Electromagnetic damping of elastic waves: Experimental results

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The passage of an elastic wave causes straining and translation in the transmitting material. If a magnetic field is applied, and the medium is an electrical conductor, some of the energy of the wave is dissipated by the flow of electrical eddy currents. Usually the amount of energy lost is very small, but it may be greatly increased if the applied field is strongly non-uniform.

Laboratory experiments are described which demonstrate this effect for standing elastic waves in a metal bar. The applied magnetic field changes from almost zero to its full strength over a distance which is short compared to the length of the standing wave. The result of this strong non-uniformity is that the energy lost due to the translation of the bar in the field greatly exceeds the energy lost due to the straining of the bar in the field.

The dependence of the attenuation of the waves by the magnetic field is investigated for variation in frequency of vibration, bar thickness, and field gradient.

Introduction

Two generally separate branches of geophysics are the study of seismic waves to determine the structure of the earth, and the explanation of the origin of the magnetic field of the earth. Dynamo theories advanced as part of the second project require high intensities of magnetic field to exist within the earth's core, and it is important to know whether the effect of such magnetic fields upon the propagation of seismic waves could be detected. Theoretical papers on the subject of magnetoelastic effects in the core have been published by Cagniard (1952), Rikitake (1952), Knopoff (1955), Yukutake (1957), Cole (1957), Knopoff and MacDonald (1958), and Kraut (1965), and in general these authors conclude that effects in the core should be negligible.

Consideration of this problem led to the laboratory experiments described in this paper, in which standing elastic waves are attenuated by an applied magnetic field. The significance of the experiments is in demonstrating and investigating the damping effect of the non-uniform region of the field. Previous experiments, reported by Robey (1953), Leonard (1953), Willett (1957), and Galkin and Koroliuk (1960), have been carried out in magnetic fields which were uniform.

That the non-uniformity of a field would have an effect upon elastic wave motion is evident from a 'first principles' point of view. Transmission of a wave causes material to be compressed, and also subjected to translational movement. In a uniform field, only the process of compression–expansion will cause eddy currents and attenuation, for no dissipative effect is associated with the rigid motion of a conductor that is wholly within a region of uniform field. In a non-uniform field, however, dissipative effects will be associated with rigid-body motion. This provides a mechanism for electromagnetic attenuation that is in addition to that of the compression process.

Apparatus

The apparatus for the experiments consisted of a metal bar suspended freely in a horizontal position, in which longitudinal elastic waves were excited by a light piezo-electric crystal glued to one end. The crystal was driven by an oscillator of variable frequency, and standing waves were detected by the variation in resistance they caused in light strain gauges, glued to the sides of the bar. The signal from the strain gauges was electronically amplified, filtered, and displayed on an oscilloscope, where it was recorded photographically. The mass of the crystal was ignored in calculating the standing wave patterns set up in the bar, and similarly the effects of the supports were ignored.

The magnetic field was applied across a section of the bar by a powerful electromagnet, mounted so that the bar passed between its pole pieces. The base of the electromagnet was mounted on

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rails, so that the position of the field could be varied along the bar. A block diagram of the apparatus is shown in Fig. 1.

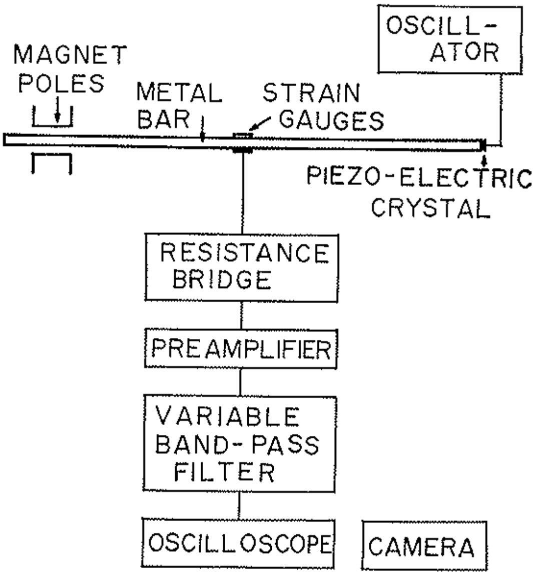

FIG. 1. A schematic diagram of the apparatus.

Method

When a magnetic field was applied to the freely vibrating bar, it caused an increase in the rate of decay of vibrations. The contribution of the magnetic field to the damping of the vibrations was estimated by observing the exponential decays with field on and field off, and subtracting the constant for one from the constant for the other. Thus if the decay with field off is according to

$$u(t) = u_0 \, e^{-\gamma_0 t}$$

where u, t and γ_0 represent amplitude of displacement, time, and decay constant respectively, and if the decay with field on is according to

$$u(t) = u_0 \, e^{-\gamma t}$$

then because

$$e^{-\gamma t} = e^{-\gamma_0 t} \, e^{-\gamma_m t}$$

we may determine

$$\gamma_m = \gamma - \gamma_0$$

where γ_m is the decay constant due to the magnetic field.

A decay constant was determined experimentally by first exciting the required resonant harmonic in the bar. When the standing wave was established in steady resonance, crystal power was switched off, and the strain-gauge signal immediately swept across the oscilloscope screen. The sweep was adjusted to display a major portion of the exponential decay of the signal, which was photographed on time exposure. The amplitude of the signal was then plotted against time on semi-logarithmic graph paper, and the slope of the resulting line was proportional to the exponential decay constant.

In some experiments, when only the applied field was varied, changes in decay constant were estimated more rapidly by observing the variation in amplitude of the resonant standing wave. For a bar driven steadily in resonance, the amplitude of the oscilloscope signal is inversely proportional to the free decay constant of the same system.

The Experiments

Experiment 1. The Effect of a Non-uniform Region of Magnetic Field

The magnetic field was created by an electromagnet, and consisted of a uniform region in the gap between the pole pieces, surrounded by a bordering region of strong non-uniformity. The purpose of this experiment was to compare the damping effects of the uniform and non-uniform regions of field.

Starting from a remote position, the magnetic field was gradually brought in to bear across one end of the bar. In this way first one region of non-uniformity, then the uniform region, and then another region of non-uniformity came across the bar, and their separate effects could be observed. Except for magnet position, all parameters were kept constant during the experiment.

The results of this experiment are given in Fig. 2. The graph shows the variation of attenuation with magnet position. It can be seen that the effects of the regions of non-uniformity at the edge of the pole pieces dominate any effect that might be due to the region of uniform field between the pole pieces.

Experiment 2. Variation of the Position of the Magnet along the Bar

Decay constants were measured with the magnetic field applied at different positions along

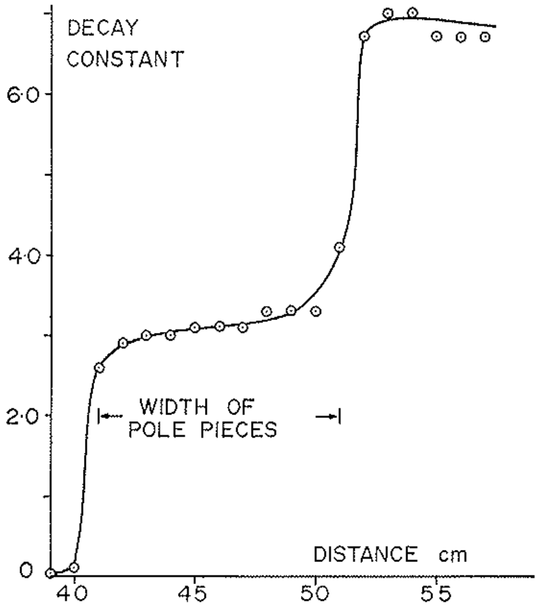

FIG. 2. Experiment 1. The damping effect of the non-uniform field at the edges of the magnet pole pieces. The decay constant is in uncalibrated units.

the bar, which was excited in its fundamental longitudinal mode of vibration. In addition to recording a decay photographically, the energy loss of the vibration was measured by the steady-driven method.

The two sets of experimental data are presented in Fig. 3. They define a simple curve, which has a minimum in the center and a maximum at each end. This is taken to mean that the effect depends primarily upon the amplitude of vibration of the bar in the field, which also has a maximum at each end of the bar and is zero in the center.

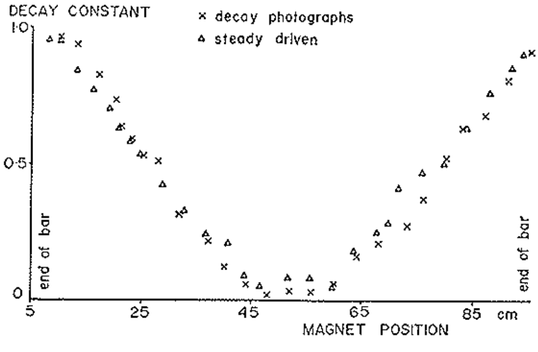

FIG. 3. Experiment 2. Variation of the position of the magnet along the bar. The two sets of data have been matched together by multiplying each by an arbitrary vertical scale factor.

The effect does not appear to be due in any significant way to the compression-expansion cycle of the bar, as the amplitude of this has a maximum at the center, and is zero at each end.

Experiment 3. Variation of Frequency

The frequency of resonant vibration in the longitudinal mode of a bar free at both ends is given by

$$\omega_n = n\pi\alpha/L$$

where n is the order of the harmonic, α is the longitudinal elastic velocity, and L is the length of the bar. In a bar of a particular material, the frequency can therefore be varied by changing either the order of the harmonic or the length of the bar.

In this experiment the frequency of vibration was increased by both methods, starting at the lowest frequency practical, which was the fundamental mode of a bar 3.6 m long. Other parameters were kept constant, and a region of non-uniform field was applied across the end of the bar. The upper limit of frequency was reached when the magnetic damping became small compared with the natural damping.

Bars of aluminum, brass, and copper, of rectangular cross-section $\frac{1}{4}$ inch by $\frac{1}{8}$ inch, were tested in the frequency experiment, and the same magnetic field was applied in each case. Results are shown in Fig. 4. Shortening a bar also had the effect of reducing its mechanical energy, and to compensate for this, each measured decay constant has been normalized according to

$$\gamma_N = (L/3.6)\gamma_m$$

where L is the length of the bar in meters.

It is clear from Fig. 4 that the attenuation is approximately proportional to the inverse of the frequency, although the brass bar deviates from this behavior at the low end of the frequency range.

Experiment 4. Variation of the Thickness of the Bar

In addition to the frequency test carried out on the aluminum bar $\frac{1}{8}$ inch thick, similar tests were carried out on bars of the same metal $\frac{3}{16}$ inch and $\frac{1}{4}$ inch thick. Results for the three bars are shown in Fig. 5. For any particular frequency, we note that increasing the bar thickness has the effect of decreasing the attenuation. This is evidence that a 'skin effect' is being observed, as the energy loss cannot be occurring uniformly across the thickness of the bar.

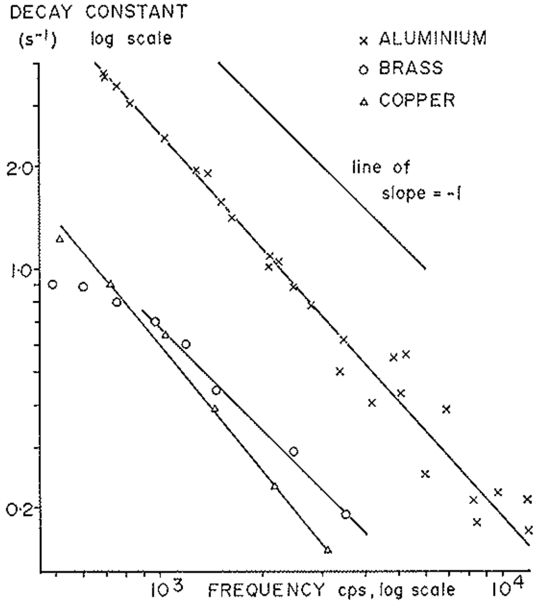

FIG. 4. Experiment 3. The frequency dependence of the electromagnetic damping of standing waves in metal bars. Each bar is $\frac{1}{8}$ inch in thickness.

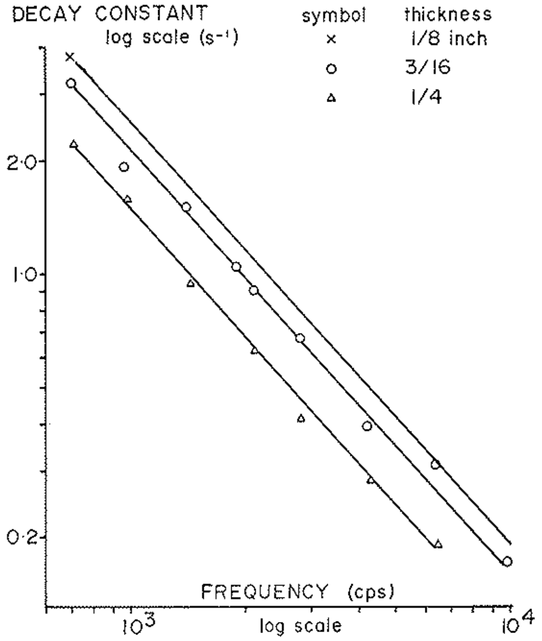

FIG. 5. Experiment 4. The frequency dependence of the electromagnetic damping for different thicknesses of aluminum bar. The unbroken line, for thickness $\frac{1}{8}$ inch, has been transcribed from Fig. 4.

The intercepts of the lines drawn on Fig. 5 are such that, at any frequency, the decay constants for the three bars are in the proportions $1 : 0.84 \pm .02 : 0.59 \pm .02$.

Experiment 5. Variation of Field Gradient

In the previous experiments the magnetic field was kept constant, even though its position relative to the bar was sometimes varied.

In Experiment 5, the magnetic field only was varied. This was achieved by increasing the current that flowed through the energizing coils of the magnet. The gap between the pole pieces was kept constant and for each different magnet current, the magnetic field profile across the median plane between the pole pieces was measured using a rotating coil probe. The steepest slope on the plot of each profile thus obtained was measured to give $p_{\max}$, the maximum gradient of the applied field. This was chosen as an index for the non-uniformity of the field because it could be measured objectively.

Decay constants for an aluminum bar were measured for six different magnet currents, and the results of the experiment are shown in Fig. 6. The proportionality $\gamma_m \propto p_{\max}^2$ is observed.

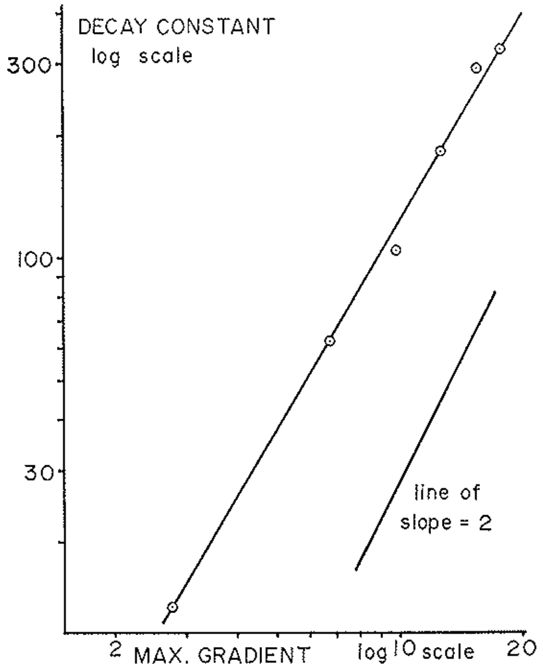

FIG. 6. Experiment 5. The dependence of the decay of vibrations in a bar upon the maximum gradient of the applied magnetic field.

Conclusions

The series of experiments has demonstrated the attenuating effect of a non-uniform magnetic field on elastic vibrations in a good electrical conductor, where the change in applied field takes place over a distance of less than one wavelength. In a following publication we plan to present a theoretical check on the qualitative characteristics of the experiments.

The results raise the question of whether the damping by the core field of standing waves in the earth has yet been sufficiently well investigated, and it is tempting to adapt the results of this paper to a crude theory for the earth. However, the differences between the laboratory apparatus, and what is thought to be the core situation, are too great for this to be done at present.

Acknowledgments

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