JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 99, NO. B11, PAGES 21,969–21,970, NOVEMBER 10, 1994

Reply

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In their comment, *Farra et al.* [1994] (hereinafter referred to as FMV) point out that the erroneous solutions to the equations for ray perturbation theory presented by us are not due to a deficiency of Hamiltonian ray perturbation theory, as we suggested [*Snieder and Sambridge,* 1993] (hereinafter referred to as S&S.) Instead, the unphysical behavior of the rays in the examples shown by us is due to the incorrect application of theory developed for initial value problems to two-point boundary value problems. Our research was partly spurred by remarks from colleagues who failed to obtain accurate ray estimates using the theory of FMV. The issue of the correct application of the stretch factor (S&S) or, alternatively, of the importance of the projection operation (FMV) for the solution of two-point ray-tracing problems has been made explicit by this discussion.

It is gratifying to see that FMV explicitly showed that Hamiltonian and Lagrangian ray perturbation theory lead to the same ray perturbation and that the ambiguities associated with the choice of the stretch factor (S&S) shows up in Hamiltonian ray perturbation theory in the choice of the sampling paramameter ξ of FMV. The proof of FMV that the projection leads to first order in the ray perturbation $\mathbf{r}_1$ to the same result as the use of a stretch factor shows that for the ray deflection one can use either method. The choice of a Hamiltonian or a Lagrangian formulation is then a matter of taste; there is no objective criterion why one method would be superior to the other. The statement of FMV that they prefer to solve two first-order equations rather than one second-order equations is unrelated to the choice of a derivation based on either Hamiltonian or Lagrangian techniques; one can write the second-order equations derived by S&S in many ways as a system of coupled first-order equations. Alternatively, one can combine the system of first-order equations derived by FMV as a single second-order equation. Using a judicious choice of the parameterization and the stretch factor, we found an efficient formulation using a second-order differential equation that leads after discretization to a sysyem of tridiagonal equations that can be solved with great efficiency [*Pulliam et al.*, 1993]. It is an order N process, with N being the number of sampling points along the ray, whereas the calculation of the propagators of FMV is an order N^2 process.

As shown by FMV, Hamiltonian and Lagrangian ray perturbation theory lead to leading order to the same perturbation of the ray position. It is arbitrary which formulation one employs for the computation of the perturbation in the ray position. This does not hold for the travel time. Since the travel time is the Lagrangian for the ray-tracing problem, one needs by definition to apply perturbation theory to the Lagrangian when one wants to construct the perturbation of the travel time. Although FMV show the first-order equivalence of stretching and projection for the ray position, they do not show whether these strategies lead to the same second-order travel time perturbation T_2 derived by S&S. Consider two solutions $\mathbf{r}_1$ for the ray perturbation that differ in second-order in the perturbation parameter ε. Such a second-order difference in $\mathbf{r}_1$ may lead according to equation (44a) of S&S to related travel time changes that are different in second-order. The associated second-order travel time perturbations may thus be different. One can easily see that for two-point ray-tracing problems the travel times corresponding to the different ray perturbations are identical to second-order. The reason for this is that only the values of the perturbation $\mathbf{r}_1$ at the endpoint enters equation (44a) for the first-order travel time perturbation of S&S. Different solution for the ray perturbation which have the same end points therefore lead to the same travel time perturbation (up to second-order). This is, however, not necessarily the case for application where one specifies at one (or more) of the end points the ray direction $\dot{\mathbf{r}}_1$ rather than the ray end points. In that situation the ray deflection computed by projection and by projection is not guaranteed to lead to the same second-order travel time perturbation.

We would like to thank FMV for clarifying the correpondence between Lagrangian and Hamiltonian ray perturbation theory.

Paper number 94JB01796.
0148-0227/94/94JB-01796$02.00

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(Received June 20, 1994; accepted July 6, 1994.)