In the last two lectures, the effects of the source rupture process on the pattern of radiated seismic energy was discussed.

However, even before earthquake mechanisms were studied, the priority of seismologists, after locating an earthquake, was to quantify their size, both for scientific purposes and hazard assessment.

The first measure introduced was the magnitude, which is based on the amplitude of the emanating waves recorded on a seismogram.

The idea is that the wave amplitude reflects the earthquake size once the amplitudes are corrected for the decrease with distance due to geometric spreading and attenuation.
Earthquake magnitudes and moment

Introduction

Magnitude scales thus have the general form:

\[ M = \log \left( \frac{A}{T} \right) + F(h, \Delta) + C \]

where \( A \) is the amplitude of the signal, \( T \) is its dominant period, \( F \) is a correction for the variation of amplitude with the earthquake’s depth \( h \) and angular distance \( \Delta \) from the seismometer, and \( C \) is a regional scaling factor.

Magnitude scales are logarithmic, so an increase in one unit e.g. from 5 to 6, indicates a ten-fold increase in seismic wave amplitude.

Note that since a \( \log_{10} \) scale is used, magnitudes can be negative for very small displacements. For example, a magnitude -1 earthquake might correspond to a hammer blow.
The concept of earthquake magnitude was introduced by Charles Richter in 1935 for southern California earthquakes.

He originally defined earthquake magnitude as the logarithm (to the base 10) of maximum amplitude measured in microns on the record of a standard torsion seismograph with a pendulum period of 0.8 s, magnification of 2800, and damping factor 0.8, located at a distance of 100 km from the epicenter.

This standard instrument, known after its designer as the Wood-Anderson seismometer, consists of a small copper cylinder attached to a vertical metal fiber. The restoring force is supplied by tension in the fiber.
The picture on the left shows a short period Wood-Anderson torsion seismometer.

The instrument as a whole is sensitive to horizontal motions, which are detected via light reflected from a small mirror in the cylinder.

The magnitude scale devised by Richter is now referred to as the local magnitude $M_L$. 
Earthquake magnitudes and moment

Richter magnitude

In practice, the scale requires different calibration curves for regions such as stable continental interiors, as compared to the Southern California region for which the scale was originally defined.

This is because the attenuation of seismic waves with distance can be very different for different geological provinces.

The magnitude of the largest arrival (often the S-wave) is measured and corrected for the distance between source and receiver, given by the P- and S-wave differential arrival times. The scale for Southern California is defined by:

\[ M_L = \log A + 2.76 \log \Delta - 2.48 \]
Earthquake magnitudes and moment

Richter magnitude

- Richter magnitudes in their original form are no longer used because they only apply to Southern California and the Wood-Anderson seismograph is now rarely used for recording the seismic wavefield.
- However, local magnitudes are sometimes still reported, because many buildings have resonant frequencies near 1 Hz, which is close to that of a Wood-Anderson seismograph. Therefore, $M_L$ is often a good indicator of the potential for structural damage.
- A number of different global and local magnitude scales have been produced since the original formulation of Richter. Some of these will now be discussed.
Earthquake magnitudes and moment
Body and surface wave magnitudes

- For global studies of teleseismic events, the two primary magnitude scales that have traditionally been used are the body wave magnitude, $m_b$ and the surface wave magnitude, $M_s$.

- $m_b$ is measured from the early portion of the body wave train, usually that associated with the P-wave, and is defined as:

\[
m_b = \log \left( \frac{A}{T} \right) + Q(h, \Delta)\]

- In this case, $A$ is the ground motion amplitude in microns after the effects of the seismometer are removed, $T$ is the wave period in seconds, and $Q$ is an empirical term that is a function of angular distance and focal depth.

- $Q$ can be derived as a global average or for a specific region.
Earthquake magnitudes and moment
Body and surface wave magnitudes

- The plot below shows an estimate of the $Q$-factor for body wave magnitude $m_b$ derived from earthquakes in the Tonga region.
Earthquake magnitudes and moment
Body and surface wave magnitudes

- Measurements of $m_b$ depend on the seismometer used and the portion of the wave train measured.
- Common US practice is to use the first 5 s of the record, and periods less than 3 s (usually about 1 s), on instruments with a peak response of about 1 s.
- $m_b$ is usually measured out to a distance of 100°, beyond which core diffraction has a complicated effect on the amplitude.
- The surface wave magnitude, $M_s$, is measured using the largest amplitude (zero to peak) of the arriving surface waves. Gutenberg and Richter first devised a scale for teleseismic surface waves in 1936.
This was developed more extensively by Gutenberg (1945), who devised the formula

\[ M_s = \log A + 1.656 \log \Delta + 1.818 \]

where \( A \) is the horizontal component of the maximum ground displacement (in microns) due to surface waves with periods of 20 s.

Many formulae for \( M_s \) have been proposed since that of Gutenberg. Vaněk (1962) devised the formula:

\[ M_s = \log \left( \frac{A}{T} \right)_{\text{max}} + 1.66 \log \Delta + 3.3 \]

which has been officially adopted by the International Association for Seismology and Physics of the Earth’s Interior (IASPEI).
The $(A/T)_{\text{max}}$ term is the maximum of all $A/T$ values of the wave group on a record.

If Rayleigh waves with a period of 20s are used, which often have the largest amplitudes, the above expression reduces to

$$M_s = \log A_{20} + 1.66 \log \Delta + 2.0$$

noting that in general $\log[X/Y] = \log X - \log Y$, and $\log 20 = 1.3$.

As in the expression for $m_b$, $A$ is the ground motion amplitude in microns after the effects of the seismometer have been removed.
Earthquake magnitudes and moment
Limitations

- As measures of earthquake size, magnitudes have two major advantages. First, they are directly measured from seismograms without sophisticated signal processing. Second, the estimates they yield are intuitively meaningful (magnitude 5 is moderate, magnitude 6 is strong etc.).

- However, magnitudes also have several related limitations. First, they are totally empirical, and thus have no direct connection to the physics of the earthquake. The equations used are not even dimensionally correct ($A/T$ is not dimensionless, yet the logarithm of this quantity is still taken).

- A second problem is with consistency; magnitude estimates vary noticeably with azimuth, due partly to the source radiation pattern.
Earthquake magnitudes and moment

Limitations

- Different magnitude scales also yield different values, and body and surface wave magnitudes do not correctly reflect the size of large earthquakes.

- This is demonstrated in the table below, which shows significant discrepancies between $m_b$ and $M_s$

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Body wave magnitude $m_b$</th>
<th>Surface wave magnitude $M_s$</th>
<th>Fault area (km$^2$) (lengthxwidth)</th>
<th>Average dislocation (m)</th>
<th>Moment (dyn–cm) $M_0$</th>
<th>Moment Magnitude $M_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truckee, 1966</td>
<td>5.4</td>
<td>5.9</td>
<td>10x10</td>
<td>0.3</td>
<td>8.3x10$^{24}$</td>
<td>5.9</td>
</tr>
<tr>
<td>San Fernando, 1971</td>
<td>6.2</td>
<td>6.6</td>
<td>20x14</td>
<td>1.4</td>
<td>1.2x10$^{26}$</td>
<td>6.7</td>
</tr>
<tr>
<td>Loma Prieta, 1989</td>
<td>6.2</td>
<td>7.1</td>
<td>40x15</td>
<td>1.7</td>
<td>3.0x10$^{26}$</td>
<td>6.9</td>
</tr>
<tr>
<td>San Francisco, 1906</td>
<td>6.2</td>
<td>7.8</td>
<td>450x10</td>
<td>4</td>
<td>5.4x10$^{27}$</td>
<td>7.8</td>
</tr>
<tr>
<td>Alaska, 1964</td>
<td>6.2</td>
<td>8.4</td>
<td>500x300</td>
<td>7</td>
<td>5.2x10$^{29}$</td>
<td>9.1</td>
</tr>
<tr>
<td>Chile, 1960</td>
<td>8.3</td>
<td>8.3</td>
<td>800x200</td>
<td>21</td>
<td>2.4x10$^{30}$</td>
<td>9.5</td>
</tr>
</tbody>
</table>
The earthquakes with moments greater than the San Fernando earthquake all have $m_b = 6.2$, even as the moment increases by a factor of 20,000.

Similarly, the earthquakes larger than the San Francisco earthquake have $M_s \approx 8.3$, even as the moment increases by a factor of 400.

This effect, called magnitude saturation, is a general phenomenon for approximately $m_b > 6.2$ and $M_s > 8.3$.

In the above table, the 1906 San Francisco earthquake approximately represents the maximum size of continental transform earthquakes.

However, the Alaska and Chilean earthquakes had much larger rupture areas because they occurred on shallow dipping subduction thrust interfaces.
Faults associated with subducting slabs can have widths of 100s of km on which strain can build up and eventually be released seismically.

The larger fault dimensions give rise to greater slip, so the combined effects of larger fault area and more slip cause the largest earthquakes to occur at subduction zones rather than transforms.

Uncertainties in earthquake magnitude estimates have several sources. First, the Earth’s seismic structure exhibits significant lateral variations, particularly at shallow depth.

Estimation techniques used to compute magnitudes vary over time (hence pre-1964 earthquakes do not have $m_b$ values).
Different techniques (body wave, surface wave, geodetic, geological) can yield contrasting estimates of magnitude.

Fault dimensions and dislocations are average values for quantities that can vary significantly along the fault.

All of these effects are understandable given that amplitudes depend on the scalar moment, the azimuth of the seismometer relative to the fault geometry, the distance from the source, and the source depth.

In addition, because the source time function has a finite duration, depending on fault dimensions and rise time, the amplitudes vary with frequency.
The moment magnitude $M_w$ has several advantages compared to the magnitude estimates discussed so far. It is defined by:

$$M_w = \log \frac{M_0}{1.5} - 10.73$$

where $M_0$ is in dyn-cm (1 dyn = 10$^{-5}$ N).

Recall that the seismic moment was defined by:

$$M_0 = \mu \bar{D} S$$

where $\mu$ is the shear modulus of the rocks involved in the earthquake, $S$ is the rupture area, and $\bar{D}$ is the average displacement or slip on $S$. 
Earthquake magnitudes and moment

Moment magnitude

- The main benefit of $M_w$ is that the magnitude is directly tied to earthquake source processes that do not saturate.
- It turns out that $M_w$ is comparable to $M_s$ until $M_s$ saturates at about 8.2.
- The largest recorded earthquake, the 1960 Chile event, had $M_w = 9.5$. Moment magnitude has become the common measure of the magnitude of large earthquakes.
- Estimation of $M_0$ and hence $M_w$ requires more analysis of seismograms compared to $m_b$ and $M_s$. However, semi-automated programs like the Harvard CMT project now regularly compute moment magnitudes for most earthquakes larger than about $M_w = 5$. 
Earthquake magnitudes and moment
Magnitude saturation

The reason for $M_s$ and $m_b$ saturating at large magnitudes can be traced back to the source spectrum of an earthquake. The plot below shows a theoretical source spectrum.

- $T_R$ and $T_D$ correspond to the rupture and rise times respectively.
- $f_c$ is referred to as the corner frequency.
The above source spectrum assumes that the source time function is the convolution of two “boxcar” functions, corresponding to the finite length of the fault and the finite rise time of the faulting at any point.

It turns out that the spectral amplitude (obtained by applying the Fourier transform) of the source signal is given by:

\[ |A(\omega)| = M_0 \left| \frac{\sin(\omega T_R/2)}{\omega T_R/2} \right| \left| \frac{\sin(\omega T_D/2)}{\omega T_D/2} \right| \]

If the logarithm of both sides is now taken, then:

\[ \log A(\omega) = \log M_0 + \log[\text{sinc}(\omega T_R/2)] + \log[\text{sinc}(\omega T_D/2)] \]

noting that in general \( \log XY = \log X + \log Y \), and \( \text{sinc}X = 1 \) if \( X = 0 \) but otherwise \( \text{sinc}X = (\sin X)/X \).
A useful approximation for sinc\(X\) is that sinc\(X \approx 1\) for \(X < 1\) and sinc\(X \approx 1/X\) for \(X > 1\).

This explains why the theoretical source spectrum in the previous diagram has three linear segments. The flat segment extending to zero frequency gives \(M_0\).

The corner frequencies are given by \(2/T_R\) and \(2/T_D\).

It can be shown that for earthquakes with a shear velocity of about \(\beta = 4\) km/s, the rupture and rise times can be given by the approximate expressions \(T_R = 0.35L\) and \(T_D = 0.1Lf^{1/2}\). In these expressions, \(L\) is the width of a rectangular fault and \(f\) is the ratio of width to length.
The simple analysis provided above shows why $m_b$ and $M_s$ differ, and why both magnitude scales saturate.

As the fault length increases, the seismic moment, rupture time, and rise time increase. Thus, the corner frequencies move to the left (i.e. to lower frequencies).

The moment $M_0$ determines the zero frequency level, which rises as the earthquake becomes larger.

However, the surface wave magnitude $M_s$ is usually measured at a period of 20 s, and so depends on the spectral amplitude at this period.

As the moment increases, a period of 20 s will eventually lie to the right of the first corner frequency, at which point $M_s$ will no longer increase at the same rate as $M_0$. 
Earthquake magnitudes and moment
Magnitude saturation

As the moment increases still further, 20 s will eventually lie to the right of the second corner frequency, which results in $M_s$ saturating at about 8.2.

A similar effect occurs for body waves, but because $m_b$ is usually estimated from waves with a period of about 1 s, it saturates at lower moment.
Earthquake magnitudes and moment
Magnitude saturation

In this example, $M_s$ is plotted against $M_0$ and fault area ($S$). Saturation clearly occurs in both plots.

Open and closed circles denote intraplate and interplate earthquakes, respectively.
If you visit the Incorporated Research Institutions for Seismology (IRIS) webpage http://www.iris.edu, you can access a large resource of current and past earthquake data and analyses.
You will find that magnitude estimates of one kind or another are used in the description of most earthquakes, and form an integral part of the event information that is attached to most seismic datasets that are distributed.

If you look up an earthquake on the IRIS website, you may notice that definitions of magnitude exist other than the ones covered in this lecture.

One of these is $M_d$, the duration magnitude. This is based on the duration of shaking as measured by the time decay of the amplitude of the seismogram. This is sometimes done when the dynamic range of the recording instrument makes it impossible to measure peak amplitudes.