PEAT8002 - SEISMOLOGY
Lecture 12: Earthquake source mechanisms and radiation patterns II

Nick Rawlinson

Research School of Earth Sciences
Australian National University

- P-wave first-motions often cannot constrain focal mechanisms very accurately.
- Additional information can be obtained by comparing observed and synthetic waveforms and adjusting source parameters until a suitable fit is obtained. This can be done using either a forward or inverse approach.
- Waveform analysis also gives information about earthquake depths and rupture processes that cannot be obtained from first motions.
- The generation of synthetic seismograms is often done in the frequency domain, which allows efficient manipulation of harmonic waves.
- A recorded seismic wave is a function of the earthquake source, Earth structure sampled by the propagating wave, and the seismometer.

- It is useful to think of the seismogram $u(t)$ in terms of its Fourier transform $U(\omega)$, which represents the contribution of different frequencies. These two quantities are related by:

$$u(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} U(\omega)e^{i\omega t}\mathrm{d}\omega, \qquad U(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} u(t)e^{-i\omega t}\mathrm{d}t$$

- The above equations allow a seismogram or its components to be transformed from the time domain into the frequency domain or vice versa.
- Generating synthetic seismograms from earthquakes can be done using a convolution approach, where we deterministically combine contributions from known effects.

- The convolution $s(t)$ of two time series $w(t)$ and $r(t)$ in the time domain can be written:

$$s(t) = w(t) * r(t) = \int_{-\infty}^{+\infty} w(t-\tau) r(\tau) \mathrm{d}\tau$$

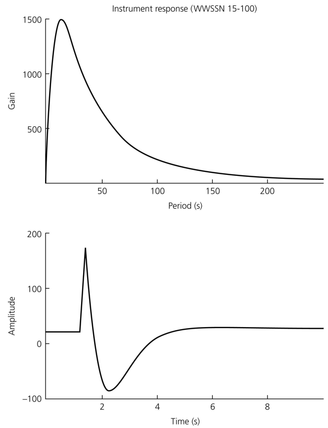
where τ applies a relative time shift to the waveforms.
- Convolution is often carried out in the frequency domain because it simply equals the product of the Fourier transforms i.e. $S(\omega) = W(\omega)R(\omega)$.
- A seismogram can therefore be written as:

$$u(t) = x(t) * e(t) * q(t) * i(t)$$

where $x(t)$ is the source-time function (i.e. the signal imparted by the earthquake), $e(t)$ and $q(t)$ represent the elastic and anelastic effects of Earth structure respectively, and $i(t)$ represents the instrument response of the seismometer.

- Equivalently, the seismogram in the frequency domain can be written:

$$U(\omega) = X(\omega)E(\omega)Q(\omega)I(\omega)$$



- Response of an 1960s analogue WWSSN long period seismometer.
- The impulse response indicates how the seismometer would respond to an input delta function.

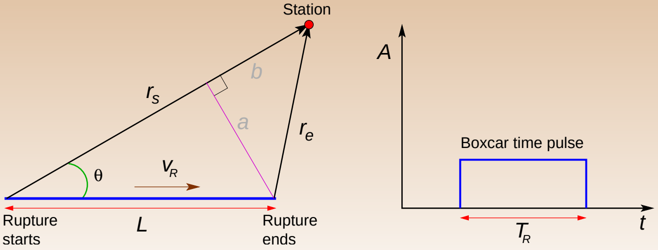
- The effects of earth structure are divided into two factors. $e(t)$ is a function of the Earth's elastic properties, which give rise to focusing, defocusing, reflection, refraction etc. of the propagating wave.
- $q(t)$ describes anelastic attenuation, which is caused by the conversion of wave energy to heat. The quality factor Q characterises this attenuation.
- The earthquake source time function $x(t)$, can be long and complex, depending on the nature of the rupture mechanism.
- In the case of a short fault that slips instantaneously, the seismic moment function $M(t) = \mu D(t) S(t)$ is a step function. Since $x(t) = \dot{M}(t)$, the source time function is simply a delta function.

- In order to synthesise greater complexity in the source time function, consider a simple case in which the rupture at each point on a rectangular fault radiates an impulse. The total radiated signal is not impulsive because the point impulses occur over a finite time period.
- If we consider the simple situation of a fault of length L oriented at an angle θ from the receiver, and rupturing at speed v_R, then the first arrival at the receiver will be at $t = r_s/v$. Here r_s is the distance from the start of the fault and v is the propagation speed of either a P-wave $(v = \alpha)$ or an S-wave $(v = \beta)$.
- The far end of the fault ruptures at a time L/v_R later, giving a seismic arrival time of $t = r_e/v + L/v_R$, where r_e is the distance from the end of the fault to the receiver.

- From the diagram below, we see that $b = r_s - L\cos\theta$ and $a = L\sin\theta$. Therefore:

$$r_e{}^2 = a^2 + b^2 = r_s{}^2 - 2r_s L\cos\theta + L^2$$

- For points far from the fault ($r >> L$), $r_e \approx r_s - L\cos\theta$.



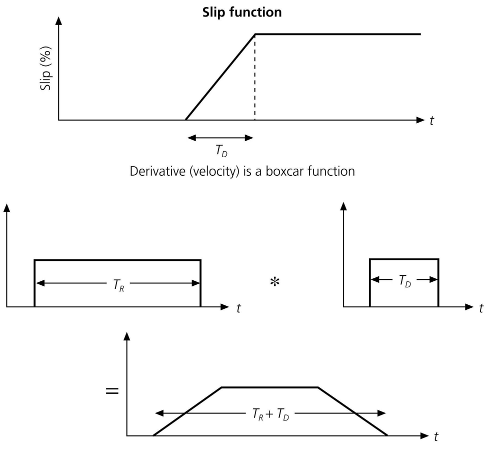
- The time pulse T_R due to the finite length of the fault is given by:

$$T_R = \frac{L}{v_R} + \frac{r_e}{v} - \frac{r_s}{v} = \frac{L}{v_R} - \frac{L\cos\theta}{v} = \frac{L}{v}\left[\frac{v}{v_R} - \cos\theta\right]$$

- T_R is known as the rupture time. v_R is typically assumed to be about 0.7-0.8 times the shear velocity β, so the ratio v/v_R is about 1.2 for shear waves and 2.2 for P-waves.
- The maximum rupture duration occurs for $\theta = 180°$, and the minimum rupture duration occurs for $\theta = 0°$.
- These expressions can be modified for different fault shapes and rupture propagation directions.

- The source time function is also influenced by the fact that even at a single point on the fault surface, slip does not occur instantaneously.
- The slip history is often represented as a ramp function that begins at time zero and ends at the rise time T_{D}. The derivative of this ramp function, which provides the source time function for a single point, is therefore a boxcar function.
- Convolving this rise time effect with the finite duration of the faulting process yields a trapezoid with length $T_{\mathrm{R}} + T_{\mathrm{D}}$.
- More complicated source-time functions than these can be derived from observational data, particularly for large earthquakes.

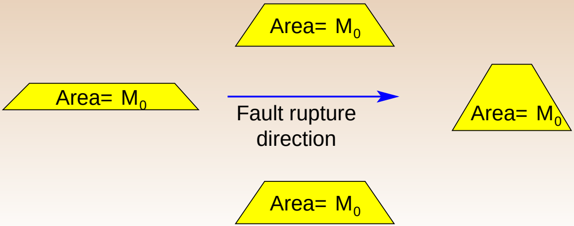
Earthquake sources
Waveform modelling



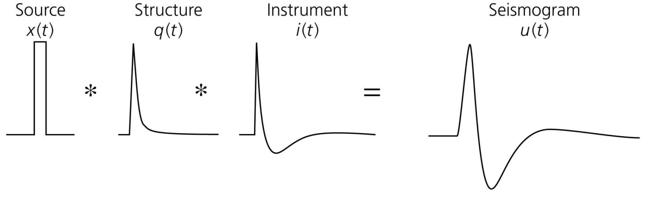
- Illustration of the effect of including rise time in the estimate of the source time function $x(t)$.
- Here it is assumed that at each point, the fault moves with a constant velocity

Earthquake sources
Waveform modelling

- The time duration of the radiated pulse has an azimuthal dependence due to the finite rupture length (i.e. it has a directivity).
- Since the same amount of energy arrives at a given distance, the amplitude of the ground motion, and hence the seismic signal, may vary depending on its location relative to the source.
- The integral of the source time function is equal to the seismic moment M_0, which is the same at all azimuths.



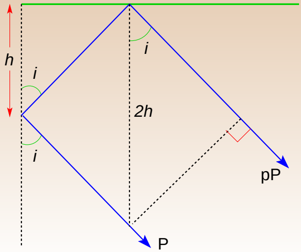
- The elastic structure operator $e(t)$ primarily reflects interactions near the Earth's surface, where the largest changes in seismic properties occurs.
- For a deep earthquake, the surface reflections and other related phases arrive much later than direct P, so these effects will not influence the first-arriving wavetrain.
- At distance ranges $30^\circ < \Delta < 90^\circ$, the effects of upper mantle triplications and core structure can also be ignored for the direct arrival. Therefore, $e(t)$ can be dropped in the convolution for $u(t)$.

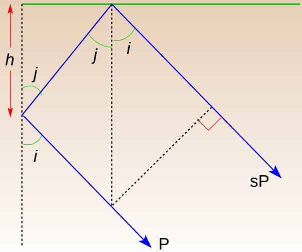


- For shallow earthquakes, reflections (pP and sP) from the Earth's surface may arrive shortly after the first break, making the direct P arrival difficult to separate out.
- The time delay of the the pP pulse relative to P for an earthquake at depth h is given by:

$$\delta t_{pP} = \frac{2h\cos i}{\alpha}$$

where i and α are the incidence angle and velocity for P-waves.





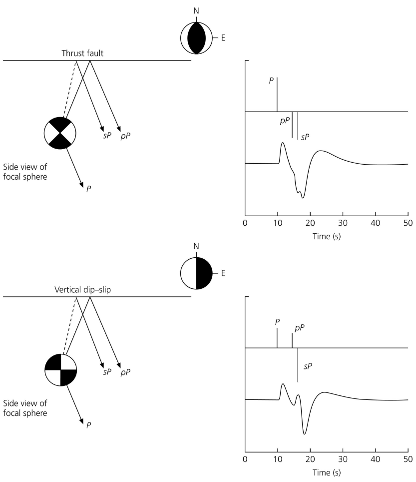
- The time delay of the sP pulse for a Poisson solid ($\lambda = \mu$) relative to P is given by:

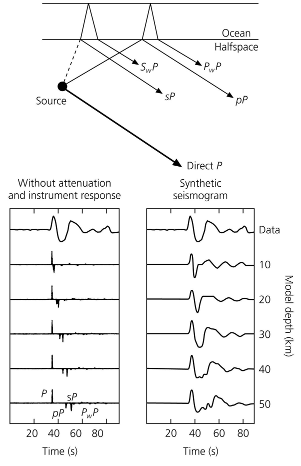
$$\delta t_{sP} = \frac{h}{\alpha}(\cos i + \sqrt{3 - \sin^2 i})$$

  noting that $\sin i/\alpha = \sin j/\beta$ and $\alpha^2 = 3\beta^2$.
- Example: for a source at 10 km depth in a medium with $\alpha = 6.8$ km/s, the time delays at $\Delta = 50^\circ$ are $\delta t_{pP} = 2.7$ s and $\delta t_{sP} = 3.8$ s.
- Source parameters can be studied by generating synthetic seismograms for various values, and finding the best fit either by trial and error forward modelling or inversion.
- Often, first motion, body wave and surface wave analyses are combined.

Earthquake sources

Waveform modelling





- A useful way to estimate the source time function is based on the so-called Green's function:

$$g(t) = e(t) * q(t)$$

  which combines the elastic and anelastic effects of propagation from the source to the receiver.
- Essentially, the Green's function describes the signal that would arrive at the seismometer if the source time function were a delta function.
- The source time function of an earthquake can be found by deconvolving the Green's function and seismometer response from the seismogram $u(t)$:

$$X(\omega) = \frac{U(\omega)}{G(\omega)I(\omega)}$$

- So far, we have assumed that earthquakes result from movement along a simple fault.
- This approach is now generalised to include other types of seismic sources by using seismic moment tensors.
- The seismic moment tensor **M** can be written in the form of a 3×3 matrix as:

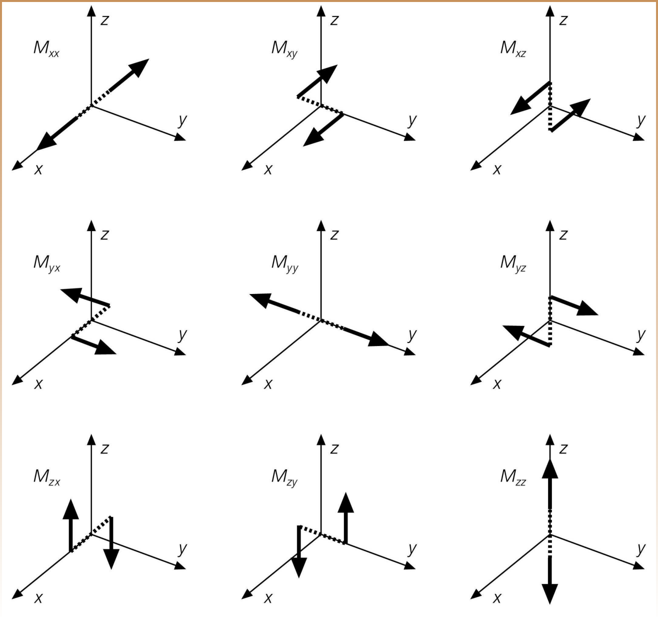
$$\mathbf{M} = \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix}$$

where each component represents one of the nine possible force couples.

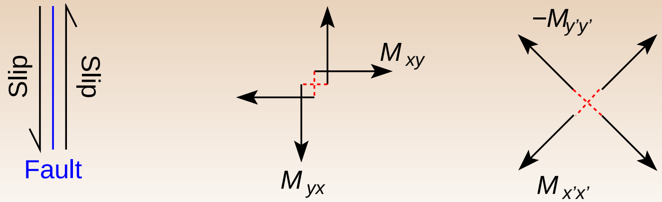
- A force couple consists of two forces acting together. For example, M_{xy} consists of two forces of magnitude f, separated by a distance d along the $y - axis$, that act in opposite directions ($\pm x$). The magnitude of $M_{xy} = fd$, which has units of Nm.
- In the case of M_{xx}, two forces of magnitude f are separated by a distance d along the $x - axis$, and act in opposite directions ($\pm x$). This type of couple is sometimes referred to as a vector dipole.
- The difference between M_{xy} and M_{xx} is that no torque is exerted in the latter case.

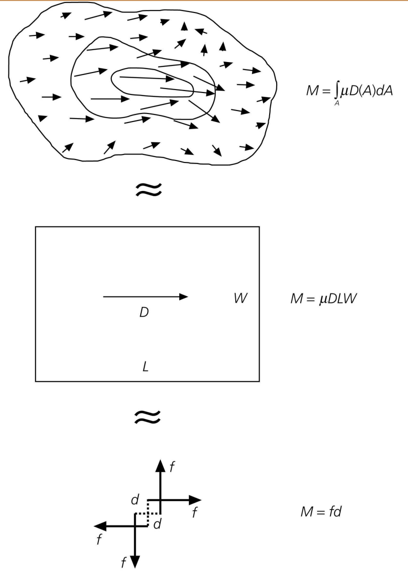
Earthquake sources

Moment tensors



- In the case of a simple planar fault with a strike direction aligned to the $y-axis$, and undergoing left-lateral pure strike-slip motion, the equivalent body forces M_{xy} and M_{yx} make up the double couple source.
- The M_{xy} couple is required in this case to ensure that angular momentum is conserved.
- The magnitude of the equivalent body force is M_0, the scalar seismic moment of the earthquake





- Thus, the moment tensor of an earthquake represents both its fault geometry, via the different components, and its size, via the scalar moment.
- The moment tensor is a simple mathematical description of the seismic waves produced by a complex rupture involving displacements varying in space and time on an irregular fault.

- For the above example, the moment tensor can be written:

$$\mathbf{M} = \begin{bmatrix} 0 & M_0 & 0 \\ M_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = M_0 \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- The moment tensor can be written in any orthogonal coordinate system. In general, the tensor will appear more complex than the above example for arbitrary orientations of slip direction and fault plane.
- For a double-couple earthquake in an arbitrary coordinate system, the components of the moment tensor are provided by the scalar moment and the components of $\hat{\mathbf{n}}$, the unit normal vector to the fault plane, and $\hat{\mathbf{d}}$, the unit slip vector.

- In index notation,

$$M_{ij} = M_0(n_i d_j + n_j d_i).$$

which can be written in full as:

$$\mathbf{M} = M_0 \begin{bmatrix} 2n_x d_x & n_x d_y + n_y d_x & n_x d_z + n_z d_x \\ n_y d_x + n_x d_y & 2n_y d_y & n_y d_z + n_z d_y \\ n_z d_x + n_x d_z & n_z d_y + n_y d_z & 2n_z d_z \end{bmatrix}$$

- In this case **M** is symmetric. Physically, this corresponds to slip on either side of the fault or auxiliary plane yielding the same seismic radiation pattern.
- The trace of **M** is $\mathrm{tr}[\mathbf{M}] = 2M_0 \hat{\mathbf{n}} \cdot \hat{\mathbf{d}} = 0$ because the slip vector lies in the fault plane and is therefore perpendicular to the normal vector.

- A non-zero trace implies a volume change (explosion or implosion); such an isotropic component does not exist for a pure double-couple source.
- The scalar moment $M_0 = \sqrt{\sum_{ij} M_{ij}{}^2}/\sqrt{2}$ is analogous to the magnitude of a vector.
- Using the definitions of the normal and slip vectors in terms of fault strike, dip and slip directions, it is straight forward to write the moment tensor for any fault.
- The reverse process of finding the fault geometry corresponding to a moment tensor is more complicated. However, this is necessary for waveform inversions that yield the moment tensor.

Earthquake sources
Moment tensors

- It turns out that the eigenvectors of the moment tensor are parallel to the T, P and null axes, which enables the fault geometry to be reconstructed from the moment tensor.

Moment tensor	Beachball	Moment tensor	Beachball
$\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$		$-\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$	
$-\frac{1}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}$	
$\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&-1\\0&0&0\\-1&0&0\end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&0\\0&0&-1\\0&-1&0\end{pmatrix}$	
$\frac{1}{\sqrt{2}}\begin{pmatrix}-1&0&0\\0&0&0\\0&0&1\end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&0\\0&-1&0\\0&0&1\end{pmatrix}$	
$\frac{1}{\sqrt{6}}\begin{pmatrix}1&0&0\\0&-2&0\\0&0&1\end{pmatrix}$		$\frac{1}{\sqrt{6}}\begin{pmatrix}-2&0&0\\0&1&0\\0&0&1\end{pmatrix}$	
$\frac{1}{\sqrt{6}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$		$-\frac{1}{\sqrt{6}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$	

- A selection of moment tensors and their associated focal mechanisms. The top row shows an explosion and an implosion.
- The next three rows are for double-couple sources.
- The bottom two rows show CLVD sources

- As illustrated in the previous slide, if all three diagonal terms of the moment tensor are nonzero and equal, the polarity of the first motions is the same in all directions.
- This triple vector dipole of three equal and orthogonal force couples is the equivalent body force system for an explosion or an implosion.
- The moment tensor therefore has the form:

$$M = E \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E\mathbf{I}$$

which has a trace of $3E$. A moment tensor with a non-zero isotropic component represents a volume change.

- Most explosions are artificial, but they can also occur naturally. For example, they may be associated with fluid and gas migration linked to magmatic processes, or with sudden phase transitions or metastable minerals. High-velocity impacts of meteorites could also be modelled with explosive sources.
- The physical process of an explosion is very different from that of an earthquake. As the initial shock wave expands and diminishes in amplitude, a point is reached when the deformations are small enough to occur elastically, yielding a spherical P-wave.
- This propagating wave interacts with interfaces within the Earth, including the free surface, and generates SV and Rayleigh waves. SH-waves are also observed, which is something that is not expected in a spherically symmetric isotropic Earth.

- One possibility to explain the presence of SH waves is the tectonic release of deviatoric stress triggered by the explosion.
- Another class of non-double-couple seismic sources are compensated linear vector dipoles (CLVDs). These are sets of three force dipoles that are compensated, with one dipole twice the magnitude of the other two:

$$M = \begin{bmatrix} -\lambda & 0 & 0 \\ 0 & \lambda/2 & 0 \\ 0 & 0 & \lambda/2 \end{bmatrix}$$

- The trace of the moment tensor is zero, so there is no isotropic component. In contrast to the beachball focal mechanisms of double-couples, the first motions for CLVDs look like baseballs or eyeballs.

- Two primary explanations have been offered fro CLVDs. One is in regard to volcanic regions, where an inflating magma dike can be modelled as a crack opening under tension. The moment tensor for such a crack turns out to be:

$$M = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda + 2\mu \end{bmatrix}$$

- This can be decomposed as follows:

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda + 2\mu \end{bmatrix} = \begin{bmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E \end{bmatrix} + \begin{bmatrix} -2\mu/3 & 0 & 0 \\ 0 & -2\mu/3 & 0 \\ 0 & 0 & 4\mu/3 \end{bmatrix}$$

where $E = \lambda + 2\mu/3$ and the second term is a CLVD.

- An alternative explanation is that CLVDs are due to near-simultaneous earthquakes on nearby faults of different geometries.
- For example, consider the sum of two double-couple sources of different orientation with moments M_0 and $2M_0$:

$$\begin{bmatrix} M_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -M_0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & -2M_0 & 0 \\ 0 & 0 & 2M_0 \end{bmatrix} = \begin{bmatrix} M_0 & 0 & 0 \\ 0 & -2M_0 & 0 \\ 0 & 0 & M_0 \end{bmatrix}$$

- Thus, adding together these two double couples yields a CLVD. In this case, the P, B and T axes of the first source are the T, P and B axes of the second

- Thus, if the first earthquake were a strike slip on a vertical fault, the second would be normal faulting on a 45° dipping fault.
- The next example shows CLVD-type focal mechanisms for earthquakes near a volcano in Iceland. The mechanism is thought to reflect reverse faulting on cone-shaped ring faults surrounding the magma chamber.
- In this model, deflation of the magma chamber increases horizontal compression, so the roof block above the magma chamber subsides with respect to the surrounding rock.

Earthquake sources

Moment tensors