooth over local extrema would reflect the width of these extrema. Since the number of regions defined by the bisectors of n stations depends quadratically on n, we expect the average area of each region to depend inversely quadradically on n and the average width to depend inversely on n. The behaviour at large numbers of observations is indeed similar to this. We suggest that the fact that we derive a stronger dependence on n in eq. (5), particularly at low numbers of observations, is due to the clustering of stations over the globe, which can have a stronger effect when small numbers of observations are used than when large number are used.

With the value of α determined, we have a tuned, continuous fitness function that can be optimized with gradient or direct search methods. In the following experiments we use this new fitness

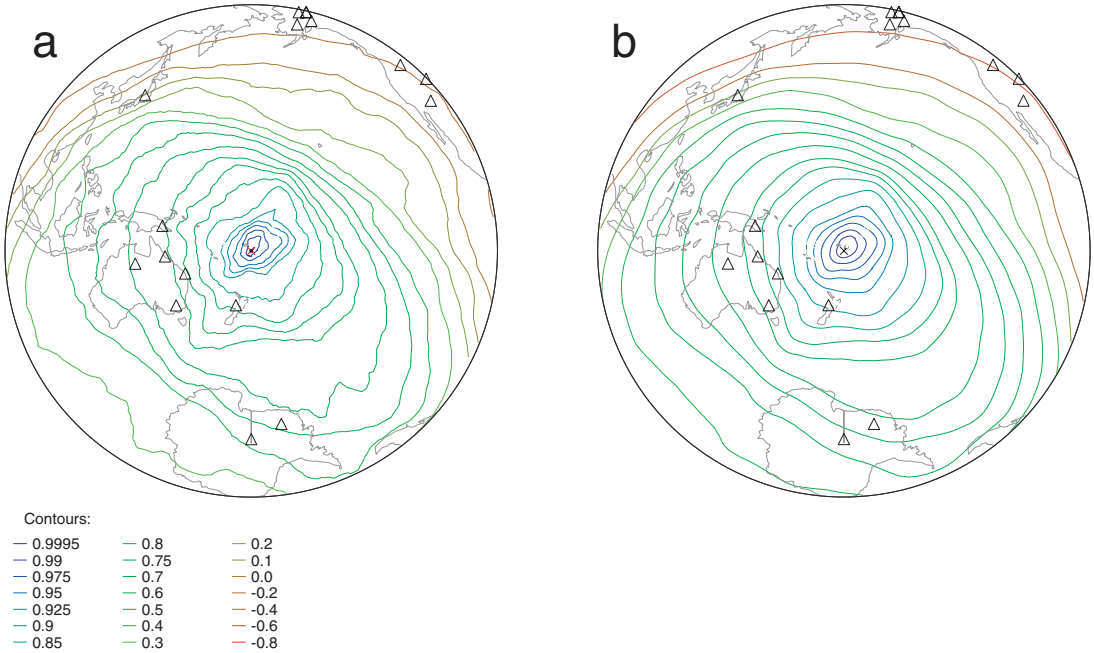

Figure 5. Contours of arrival-order fitness for for the event shown in Fig. 3(b) with (a) $\alpha = 0.3$ km, (b) $\alpha = 3.0$ km. This magnitude 4.1 (mb) event occurred at 10:13 on October 25th 1968 and was observed at 30 stations. The EHB location of this event is 19.759S, 179.980E and a depth of 462.1 km. The contour values are normalized (divided) by the maximum fitness. Over 99 per cent of the 435 constraints are satisfied. The cross shows the EHB location of this event, which is 32 km from the maximum.

function and the optimal smoothing parameter given by eq. (5) unless otherwise stated.

5 EXAMPLE APPLICATIONS OF THE ARRIVAL-ORDER METHOD

Examples of the fitness contours for two smoothing parameters (0.3 km and 3.0 km) are shown in Figs 5(a) and (b) respectively for an event near Fiji that was observed at 30 stations. The bisectors for this event are shown in Fig. 3(b). There is a trade off between smoothness and solution accuracy. In Fig. 5(a) the contours are much more jagged than in Fig. 5(b) and there is the potential for local extrema to exist, but the fitness maximum is close to the EHB location (32 km separation). Therefore, the smoothing parameter value and resulting misfit shown in Fig. 5(a) would be unsuitable for gradient optimization methods. For $\alpha = 3.0$ km, the contours are much

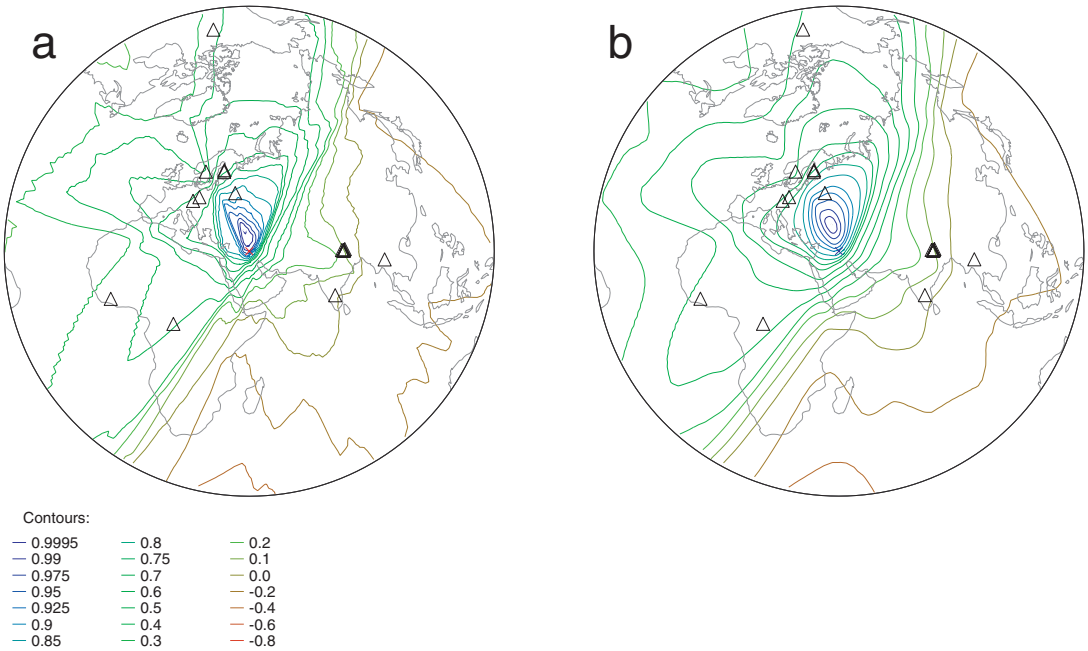

Figure 6. Same as Fig. 5, but for an event with a poor station distribution (see Fig. 3a). As before (a) $\alpha = 0.3$ km, (b) $\alpha = 3.0$ km. This magnitude 4.5 (mb) event occurred at 8:14 on January 7th 1993 and was observed at 16 stations. The EHB location of this event is 34.188N, 45.375E and a depth of 47.3 km. All 120 constraints are satisfied. The cross shows the EHB location of this event and the triangles are the stations.

smoother, but the fitness maximum is further from the EHB location (56 km). In this case an optimal smoothing parameter value is approximately $\alpha = 1.4$ km, which gives a distance of 33 km from the EHB location, and does not produce local extrema. For teleseismic P waves with slowness ranging from 4.5 to 8.5 s per degree this smoothing parameter corresponds to $\delta t = 0.1$ s.

A pathological case is shown in Fig. 6, also for damping parameters of $\alpha = 0.3$ and 3 km. This event in eastern Iraq has only 16 observations, which fall into two clusters in Europe and southern Asia. The 120 bisectors are shown in Fig. 3(a). Here the optimal damping parameter is approximately $\alpha = 4$ according to eq. (5), but should perhaps be higher due to the high degree of clustering. The solution is dragged towards the European cluster of stations because the event occurred slightly closer to it than the Asian stations. At the higher level of damping the solution lies approximately 1000 km from the EHB location. Clearly, the solution is vulnerable to bias when so few stations are present and their distribution is unfavourable. For both the arrival-order method and conventional location procedures a wider spread of stations is required to improve accuracy. Note that the arrival-order epicentre locates this event within a highly active zone in Turkey whereas the EHB location lies on the edge of a less seismically active region in Iraq.

6 ARRIVAL-ORDER EPICENTRES OF EHB CATALOGUE EVENTS

As with other location procedures the arrival-order method tends to be most accurate when the number of observations is large (i.e. >25) and least accurate when only a small number (i.e. <20) of arrival times are available. In order to investigate how well one might constrain epicentres independent from an earth model across the globe we used the arrival-order procedure to determine the epicentres of 5000 globally distributed events from the EHB data base and compared the locations to those given in the catalogue. 1250 events were selected randomly from each of the ranges 15–20, 25–30, 60–90 and 150–200 observations. The results are summarized in Fig. 7. The inaccuracy of epicentre estimates obtained when the number of observations is less than 20 is not just a result of there being fewer bisectors, but is also because the stations tended to form clusters. This leads to an uneven bisector coverage and hence poor constraints on the epicentre, similar to that shown in Fig. 6. When the number of observations exceeds 25 the arrival-order solution reproduce the

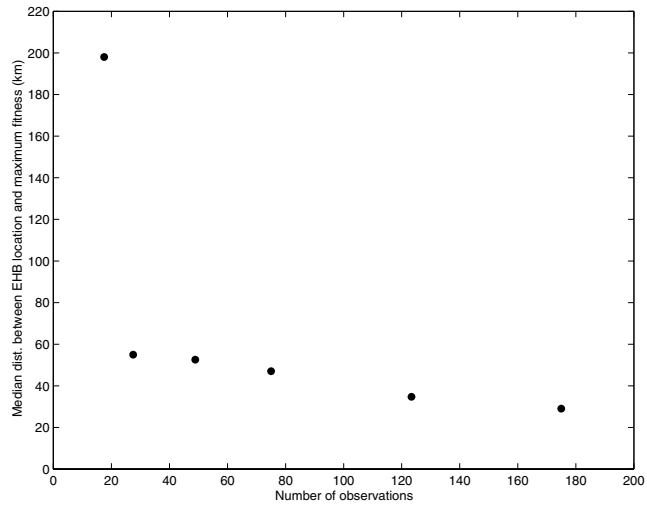

Figure 7. The median distance between EHB locations and arrival-order epicentres as a function of the number of observations. 1250 events were used in each group.

Table 1. Arrival order mislocations for the 25 explosions used by Smith & Ekstrom (1996).

Id	Year	Month	Day	Time	Latitude (degrees)	Longitude (degrees)	Arrival order mislocation (km)
A	1965	7	15	14:16:08.10	37.197	−74.352	31.39
B	1967	5	20	15:00:00.20	37.130	−116.064	17.82
C	1968	1	19	18:15:00.10	38.634	−116.215	2.21
D	1968	4	26	15:00:00.10	37.295	−116.456	16.84
E	1969	9	10	21:00:00.10	39.406	−107.948	1.78
F	1969	11	30	3:32:59.70	49.924	78.956	4.61
G	1971	4	25	3:32:59.90	49.769	78.034	26.40
H	1971	11	6	22:00:00.06	51.472	179.107	24.33
I	1972	8	16	3:16:59.80	49.765	78.059	42.17
J	1972	11	2	1:27:00.20	49.927	78.758	5.18
K	1973	5	17	16:00:00.00	39.793	−108.366	16.33
L	1977	8	10	22:00:00.10	50.956	110.983	18.07
M	1983	7	10	3:59:59.99	51.362	53.306	12.59
N	1983	7	10	4:04:59.94	51.367	53.327	10.81
O	1983	7	10	4:09:59.85	51.380	53.340	21.24
P	1983	9	24	5:00:00.03	46.783	48.315	15.17
Q	1983	9	24	5:05:00.03	46.788	48.297	11.92
R	1983	9	24	5:10:00.08	46.767	48.311	18.29
S	1983	9	24	5:15:00.14	46.749	48.303	2.71
T	1983	9	24	5:19:59.93	46.754	48.289	5.15
U	1983	9	24	5:25:00.00	46.766	48.274	54.53
V	1984	7	21	2:59:59.81	51.358	53.319	11.06
W	1984	7	21	3:04:59.71	51.390	53.351	5.55
X	1984	7	21	3:09:59.84	51.371	53.337	5.45
Y	1988	7	7	15:05:30.07	37.252	−116.377	1.52
Root Mean Square Mislocation (km)							19.93
Median Mislocation (km)							12.59

EHB epicentres with a misfit (<50 km) that is only slightly larger than the EHB location uncertainty. Of the events in the EHB catalogue 18 per cent have less than 50 observations, while only 2 per cent have less than 20 observations. Therefore, the arrival-order approach is expected to achieve a level of accuracy that approaches that of the EHB locations in approximately 82 per cent of cases.

7 THE ARRIVAL ORDER RELOCATION OF NUCLEAR EXPLOSIONS

To get a reliable measure of the accuracy of arrival order epicentres the AO method is applied to the relocation of 25 explosions. These 25 events are the same as those used by Smith & Ekstrom (1996) and Piromallo & Morelli (2001). Smith & Ekstrom (1996) found an rms mislocations for these events of 14.80 km for locations found using the JB tables, and 14.32 km for locations using PREM. Only teleseismic arrivals were used in the AO relocations.

The AO mislocations are shown in Table 1. The rms mislocation was found to be 19.93 km. Clearly, events I and U are very poorly located using the AO approach, possibly due to a geographical bias in their distribution of stations (predominantly in Europe). The rms mislocation of the remaining 23 events is 15.00 km. The epicentres of those events with very good station coverage, such as J, W and X, are well constrained by the arrival order method. So much so that the arrival order epicentres of nine events are more accurate than the locations found by Piromallo & Morelli (2001) using SP6 (Morelli & Dziewonski 1993) and 3-D station corrections. Note that comparisons were only possible for 23 of these events since Piromallo & Morelli (2001) only give mislocations for 23 events. The AO locations of 17 events lie with the 1000 sq. km inspection area prescribed by the CTBT. Therefore, we conclude that in the majority of cases, a model independent method based only on arrivals orders can achieve accuracies which approach that of standard location techniques based on a 1-D velocity model.

The rms mislocations of the PREM, IASP91 and JB table locations were less than those of the AO locations by 5.61, 6.69 and 5.13 km, respectively, for these 25 explosions (Smith & Ekstrom 1996). This is comparable to the improvement found when azimuthally varying station corrections are added to JB traveltimes (Smith & Ekstrom 1996). The difference between AO and 1-D model mislocation is also less than the improvement found using 3-D models instead of 1-D models.

8 DISCUSSION AND CONCLUSIONS

The robustness of the arrival order method, achieved via the direct use of all the bisector constraints, suggests that the method is potentially useful to determine epicentres in remote areas of the globe. The method may be suitable for those regions since there the velocity structure in the heterogeneous outer layers of Earth is often poorly known, and the reference models used in standard location procedures may not be representative (e.g. in the oceans). The method may also be useful in obtaining a model-independent starting point for other location schemes, which generally require a reasonable initial location and may be used in conjunction with an approach that determines depth and origin time independent of epicentre (e.g. Billings 1994). Regional and local observations can also be incorporated in a straight forward manner.

We have proposed the arrival order method as a way of producing a model independent reference accuracy for teleseismic location. We do not argue that the arrival order method may give more accurate epicentres than those found using 1-D models. One would hope that model based techniques would always improve upon this location, however our experiments suggest that in the majority of cases the AO epicentres are only slightly less accurate than the epicentres produced from 1-D velocity models. We have shown that reasonably accurate teleseismic epicentre determinations can be obtained without the use of a velocity model when the number of observations exceeds 25. In particular, the relocation of ground truth events has demonstrated that epicentres found using the JB tables and PREM are only 5.13 and 5.61 km more accurate respectively than those found using the AO method. The arrival order epicentres are surprisingly accurate given the simplicity of the approach and its assumptions about the velocity structure of the Earth. When a large number of observations is available (>150) the arrival-order method reproduces EHB epicentres to within about 25 km, which is comparable to formal uncertainty estimates for EHB locations in regions outside dense station networks.

ACKNOWLEDGMENTS

We are grateful to referee Mike Antolik and an anonymous referee, as well as Günter Bock, for helpful suggestions which improved presentation of the manuscript.

REFERENCES

Anderson, K.R., 1981. Epicentral location using arrival time order, *Bull. seism. Soc. Am.,* **30,** 119–130.

Antolik, M., Ekstrom, G. & Dziewonski, A.M., 2001. Global Event Location with full and Sparse Data Sets Using Three-dimensional Models of Mantle *P*-wave Velocity, *Pure appl. Geophys.,* **158,** 291–317.

Billings, S.D., 1994. Simulated Annealing for Earthquake Location, *Geophys. J. Int.,* **118,** 680–692.

Boschi, L. & Dziewonski, A.M., 1999. High and low resolution images of the Earth's mantle: Implications of different approaches to tomographic modeling, *J. geophys. Res.,* **104,** 25 567–25 594.

Crosson, R.S., 1976. Crustal structure modelling of earthquake data 1. Simultaneous least squares estimation of hypocenter and velocity parameters, *J. geophys. Res.,* **81,** 3036–3046.

Dziewonski, A.M. & Anderson, D.L., 1981. Preliminary Reference Earth Model, *Phys. Earth planet. Inter.,* **25,** 297–356.

Engdahl, E.R., van der Hilst, R.D. & Bullen, R.P., 1998. Global teleseismic earthquake relocation with improved traveltimes and procedures for depth determination, *Bull. seism. Soc. Am.,* **88,** 722–743.

Jeffreys, H. & Bullen, K.E., 1940. *Seismological Tables,* British Association for the Advancement of Science.

Kennett, B.L.N. & Engdahl, E.R., 1991. Traveltimes for global earthquake location and phase identification, *Geophys. J. Int.,* **105,** 429–465.

Kennett, B.L.N., Engdahl, E.R & Buland, R., 1995. Constraints on seismic velocities in the Earth from travel times, *Geophys. J. Int.,* **122,** 108–124.

Morelli, A. & Dziewonski, A.M., 1993. Body Wave Travel Times and a Spherically Symmetric *P*- and *S*-wave velocity model, *Geophys. J. Int.,* **112,** 178–194.

Nicholson, T.A., Sambridge, M.S. & Gudmundsson, O., 2002. Hypocentre Location by Pattern Recognition, *J. geophys. Res.,* **107**, 10.1029/2000JB000035 19 June 2002.

Piromallo, C. & Morelli, A., 2001. Improving Seismic Event Location: An Alternative to Three-dimensional Structural Models, *Pure appl. Geophys.,* **158,** 319–348.

Sambridge, M.S. & Kennett, B.L.N., 1986. A novel method of hypocentre location, *Geophys. J. R. astr. Soc.,* **87,** 679–697.

Smith, G.P. & Ekstrom, G., 1996. Improvinsmic event location using a three-dimensional Earth model, *Bull. seism. Soc. Am.,* **86,** 788–796.

Su, W. & Dziewonski, A.M., 1993. Joint 3-D Inversion for *P* and *S* Velocity in the Mantle, *EOS,* **74,** 557.

Van der Hilst, R.D., Widiyantaro, S. & Engdahl, E.R., 1997. Evidence for deep mantle circulation from global tomography, *Nature,* **186,** 578–584.