*** N.B. The material presented in these lectures is from the principal textbooks, other books on similar subject, the research and lectures of my colleagues from various universities around the world, my own research, and finally, numerous web sites. I am grateful for some figures I used in this lecture to P. Wu, D. Russell and E. Garnero. I am thankful to many others who make their research and teaching material available online; sometimes even a single figure or an idea about how to present a subject is a valuable resource. Please note that this PowerPoint presentation is not a complete lecture; it is most likely accompanied by an in-class presentation of main mathematical concepts (on transparencies or blackboard).***
No **attenuation** (decrease in amplitude with distance due to spreading out of the waves or absorption of energy by the material), **dispersion** (variation in velocity with frequency), nor **anisotropy** (velocity depends on direction of propagation) is included.
Compressional Wave (P-Wave) Animation

Deformation propagates. Particle motion consists of alternating compression and dilation. Particle motion is parallel to the direction of propagation (longitudinal). Material returns to its original shape after wave passes.
Deformation propagates. Particle motion consists of alternating transverse motion. Particle motion is perpendicular to the direction of propagation (transverse). Transverse particle motion shown here is vertical but can be in any direction. However, Earth’s layers tend to cause mostly vertical (SV; in the vertical plane) or horizontal (SH) shear motions. Material returns to its original shape after wave passes.
## Seismic Body Waves

<table>
<thead>
<tr>
<th>Wave Type (and names)</th>
<th>Particle Motion</th>
<th>Other Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>P</strong>, <strong>Compressional</strong>, <strong>Primary</strong>, <strong>Longitudinal</strong></td>
<td>Alternating compressions (&quot;pushes&quot;) and dilations (&quot;pulls&quot;) which are directed in the same direction as the wave is propagating (along the raypath); and therefore, perpendicular to the wavefront.</td>
<td>P motion travels fastest in materials, so the P-wave is the first-arriving energy on a seismogram. Generally smaller and higher frequency than the S and Surface-waves. P waves in a liquid or gas are pressure waves, including sound waves.</td>
</tr>
<tr>
<td><strong>S</strong>, <strong>Shear</strong>, <strong>Secondary</strong>, <strong>Transverse</strong></td>
<td>Alternating transverse motions (perpendicular to the direction of propagation, and the raypath); commonly approximately polarized such that particle motion is in vertical or horizontal planes.</td>
<td>S-waves do not travel through fluids, so do not exist in Earth’s outer core (inferred to be primarily liquid iron) or in air or water or molten rock (magma). S waves travel slower than P waves in a solid and, therefore, arrive after the P wave.</td>
</tr>
</tbody>
</table>
The frequency of your birthday
LECTURE 5 - Wave Equation

\[ \frac{\partial^2 \phi}{\partial t^2} - c^2 \nabla^2 \phi = 0 \]

is a differential wave equation, describing the elastic waves.

C is the wave speed.

- 1D wave equation can be written as:

\[ \frac{\partial^2 u}{\partial x_1^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} \]

The general solution is:

\[ u(x_1, t) = f(x_1 - ct) + g(x_1 + ct) \]

These disturbances propagate with velocity \( c \) in +x_1 and -x_1 direction.

Let's assume that the solution has a form that separates spatial from temporal dependence:

\[ u(x_1, t) = X(x_1) \cdot T(t) \]

From the equations of motion that we derived, you can see that the speed at which the disturbance travels is:

\[ \sqrt{\frac{\lambda + 2\mu}{\rho}} \] for compressional waves

\[ \sqrt{\frac{\mu}{\rho}} \] for shear waves
LECTURE 5 - Wave Equation

\[ T(t) \frac{\partial^2 X(x_1)}{\partial x_1^2} = \frac{X(x_1)}{c^2} \frac{\partial^2 T(t)}{\partial t^2} \quad ; \quad \partial \to \partial \]

\[ c^2 \frac{1}{X(x_1)} \frac{d^2 X(x_1)}{dx_1^2} - \frac{1}{T(t)} \frac{d^2 T(t)}{dt^2} = 0 \]

- FUNCTION OF X_1 ONLY
- FUNCTION OF t ONLY \Rightarrow MUST BE CONSTANT

LET CONSTANT BE EQUAL TO \(-\omega^2\). WE THEN GET

\[ \frac{d^2 X(x_1)}{dx_1^2} + \frac{\omega^2}{c^2} X(x_1) = 0 \]
\[ \frac{d^2 T(t)}{dt^2} + \omega^2 T(t) = 0 \]

\[ \left\{ \begin{array}{l} X(x_1) = A_1 e^{i\omega x_1} + A_2 e^{-i\omega x_1} \\ T(t) = B_1 e^{i\omega t} + B_2 e^{-i\omega t} \end{array} \right. \]

THESE EQUATIONS HAVE THE FORM SATISFIED BY SIMPLE HARMONIC FUNCTIONS.

\[ \omega = 2\pi f \text{ - angular frequency} \]
\[ T = \frac{2\pi}{\omega} \text{ - period} \]
\[ k = \frac{\omega}{c} \text{ - wavenumber} \]

THUS, THE SOLUTION FOR \( u(x_1,t) \) BECOMES:

\[ u(x_1,t) = C_1 e^{i\omega(t+x_1\xi)} + C_2 e^{i\omega(t-x_1\xi)} + C_3 e^{-i\omega(t+x_1\xi)} + C_4 e^{-i\omega(t-x_1\xi)} \]

WHEN APPROPRIATE BOUNDARY CONDITIONS ARE APPLIED, IMAGINARY PARTS \( \to 0 \) GROUND DISPLACEMENTS ARE REAL FUNCTIONS.
Ground displacements recorded on seismograms

Ground displacement at HRV (Harvard) station, after March 3, 1985 Chilean earthquake
Amplitude of Body Waves & Surface Waves

- Surface waves have much larger amplitudes than Body waves because the energy is spread over a circle \((2\pi x)\) rather than a half sphere \((2\pi x^2)\)
- Surface wave energy decays with \(1/x\), surface wave amplitude decays as \(1/\sqrt{x}\)
- Body wave energy decays with \(1/x^2\), Body wave amplitude decays as \(1/x\)
A simulation of the San Simeon earthquake, CA, through a model of 3D structure. This is achieved using a numerical finite difference method on a grid of points.

The main wave front is visibly refracted or bent by contrasts in the velocity across both the Hayward and San Andreas faults.

Concentrations of high amplitude standing waves persist throughout the movie around San Jose and in San Pablo Bay. These areas are low-velocity sedimentary basins and cause the amplitudes of ground motion to be amplified as well as extend the duration of the motions.

Both of these factors increase the level of hazard to structures.

Courtesy of Prof. Douglas Dreger, UC Berkeley and Dr. Shawn Larsen, LLNL
Global seismic velocity field after large earthquakes

Source: Vala Hjorleifsdottir and Jeroen Tromp, Caltech
The nature of seismic waves

- Wave paths are “bent” when going deeper in Earth.
- P and S-wave seismic velocities generally increase with depth.
The nature of seismic waves

- wave paths are reflected or refracted when crossing from one material into another
Ray geometry: ray parameter

Let's consider the following geometry of a ray path:

Laterally homogeneous Earth (radial symmetry or 1D)

\[ \Delta - \text{epicentral distance} \]
\[ T - \text{travel time} \]

1) Sine rule
\[ \frac{r_0}{r_1} = \frac{\sin(\pi - i'_0)}{\sin i'_0} = \frac{\sin i'_0}{\sin i_0} \]

2) Snell's law
\[ \frac{v_0}{v_1} = \frac{\sin i_0}{\sin i_1} \]

From 1) and 2)
\[ \frac{v_0}{v_1} \sin i_1 = \frac{r_0}{r_1} \sin i'_0 = \frac{r_0}{r_2} \sin i_2 \ldots \]

\[ p = \frac{r}{v_1} \sin i_1 = \frac{r_0}{v_0} \sin i_0 = \frac{r_1}{v_2} \sin i_2 \ldots \]

\[ p = \frac{r}{v} \] Ray parameter is constant along the path

\[ \frac{r}{v} = \eta \Rightarrow p = \eta \sin i \Rightarrow p = \frac{r_M}{v_M} \]

\[ v_M - \text{velocity at the bottoming point} \]
Ray theory: travel time and epicentral distance

Now, let's derive $T$ and $\Delta$ using the ray parameter:

\[ \cos i = -\frac{d\tau}{ds} \]
\[ ds = -\frac{dt}{\cos i} \]
\[ v = \frac{ds}{dt} = \frac{-dt}{\cos i} \]
\[ dt = \frac{ds}{v} = \frac{-dr}{v \cos i} \]

\[ \frac{T}{2} = -\int_{t_0}^{t_M} \frac{dt}{v \cos i} \]

From $p = \gamma \sin i$, where $\gamma = \frac{1}{v}$
\[ \sin i = \frac{p}{\gamma} \]
\[ \cos i = \sqrt{1 - \frac{p^2}{\gamma^2}} \]
\[ T = -2 \int_{t_0}^{t_M} \frac{dt}{v \cos i} = -\int_{t_0}^{t_M} \frac{p}{\gamma \sqrt{1 - \frac{p^2}{\gamma^2}}} dt \]

**TRAVEL TIME**

\[ T = 2 \int_{t_0}^{t_M} \frac{\gamma^2}{\gamma^2 - p^2} dt \]

**EPICENTRAL DISTANCE**

\[ \Delta = 2 \int_{t_0}^{t_M} \frac{p}{\gamma \sqrt{\gamma^2 - p^2}} dt \]
Mode conversions and partitioning of seismic energy at a boundary

- S waves can be decomposed to SV (the motion within the vertical plane) and SH components (the motion perpendicular to the vertical plane)
- P can be transformed to SV and vice versa

Wave propagation:
- Stresses and displacements must be transmitted across the interface

- Free surface: all the stresses must be zero; no restrictions on displacements
- Solid-solid interface: all the stresses and displacements must be continuous
- Solid-liquid interface: tangential stresses must vanish; normal displacements and normal stresses must be continuous
EMSC8002 LECTURES 5/6 - Rays in spherically symmetric media: 1) Wiechert-Herglotz inversion for 1D velocity structure of the Earth

Hrvoje Tkalčić

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The law of reflection

- We have shown that the Snell’s law can be derived from Fermat’s principle.
- Let’s show the same for the law of reflection now.
- A bird in point A wants to feed her youngsters in point B choosing the path that will take a minimum time (which is exactly how waves travel)
- Q: Assuming that fishes are all over the lake, where does the bird touch the lake surface to catch a fish on its way to the nest?
- A: In point C, so that $\varphi_1 = \varphi_2$
- How do we show that the path ACB is the one that requires minimum time? Consider $A'$ - an image of A with respect to the surface - it is visible that any path AKB must be longer than ACB because any two sides of a triangle are longer than the third side ($A'K+KB$ is longer than $A'CB$).
Seismic “phases” and their nomenclature (some simple cases)
Construction of travel time curves (hodochrones)
Wiechert-Herglotz inversion

\[ \Delta = 2 \int_{\gamma_M}^{\gamma_0} \frac{p \, dt}{\sqrt{\gamma^2 - p^2}}; \gamma = \frac{r}{\nu} \]

**We derived:**

**We consider:**

\[ \gamma = \gamma(t) \]

\[ \Delta = \Delta(p) \]

\[ \eta = \gamma(t) \quad \leftrightarrow \quad t = t(\eta) \]

\[ \Delta = \int_{\gamma_M}^{\gamma_0} 2p \frac{dt}{t(\eta)\sqrt{\gamma^2 - p^2}} \frac{d\eta}{d\gamma} \]

**The deepest ray epicentral distance:**

\[ \Rightarrow \Delta_1 = \int_{\gamma_M}^{\gamma_0} \frac{2p_1}{t(\eta)\sqrt{\gamma^2 - p^1}} \frac{dt}{d\gamma} \frac{d\eta}{d\gamma} \cdot \frac{1}{\sqrt{\gamma^2 - \gamma_M^2}} \]
Herglotz-Wiechert inversion

\[ \Delta = \int_{p_1}^{p_0} \frac{2p}{\sqrt{p^2 - \gamma_{H_1}^2}} \frac{1}{\sqrt{p^2 - \gamma_{H_1}^2}} \, dp \]

\[ \int_{p_1}^{p_0} \frac{\Delta}{\sqrt{p^2 - \gamma_{H_1}^2}} \, dp = \int_{p_1}^{p_0} \frac{2p}{1 + \gamma_{H_1}^2} \frac{1}{\sqrt{p^2 - \gamma_{H_1}^2}} \, dp \]

\[ X = \int_{p_1}^{p_0} \Delta \, dp = \Delta \left( \text{arccosh} \frac{p}{\gamma_{H_1}} \right) = \Delta \left( \text{arccosh} \frac{p_0}{\gamma_{H_1}} \right) - \int_{\Delta(p_1)}^{\Delta(0)} \text{arccosh} \frac{p}{\gamma_{H_1}} \, d\Delta \]

\[ X = 0 \cdot \text{arccosh} \frac{p_0}{\gamma_{H_1}} - \Delta_1 \cdot \text{arccosh} \frac{p_0}{\gamma_{H_1}} - \int_{\Delta(0)}^{\Delta_1} \text{arccosh} \frac{p}{\gamma_{H_1}} \, d\Delta = \int_{\Delta_1}^{0} \text{arccosh} \frac{p}{\gamma_{H_1}} \, d\Delta \]

\[ X = \int_{0}^{\Delta_1} \text{arccosh} \frac{p}{\gamma_{H_1}} \, d\Delta \]
Wiechert - Herglotz inversion

THE RIGHT HAND SIDE:

\[
y = \int_{p_1}^{p_0} \int_{\frac{2p}{\gamma_N^2 - p^2}}^{p_0} \frac{1}{\sqrt{r^2 - p^2}} \frac{dr}{d\gamma} d\gamma = \int_{\frac{2p}{\gamma_H^2}}^{p_0} \frac{dp}{\gamma_H^2 - p^2} - \frac{2p}{\gamma_H^2 - p^2}
\]

SUBSTITUTION:

\[
(\gamma_H^2 - p^2)(p^2 - \gamma_N^2) = \left[ p^2 - \frac{1}{y} (\gamma_H^2 + \gamma_N^2) \right] - \frac{1}{y} (\gamma_H^2 - \gamma_N^2)^2
\]

Now equalizing the two sides:

\[
\int \frac{1}{2} \left( \frac{\gamma_N^2 - \gamma_H^2}{\gamma_H^2 - p^2} \right) dx = \int \frac{dx}{1 - x^2} = \arcsin x \quad \left|_{-1}^{1} \right. = \pi
\]

Gustav Herglotz, 1906
Herglotz-Wiechert Inversion

This is to show how $p$ can be empirically determined for a given epicentral distance $\Delta$. Works well when $v$ increases with depth ($\frac{dt}{d\Delta} < 0$) for a given epicentral distance $\Delta$.
Observed and theoretical travel time curves

Kennett et al., 1991

IASP91 Travel Times
Surface Focus

Observed at high-frequency
1-D structural models of the Earth

PREM (Dziewonski & Anderson, 1981)

* It can be shown how the radial density profile can be determined - see the appendix of Fowler’s “Solid Earth”
Probing the Earth with seismology: European discoverers of seismic discontinuities

Andrija Mohorovičić (1857-1936)
Crust-Mantle boundary 1910

Beno Gutenberg (1889-1960)
Mantle-Core boundary 1914

Inge Lehmann (1888-1993)
Inner Core 1936

Recipe for longevity: study the inner core!
EMSC8002 LECTURE 6 - Rays in spherically symmetric media: 2) Mohorovičić’s inversion and the discovery of the crust-mantle boundary a.k.a. “Moho”

Hrvoje Tkalčić

"Andrija Mohorovičić" by a surrealistic painter, Carlo Billich

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1909 Earthquake and the Mohorovičić’s assumption

A. Mohorovičić (1910) – Discovered Crust-Mantle Boundary
He created an empirical travel-time curve, based on his observations of a Croatian 1909 earthquake.

He clearly observed two phases (arrivals) between ~280 km and 760 km (also two S phases).

<table>
<thead>
<tr>
<th>Δ[km]</th>
<th>Tn[min sec]</th>
<th>B[min sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0 18.1</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>0 36.1</td>
<td>0</td>
</tr>
<tr>
<td>280</td>
<td>0 43.4 0 50.4</td>
<td>0 50.4</td>
</tr>
<tr>
<td>400</td>
<td>1 04.6 1 11.8</td>
<td>1 11.8</td>
</tr>
<tr>
<td>600</td>
<td>1 30.0 1 47.3</td>
<td>1 47.3</td>
</tr>
<tr>
<td>760</td>
<td>1 40.4 2 15.1</td>
<td>2 15.1</td>
</tr>
<tr>
<td>800</td>
<td>1 54.9 2 06.3</td>
<td>2 06.3</td>
</tr>
<tr>
<td>900</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

He assumed: \( T = \frac{2}{V_0} \int \frac{dx}{\sqrt{V_0^2 \cos^2 x - \rho^2}} \)

We derived: \( T = \frac{2}{V_0} \int \frac{dx}{\sqrt{V_0^2 \cos^2 x - \rho^2}} \)
Mohorovičić’s method
Mohorovičić’s method

\[ \Delta = \frac{2}{k+1} \arccos \frac{\rho V_0}{\eta_0} \]

\[ \Delta = 2 \left[ \frac{1}{k+1} \left( \frac{\rho V_0}{\eta_0} \right)^k \arccos \frac{\rho V_0}{\eta_0} \right] = \frac{2}{k+1} \arccos \frac{\rho V_0}{\eta_0} \]

\[ \Delta = 2 \int_{t_0}^{t_0} \frac{\tan^{-1} \frac{v}{\eta_0}}{\eta_0} \, dt \]
Mohorovičić’s method

\[ \Delta = \frac{2}{k+1} \arccos \left( \frac{v}{v_0} \right) = \frac{2}{k+1} \arccos (\sin i_0) = \frac{2}{k+1} \left( \frac{\pi}{2} - \arcsin (\sin i_0) \right) \]

\[ \Delta = \frac{2}{k+1} \left( \frac{\pi}{2} - i_0 \right) = \frac{\pi}{k+1} - \frac{2}{k+1} i_0 \Rightarrow i_0 = \frac{\pi}{2} - \frac{k+1}{2} \Delta \]

Substitution to \( T \)

\[ T = \frac{2}{1+k} \frac{4}{v_0} \cos i_0 = \frac{2}{1+k} \frac{4}{v_0} \cos \left[ -\frac{k+1}{2} \Delta + \frac{\pi}{2} \right] = \frac{2}{v_0 (1+k)} \sin \left[ \frac{k+1}{2} \Delta \right] \]

\[ T = \frac{2}{v_0 (1+k)} \sin \left[ \frac{k+1}{2} \cdot \Delta \right] \]

This equation can be solved for two empirical values of \( \Delta \) and \( T \), for \( k \) and \( v_0 \).
The depth of the discontinuity

Using the empirical values, Mohorovičić obtained

\[ k = 3.6 \]

\[ v_0 = 5.56 \text{ km/s} \]

From

\[ v = v_0 \left( \frac{t}{t_0} \right)^k \]

\[ v_m = v_0 \left( \frac{t_0}{t_m} \right)^k \]

\[ P = \frac{t_0}{v_0} \sin i_0 - \frac{t_m}{v_m} \]

\[ \frac{t_m v_0}{t_0 v_m} = v_0 \left( \frac{t_0}{t_m} \right)^k \]

\[ t_m = t_0 \sqrt{\frac{t_0}{t_m}} \sin i_0 \]

For \( \Delta = 760 \text{ km} \) (\( P \) disappears)

\[ i_0 = \frac{\pi}{2} - \frac{k + 1}{2} \Delta = 74.16^\circ \]

\[ t_m = 6318 \text{ km} \]

\[ H_{\text{Moho}} = t_0 - t_m = 6371 \text{ km} - 6318 \text{ km} = 53 \text{ km} \]

Voilà!

Andrija Mohorovičić 1910
The discontinuity in seismic wave speeds

Abrupt change in the composition and density of rocks results in a sharp change in seismic wave speeds.

Somewhat arbitrary values on this scheme, but generally OK.
The depth of Moho (crustal thickness)
Moho in popular culture

- The Mohorovičić Discontinuity is mentioned in one particular computer game, an RTS called *Total Annihilation*. Players can build a "Moho Mine" in order to mine metal at or close to the Mohorovičić Discontinuity. Due to the size of the structure, the public being unfamiliar with the Mohorovičić Discontinuity, and an expansion structure called the "Moho Metal-Maker", "Moho" is misinterpreted as meaning "big."
- The Mohorovičić Discontinuity is also mentioned in the novel *Abduction* by Robin Cook, in which a team of scientists are abducted by inhabitants of an underground civilization.
- In the cartoon *Inhumanoids* the monster, D-Compose's kingdom of Skellweb lies within the Moho.
- In *Star Control 2*, one of the "ramblings" of the odd Mycon race is referring to the Deep Children as "Dwellers in the Mohorovichic." This is a reference to the fact that a "Deep Child" will burrow deep into the surface of a planet to begin de-terraforming.
- *Deep Storm: A Novel* by Lincoln Child details an expedition where a team of scientists attempts to drill through the ocean floor to the Mohorovičić Discontinuity.
- In *The Mohole Mystery* by Hugh Walters lethal microbes and belligerent egg-shaped creatures inhabit the Mohorovičić Discontinuity when a manned rocket propelled capsule is sent down to investigate.
- In the *Mars trilogy* by Kim Stanley Robinson, the colonizers of Mars dig deep "moholes" to allow outgassing from the planet's interior as a means to increase the atmospheric pressure - thus contributing to the terraforming of the planet.
- In *Sid Meier's Alpha Centauri* game, the Mohorovičić Discontinuity is mentioned with the technology advance "Industrial Automation." Another technology advance, "Ecological Engineering," allows terraforming units to dig "Thermal Boreholes," which are equivalent to moholes and which, in the game, produce both energy and minerals.
Reaching Moho

- **Project Mohole** was an ambitious attempt to drill through the Earth's crust into the Mohorovičić discontinuity, and to provide an Earth science complement to the high profile Space Race. It was led by the American Miscellaneous Society with funding from the National Science Foundation. Phase One was executed in spring 1961. Off the coast of Guadalupe, Mexico, five holes were drilled, the deepest at 183 m below the sea floor in 3,500 m of water. This was unprecedented: not in the hole's depth but because of the depth of the ocean and because it was drilled from an untethered platform. The **Mohole** project failed due to poor management and cost overruns.

- The **Kola Superdeep Borehole** (KSDB) was the result of a scientific drilling project of the former USSR. The project attempted to drill as deep as possible into the Earth's crust. Drilling began on May 24, 1970 on the Kola Peninsula, using an "Uralmash-4E" and later an "Uralmash-15000" drilling device. A number of boreholes were drilled by branching from a central hole. The deepest, SG-3, was completed in 1989, creating a hole 12,262 metres (the deepest hole ever made by humans). However, due to higher than expected temperatures at this depth and location, 180º C instead of expected 100º C, drilling deeper was deemed infeasible and the drilling was stopped in 1992.

- **Chikyu Hakken** (地球発見), Japanese for "Earth Discovery", is a mission primarily led by the **Japan Agency for Marine-Earth Science and Technology**, or JAMSTEC. The half-billion dollar plus project aims to be the first to drill seven kilometers beneath the seabed and into the Earth's mantle.
Deformation propagates. Particle motion consists of elliptical motions (generally retrograde elliptical) in the vertical plane and parallel to the direction of propagation. Amplitude decreases with depth. Material returns to its original shape after wave passes.
Deformation propagates. Particle motion consists of alternating transverse motions. Particle motion is horizontal and perpendicular to the direction of propagation (transverse). To aid in seeing that the particle motion is purely horizontal, focus on the Y axis (red line) as the wave propagates through it. Amplitude decreases with depth. Material returns to its original shape after wave passes.
Rayleigh Wave vs Water Wave

Particle motion:
Elliptical retrograde near the surface.
As depth increases, the width of the elliptical path decreases.
Below depth of 1/5th of a wavelength, motion becomes prograde.

Particle motion (water depth > wavelength):
Circular prograde.
Radius decreases with depth.