Thin-Sheet EM Modelling of the Tasman Sea

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Abstract

The Tasman Project of Seafloor Magnetotelluric Exploration (TPSME) took place between December 1983 and April 1984 (Filloux *et al.*, 1985; Ferguson *et al.*, 1985; Lilley *et al.*, 1989). Seven magnetotelluric and two (additional) magnetometer sites spanned a range of tectonic features across the Tasman Sea. Initial analysis by Ferguson (1988) indicated large-scale three-dimensional induction effects to be present in the data. It was concluded that the most probable causes were the continental margin effect and changes in bathymetry.

In the present paper, a method is presented of modelling the salt water of the Tasman Sea and adjoining oceans as a thin sheet of variable lateral conductance, which overlies a series of uniform layers representing the solid Earth. The theory and a suitable computer algorithm were developed in a group led by J. T. Weaver at the University of Victoria, B.C., Canada. Many of the features present in the TPSME data are reproduced by this method, and with a greater understanding of induction processes in the ocean which is thus obtained, it is possible to remove three-dimensional effects from observed data. The TPSME data are then solely a measure of the response of the Earth directly beneath the observing sites, and one-dimensional modelling techniques may be used to determine the conductivity structures.

Key words: Electromagnetic induction, thin sheet modelling, Tasman Sea.

Introduction

The processes of electromagnetic induction, due to fluctuations in the Earth's magnetic field, can be used to gain an understanding of the electrical conductivity structure of the Earth. This conductivity may vary by several orders of magnitude and such changes can be related to major tectonic features, with additional information from other geophysical techniques.

Measurement of the electric and magnetic fields on the ocean floor presents a great challenge to the geophysicist. Instruments are required which will record very accurately at depths of 5000 m or greater, for a matter of months, and such observations are very much at a pioneering stage. The conductivity of sea-water is several orders of magnitude greater than the underlying crust and upper mantle. The sea-water thus acts as a low pass filter: high frequency components of the fluctuations in the magnetic field of the Earth are strongly attenuated with depth in the ocean. Thus data recorded on the ocean floor may yield much detail on large-scale tectonic features such as subducting plates, spreading ridges, and the nature and thickness of the asthenosphere; however little or no detail of crustal structures can be resolved by the method.

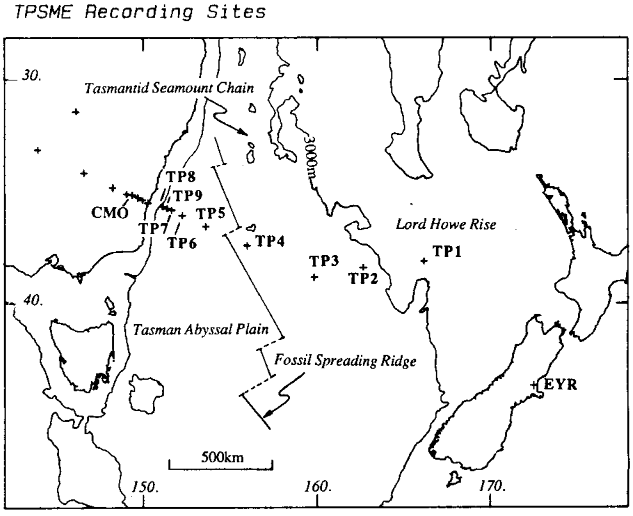


FIGURE 1
Recording sites for the Tasman Project of Seafloor Magnetotelluric Exploration.

The Tasman Project took place between early December 1983 and late April 1984. A series of electric and magnetic instruments, developed by J.H. Filloux of Scripps Institution of Oceanography, were deployed across the Tasman Sea (Figure 1). The sites spanned a range of major tectonic features which include the fossil spreading ridge, the Tasmantid Seamount Chain with Mount Gascoigne near site TP4, and the Lord Howe Rise, an area of submerged continental crust. The spacing of the instruments was closest near the southeast Australian coastline, to study the geomagnetic coast effect there and also for purposes of oceanographic research. Additional magnetic field recordings were available from a series of magnetometers on the Australian mainland, and from the magnetic observatories at Canberra (CMO), and Eyrewell (EYR) in New Zealand.

One-dimensional modelling was performed by Ferguson (1988) for sites TP3, 4 and 5 from the centre of the abyssal plain. In addition, Ferguson's analysis revealed strong three-dimensional induction effects, and a close examination of the data revealed that the most probable cause was induced electric currents flowing in the Tasman Sea. The conductivity of sea water is so great in comparison to the underlying Earth that the physical shape of the ocean is important. The changes in bathymetry and the effect of the continental margins of Australia and New Zealand will strongly influence the flow pattern of the induced electric currents.

In the present paper, to gain an understanding of electromagnetic induction within the Tasman Sea, a model is developed which incorporates a surface thin sheet of variable lateral electrical conductance. Underlying the sheet is either a half-space of uniform conductivity or a layered structure. Such oceanic thin sheet modelling is the same in principal as that presently being developed for EM exploration problems (Smith and West, 1987), though the oceanic problem is on a larger scale (the deep ocean is the 'thin sheet'), and consequently lower frequencies are considered. The present exercise uses a theory and suitable computer algorithm developed by J. T. Weaver and colleagues (McKirdy *et al.* 1985).

Thin Sheet Modelling

The thin sheet of the present model has thickness negligible in comparison to the electromagnetic skin depth in the underlying structure, so that a number of simplifying assumptions can be made (Figure 2). It is easy to show that the normal component of the magnetic field is continuous across the sheet. Similarly, the horizontal electric field is also continuous. The horizontal magnetic field is discontinuous by a factor dependent on the conductance value in the sheet, and on the horizontal electric field strength at that point. This relationship can be expressed by Price's (1949) equation:

$$2i\tau(\mathbf{r})E(\mathbf{r}) = \mathbf{iz} \times [\,B(\mathbf{r},0+) - B(\mathbf{r},0-)\,],\ \mathbf{r} = (x,y)$$

where $\tau(\mathbf{r})$ is the variable lateral conductance value at point $\mathbf{r}$, $E(\mathbf{r})$ the horizontal electric field, and the expression in square brackets denotes the difference in horizontal magnetic field values ($B(\mathbf{r},z)$) between the bottom and the top of the sheet. The horizontal magnetic fields may be expressed in terms of the integral sum of the electric field values across the sheet.

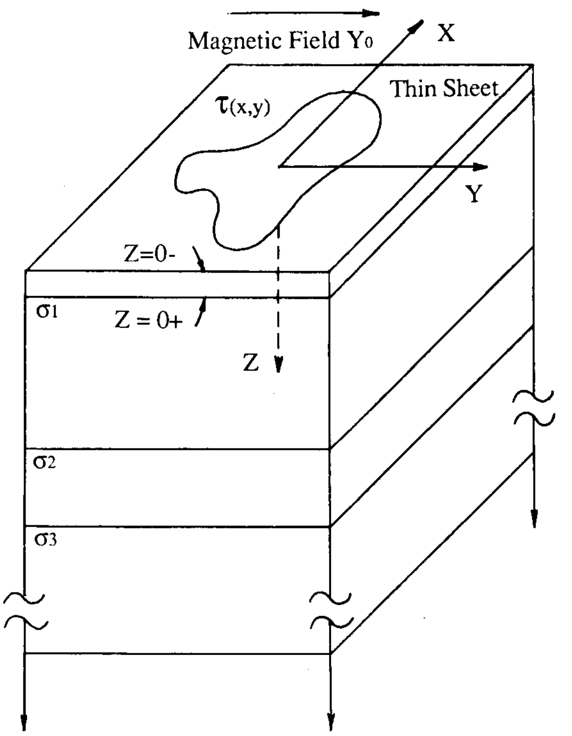


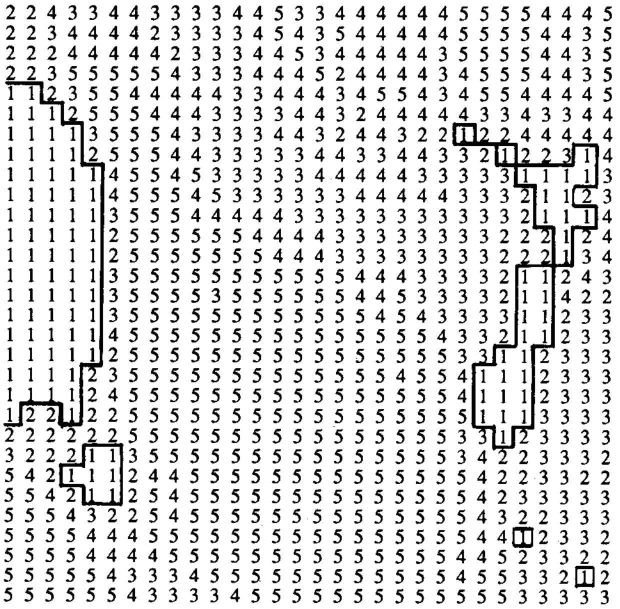
FIGURE 2
A schematic illustration of a thin sheet model. The inducing magnetic field may be polarised parallel to the Y axis (as shown) or parallel to the X axis.

Price's equation cannot be solved analytically. Instead, a numerical method is adopted, which is shown to be satisfactory by comparison with two-dimensional analytic solutions (McKirdy *et al.* 1985). In this method the thin-sheet conductance values are defined at a series of discrete points on a fixed grid. Thus, the electric field at any node on the grid is defined in terms of the electric field values at all other nodes on the grid, and the problem is solved by standard iterative methods. Once the electric field at each node has been determined, the horizontal magnetic fields at the bottom of the sheet can be calculated.

A Tasman Sea Model

The necessity of using a numerical method to solve Price's equation places a number of conditions on the model. The grid node spacing and the sheet thickness must be small compared to the electromagnetic skin depth in the underlying layer. In addition, the thickness of the thin sheet must be small, to second order, compared to the skin depth within the sheet. Profiles of the conductance values at nodes along the edge of the grid are assumed to continue laterally to infinity in all directions.

A grid constructed for the Tasman Project area is shown in Figure 3, with the corresponding conductance values for the grid listed in Table 1. The sheet thickness in this example is 10 km to include most of the crustal structures. Underlying the sheet is a simple layered model, the parameters of which are given in Table 2. The sharp rise in conductivity at 400 km



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5	5	5	5	5	5	4	3	3	3	3	4	5	5	5	5	5	5	5	5	5	5	5	5	5	3	3	3	3	3

FIGURE 3
The grid structure for the Tasman Project area with the coastlines of Australia and New Zealand outlined. Details of the grid conductance values are given in Table 1 and of the underlying structure in Table 2. The grid spacing is approximately 125 km.

TABLE 1
Conductance values and bathymetric descriptions for the thin sheet grid shown in Fig. 3.

Grid Value	Description	Mean Depth (m)	Conductance (S)
1	Continental crust	-	100.0
2	Continental shelf	250	922.5
3	Shallow ocean	1250	4212.5
4	Lord Howe Rise	2750	9147.5
5	Deep ocean	4250	14082.5

TABLE 2
Geoelectric, one-dimensional structure underlying the thin sheet model shown in Fig. 3.

Depth (km)	Thickness (km)	Conductivity (S/m)
0 - 200	200	0.001
200 - 250	50	0.2
250 - 400	150	0.03
400 +	-	1.0

is a well known geophysical discontinuity and the layer of high conductivity at a depth of 200 km was included as it is present in Ferguson's (1988) one-dimensional models. These rises in conductivity are found to be required in the thin sheet model to obtain a resonable match of the model response to the observed data. Such requirements are significant as they indicate that even this early stage of modelling produces constraints on the sub-sheet electrical conductivity structure. An initial model, underlain by a resistive half-space, produced unrealistically high flows of electric current in the Tasman Sea.

Magnetic and electric field values are derived at the base of the thin sheet at each node, for polarisations (of the modelled inducing field) in the x and y directions (Figure 3). The conductance values calculated for the grid nodes are approximately proportional to the ocean depth. Thus the assumption is made that the values computed are appropriate for the ocean floor.

A standard magnetotelluric impedance tensor is thus derived numerically at each node of the grid, and then rotated for alignment with its predominant two-dimensional structure. The impedance elements are converted into apparent resistivity and phase values, parallel (E-pol) and perpendicular (B-pol) to this structure. A skew angle is also derived from the tensor elements, to provide a measure of apparent three-dimensionality.

Results

Figure 4 illustrates the numerical model E-pol and B-pol apparent resistivity values at the approximate locations of the TPSME sites, each rotated to its appropriate azimuth, and with its associated skew angle printed as a number. The TPSME data are shown for comparison, at an inducing period of 9 hrs. The thin sheet model is shown to reproduce the orientation and trend of the TPSME data in a reasonably consistent manner, however there is a difference in magnitude. The magnitude of the calculated apparent resistivities is dependent upon the underlying structure, suggesting that this part of the model can yet be further improved. The TPSME sites on the Tasman abyssal plain (TP3 and TP4) are best modelled by the thin sheet method, and the coastline sites (TP6 and TP7) show the greatest deviation between model results and real observations. This result is considered to be largely a function of the grid spacing, as the number of grid points taken in the thin sheet model is limited by computer resources. Possibly the coastal sites may be modelled better by using a standard two-dimensional algorithm which allows a variable grid spacing (Kellett *et al.*, 1988; White *et al.*, 1989).

Another standard method of presenting lateral changes in conductivity is by calculating a Parkinson arrow (Parkinson,

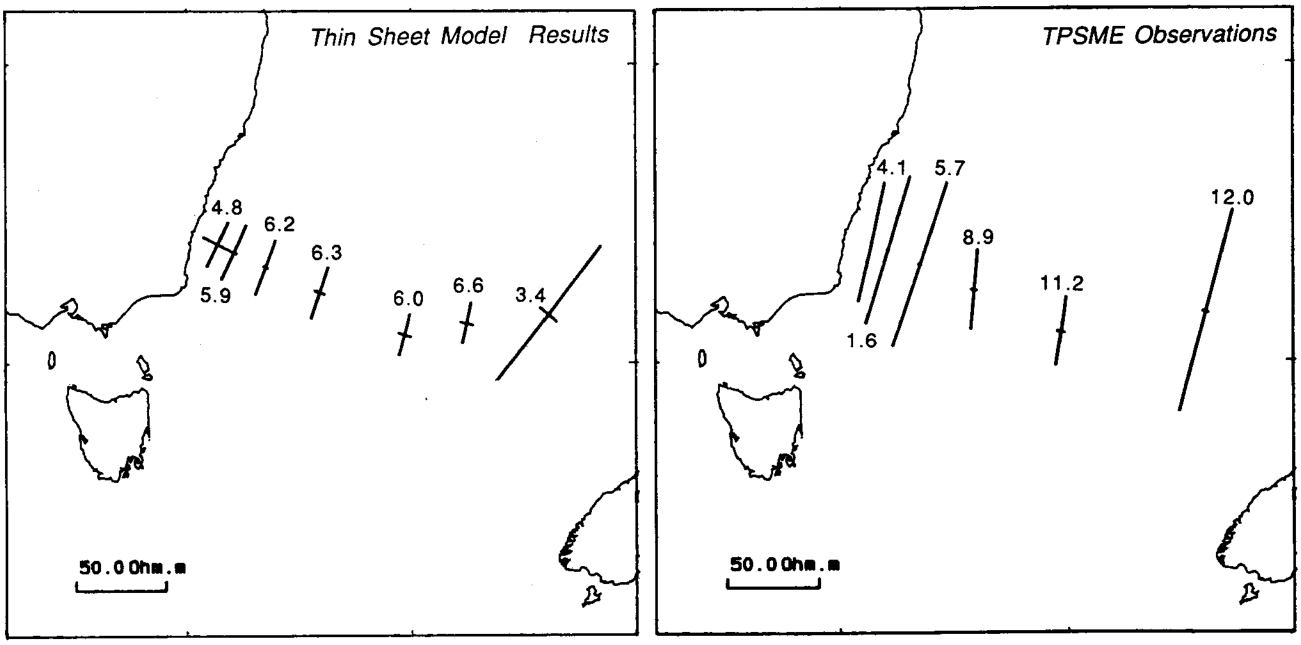

FIGURE 4
Comparison between the apparent resistivity axes for the thin sheet model and the measured data, for an inducing period of 9 hrs. The E-pol and B-pol apparent resistivities are plotted as a cross at each TPSME site. (Observed data from Ferguson 1988).

1959). The vertical component of the fluctuations in the magnetic field at and near a lateral conductivity boundary may be correlated with fluctuations in the horizontal magnetic field. This relationship may be expressed by the scalar transfer functions A and B in

$$Z = AH + BD,$$

where H, D and Z denote the north, east and downward components of the Earth's magnetic field. These quantities are complex and frequency dependent, so that real and quadrature (imaginary) arrows may be plotted from the magnitude and phase of the component pairs (-Re(A), -Re(B)) and (Im(A), Im(B)) respectively, for each inducing period.

Figure 5 illustrates Parkinson arrows derived for the seafloor TPSME sites, using the calculated magnetic field values for the bottom of the thin sheet model, together with values obtained by Ferguson (1988) from the observed data. The arrows show a reasonable degree of consistency. The orientation and magnitude of the arrows for the coastline sites are modelled well, particularly Canberra Magnetic Observatory and the coastline sites. In general, the real components of the Parkinson arrows show a better agreement between observation and model than do the quadrature parts.

Conclusion

Thin sheet modelling allows complex three-dimensional electromagnetic induction effects to be studied in a relatively straightforward manner, and many of the characteristics of the TPSME data are reproduced by this method. The results described may be used to 'de-distort' the observed sea-floor magnetotelluric observations, rendering them amenable to standard one-dimensional inversion routines. Consequently it should be possible to remove the influence of three-dimensional induction effects from the TPSME data, and to determine a model of the electrical conductivity structure of the Tasman Sea to a depth of several hundred kilometres.

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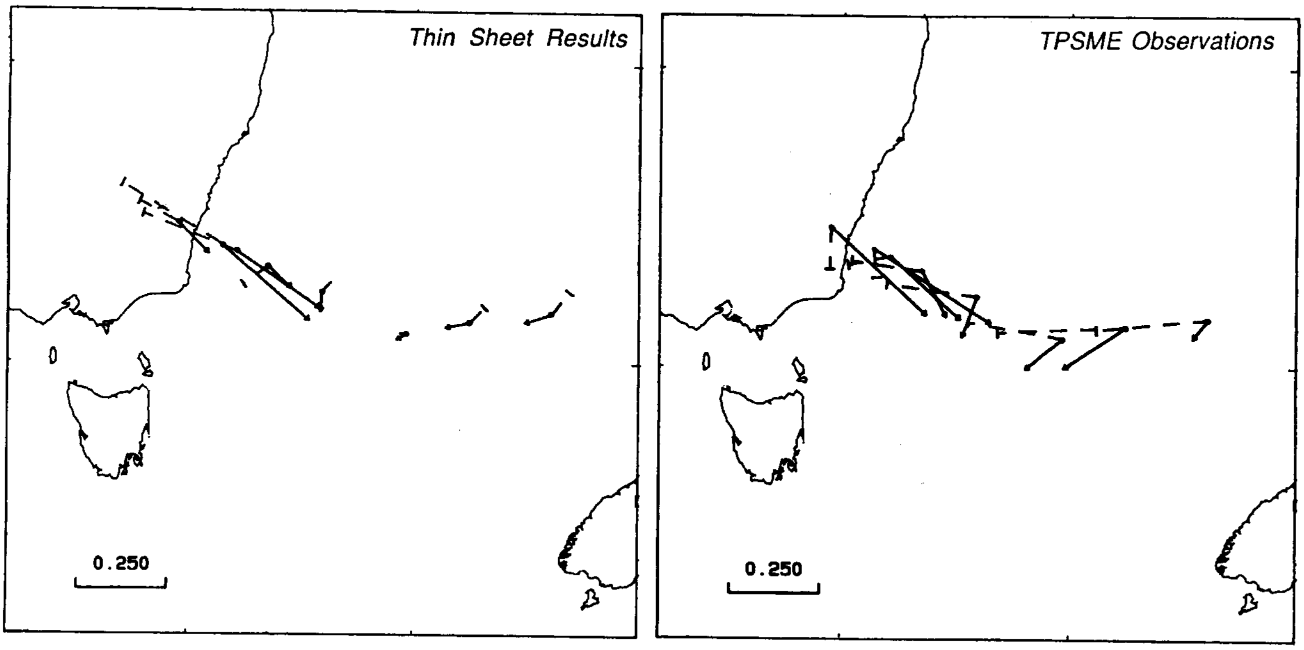

FIGURE 5
Comparison between the real and quadrature Parkinson arrows for the thin sheet model and the measured data, for an inducing period 9 hrs. The solid and dashed lines correspond to the real and quadrature components respectively. (Observed data from Ferguson 1988).