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The Analysis of Daily Variations Recorded by Magnetometer Arrays

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Summary

A procedure is proposed for the analysis of daily variations recorded by magnetometer arrays, based upon the analysis of each quiet day as an individual and complete event. Such an approach is justified by the argument that any particular observatory has no premonition of variation to occur on the next day, and records only an insignificant signal from the variation of the day before. Many of the factors affecting the variation on an individual day are independent of the rotation of the Earth, and are also therefore independent of the strong 24-hr, 12-hr, 8-hr and 6-hr periodicities usually associated with daily variation analyses.

Data for individual days are transformed into the frequency domain and plotted on polar diagrams. For an array area, the separation of anomalous fields from regional fields is based upon determining the latter as a smooth curve on a polar diagram. If a sufficient number of days are available for analysis, and they are sufficiently varied, all nine elements of a transfer function matrix may be sought, relating locally anomalous fields to regional fields.

1. Introduction

The analysis of daily variations has been based traditionally on established or permanent observatories which often have available data from long periods of recording. In these traditional studies the objective has been to seek periodicities, which can be linked empirically with other phenomena.

Magnetometer array studies have produced a new class of daily-variation records: a few days of data from many instruments, rather than many days of data from a few instruments. The purpose in analysing daily variations recorded by magnetometer arrays differs from the traditional objective. No longer are periodicities in the data being sought for their own sake: the simple purpose of magnetometer array studies in using daily variations is to exploit them as naturally-occurring source fields of long period, which cause electromagnetic induction in the Earth.

This paper examines the justification for and the consequences of considering each day to be a separate and distinct event, isolated from other daily variations preceding it and following it. One advantage to array analysis of this approach is that the variation in morphology from one quiet day to the next may be used to check the sensitivity of induced anomalous fields to primary-field polarization.

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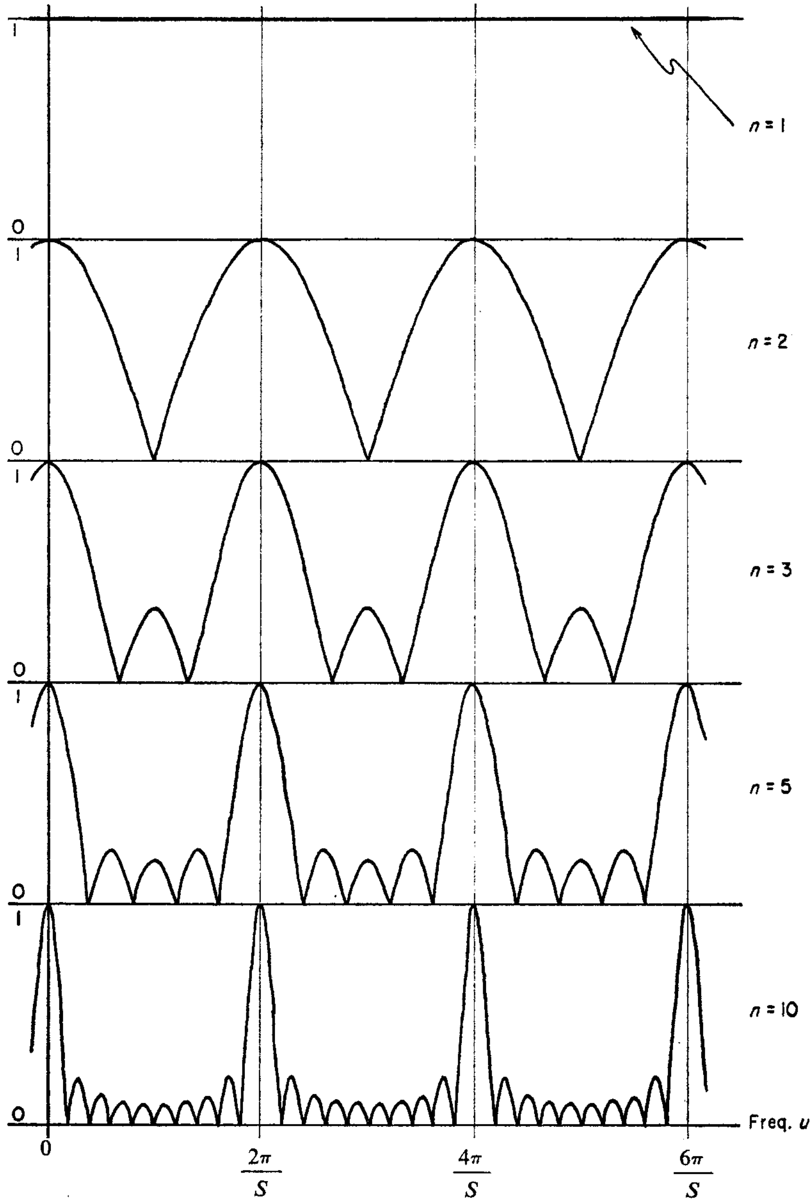

FIG. 1. The function $|(1/n)\cdot\sin(nuS/2)\cdot\mathrm{cosec}\,(uS/2)|$ plotted against angular frequency u for $n = 1, 2, 3, 5$ and 10.

2. A simple mathematical exercise

The particular Fourier transform used in this paper is

$$g(u) = \int_{-\infty}^{\infty} f(t)\,\mathrm{e}^{-iut}\,dt. \tag{1}$$

The Fourier transform of an isolated square pulse of amplitude A in a time series $f(t)$, where

$$f(t) = A, \qquad S - T/2 \leqslant t \leqslant S + T/2$$
$$f(t) = 0, \qquad \text{all other } t,$$

is

$$g(u) = (2A/u).\sin(uT/2).(\cos uS - i \sin uS). \tag{2}$$

If the time series consists of n such events at regular intervals S, then its Fourier transform may be reduced to the form

$$|g(u)| = |(2A/u).\sin(uT/2).\sin(nuS/2).\operatorname{cosec}(uS/2)|. \tag{3}$$

For the limiting case where $T \to 0$ and $AT = K$ constant,

$$|g(u)|_{T\to 0} = K|\sin(nuS/2).\operatorname{cosec}(uS/2)|$$

The function $|\sin(nuS/2).\operatorname{cosec}(uS/2)|$, normalized by division by n, is plotted in Fig. 1 for five different values of n. Equation (3) and Fig. 1 also occur in the theory of diffraction of light by a Fraunhofer grating, where the series of openings in space which form the grating are analogous to the series of pulses in time which cause Fig. 1. Note that as n gets larger in equation (3), the spectrum of the basic square pulse becomes 'modulated' and dominated by the periodicities of the repetition time S and its subharmonics. Peaks, which approach lines for large n, occur at

$$u = 2m\pi/S \quad \text{where} \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots$$

and zeros occur at $u = 2m\pi/nS$ where m and n are both integers but m/n is not an integer.

A hypothetical example of such a sequence of pulses might be a square pulse function which is triggered every morning by the rising of the Sun: perhaps the stress on a door step as a man stands there steadily and regularly for time T before setting off to work. The harmonic analysis of records of this stress observed over several days or weeks would show periodicities at 24 hr and its subharmonics, and while these may be interpreted in terms of the triggering process causing the stress, they do not give any information regarding the fundamental process of the man standing on the step.

This example emphasizes that notwithstanding the very strong tradition of seeking 24-hr, 12-hr, 8-hr and 6-hr peaks in the spectra of geomagnetic observatory records, such periodicities may not by themselves be informative about the fundamental process occurring during a single day's quiet variation, but merely reflect the forcing effect of the rotation of the Earth upon a possibly otherwise separate process. The point is demonstrated in another way by the fact that no conventional interpretation considers the 24-hr, 12-hr, 8-hr and 6-hr harmonic peaks to represent different current systems in the ionosphere. While separation of a disturbance into different frequency components by harmonic analysis is fundamental mathematically, it may be artificial physically.

3. Basic physics

It is generally accepted that quiet-day variation is caused by primary electric currents which flow external to the Earth (Chapman & Bartels 1940; Matsushita & Campbell 1967). The electrical conductivity of the ionosphere is strongly dependent on solar radiation, which restricts the currents to the day-time hemisphere, suggesting the simple model of an electric current system which remains steady relative to the Sun while the Earth rotates beneath it. Atmospheric oscillations, especially of 12-hr period, are thought to contribute to the magnetic daily variation. Their effect also, however, will be controlled by the electrical conductivity of the ionosphere, and so should be complete day-by-day rather than appearing as a sinusoidal effect of 12-hr period.

With this model once a terrestrial observatory has traversed across beneath the current system and passed into night-time the event of the day's variation is over, and the next day is a new start. The observed steadiness of field components during nights between quiet days supports the view that the ionospheric daily-variation currents are effectively shielded from the night-time side of the Earth, and also indicates that the decay of any induced currents remaining from the day-time variation is very slow, such that over 12 hr the change in field is barely perceptible.

It is therefore proposed that each quiet day can be taken alone and considered from the point of view of induction in the Earth as an isolated and aperiodic event. That successive quiet days can differ considerably from each other (Matsushita 1975) supports this idea. In computing a Fourier transform according to equation (1), for practical purposes the signal for a particular quiet day can be considered to be zero from $-\infty$ till the commencement of the day, and then after the day is over zero again to $+\infty$. That is, in an actual computation, zeros can be added to the signal of the day's variation without the physical process which actually occurred in the event of the single day being seriously misrepresented.

Such a process will produce spectra different from those usually associated with sequences of quiet days. Fig. 2(a) shows the amplitude spectrum of the vertical component of a quiet day recorded in South Australia, computed by adding 1999 zeros to a series of 49 half-hour values for the actual day, and then estimating the Fourier transform by discrete harmonic analysis of this longer series of 2048 points. Fig. 2(b) shows a similar spectrum for a second (and succeeding) quiet day, and Fig. 2(c) the spectrum when the days are analysed together. Note that Fig. 2(c) has very much the shape of either Fig. 2(a) or Fig. 2(b) modulated by the $n = 2$ curve of Fig. 1, and that while accuracy has increased at the peaks, information at the troughs has in fact been lost, by a sort of interference effect analogous to Fraunhofer diffraction.

Except at very long and very short periods, there appears no reason to doubt the validity of the entire spectra for the single days shown in Fig. 2(a) and (b). Thus information has been obtained from analysing the data of single days, and covers periods and possibly horizontal polarizations other than those of 24 hr and its subharmonics.

4. Some points arising for discussion

4.1 *Units of spectral estimates*

A single quiet day can be harmonically analysed as a periodic event, and standard Fourier coefficients obtained; or it can be regarded as an aperiodic event, and a Fourier transform obtained. It is relevant to note the relationships between the value of a Fourier coefficient and the value of a Fourier transform at the same frequency. The Fourier transform unit occurring in this paper will be the nT-day (equivalent to a nT/radian/day), and the Fourier coefficient unit simply the nT.

For a data series of n points x_j, $[j = 0, 1, 2, \ldots, (n-1)]$, standard Fourier coefficients (Jeffreys & Jeffreys 1956, p. 430) are given by

$$A_0 = (1/n) \sum_{j=0}^{n-1} x_j,$$

$$A_r = (2/n) \sum_{j=0}^{n-1} x_j \cos(2\pi jr/n); \quad B_r = (2/n) \sum_{j=0}^{n-1} x_j \sin(2\pi jr/n),$$

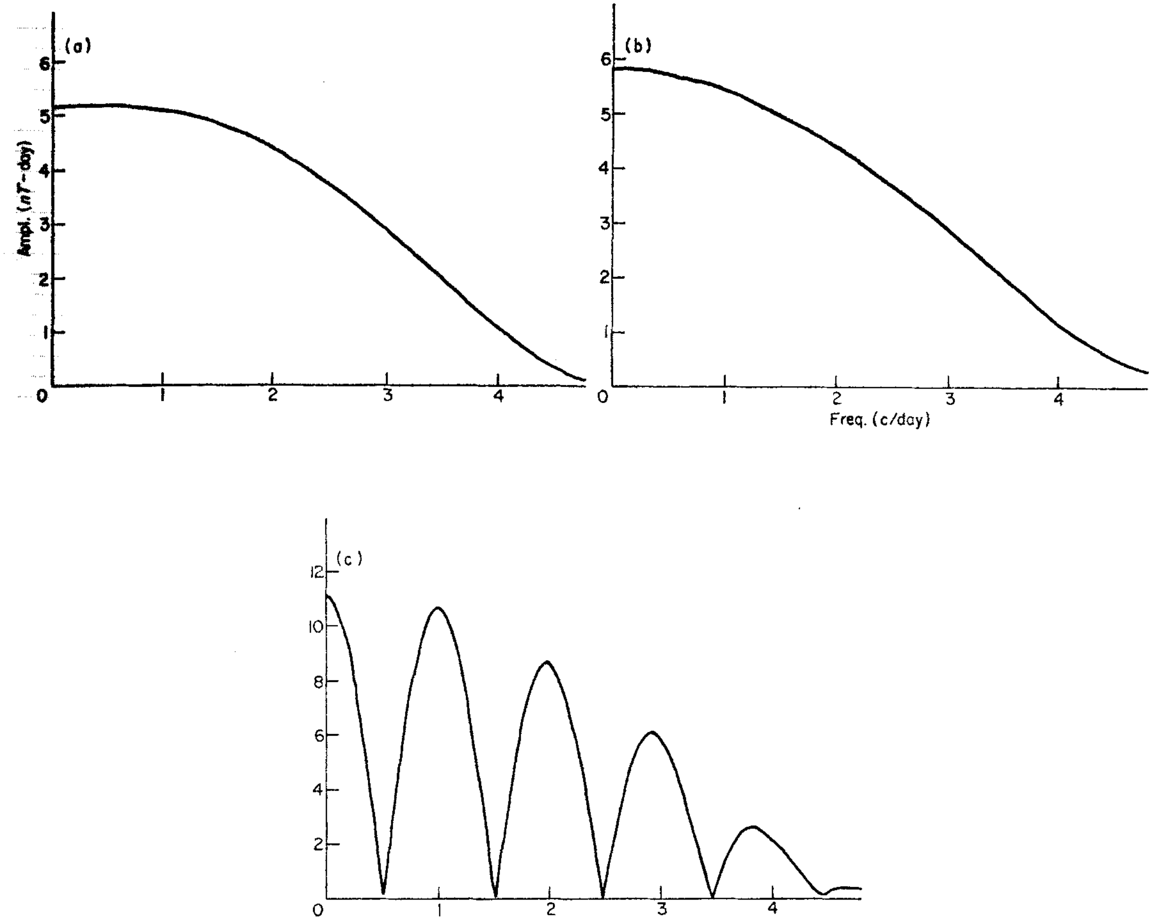

FIG. 2. Fourier transform amplitudes computed for the vertical component of variation recorded at station B9 of Fig. 5: (a) for the day 1970 October 8, 1400 GMT to October 9, 1400 GMT; (b) for the day 1970 October 9, 1400 GMT to October 10, 1400 GMT; (c) for both days together.

where r is an integer greater than zero and less than $n/2$, and, if n is even,

$$A_{n/2} = (1/n) \sum_{j=0}^{n-1} (-1)^j x_j; \quad B_{n/2} = 0,$$

such that the data series may then be expressed

$$x_j = A_0 + \sum_{r=1}^{m} [A_r \cos(2\pi rj/n) + B_r \sin(2\pi rj/n)]$$

where $m = \frac{1}{2}n$ if n is even, and $m = \frac{1}{2}(n-1)$ if n is odd.

To estimate the function $g(u)$ of equation (1) from discrete data, under the condition that the data series x_j is aperiodic and is zero both before and after the particular n values in question, equation (1) is written in the approximate form:

$$G(u) = \sum_{j=0}^{n-1} x_j (\cos ut_j - i \sin ut_j) \Delta t$$

where the data points x_j occurred at times t_j, and $G(u)$ is the approximate estimate of $g(u)$. If the time zero is taken to correspond to the commencement of the data series, and the data points are at intervals Δt apart in time, then $t_j = j\Delta t$. Consider now only those values of u which may be expressed

$$u_r = 2\pi r/n\Delta t, \quad \text{where} \quad r = 0, 1, 2, 3, \ldots, m,$$

then

$$G(u_r) = \Delta t \sum_{j=0}^{n-1} x_j [\cos(2\pi rj/n) - i \sin(2\pi rj/n)]$$

Comparing this expression for $G(u_r)$ with the expressions for the Fourier coefficients above shows that

$$G(u_0) = n\Delta t A_0$$

$$\{G(u_r)\}_{\text{real}} = (n\Delta t/2)A_r; \quad \{G(u_r)\}_{\text{imag}} = -(n\Delta t/2)B_r \tag{4}$$

and, for n even,

$$G(u_{n/2}) = n\Delta t A_{n/2}.$$

Note that these expressions would change if:

(i) A_0 and $A_{n/2}$ were defined differently, as is the case for some computing routines;

(ii) The Fourier transform were defined with some constant factor concerning π outside the integral on the right-hand side of equation (1), as is sometimes the case.

(iii) The Fourier transform were defined with a positive rather than a negative exponent in the term e^{-iut}; (the negative exponent gives rise to the minus sign in the expression above for $\{G(u_r)\}_{\text{imag}}$. It is a possible source of confusion in the estimation of phase angle).

4.2 *Spectra from Bennett & Lilley* (1973)

Bennett & Lilley (1973) analysed three successive quiet days recorded in south-east Australia, and presented Fourier amplitude spectra, an example of which is shown in Fig. 3(a). For the data of three days a spectrum modulated by the $n = 3$ curve of Fig. 1 might be expected, and inspection of Fig. 3(a) shows the Bennett & Lilley spectrum to indeed have this shape. Because three days of data give spectra with relatively wide spectral peaks, one may have confidence that the main features of the spectrum have been resolved.

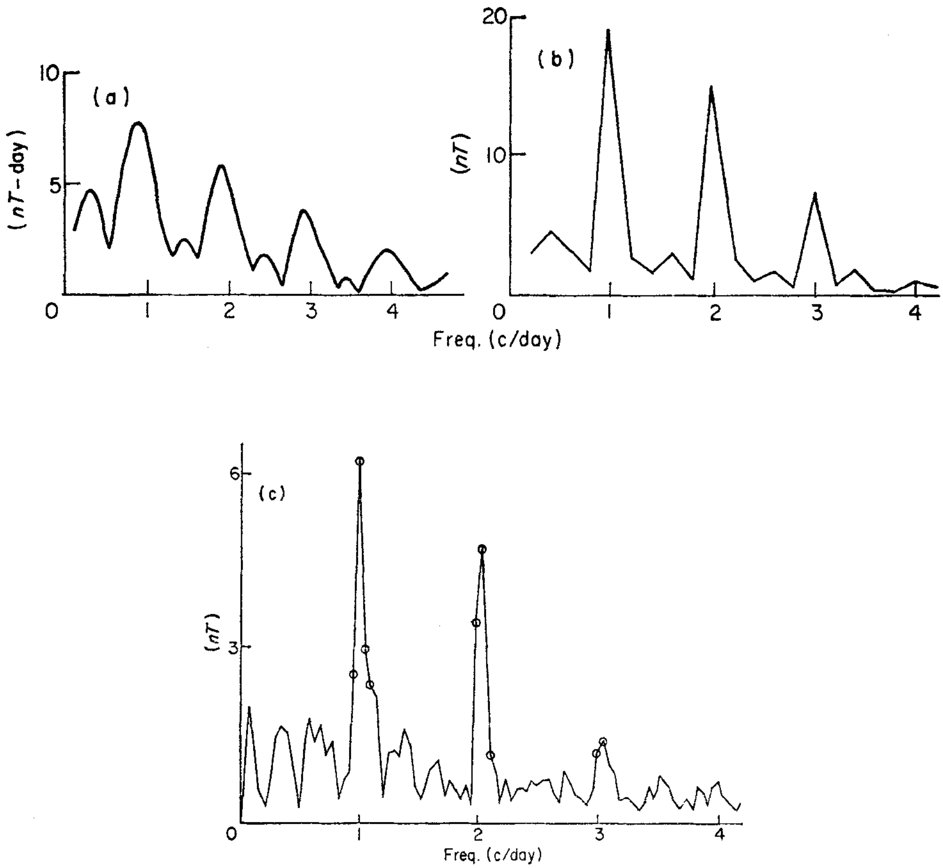

FIG. 3. (a) Fourier transform amplitude from Bennett & Lilley (1973) for three quiet days recorded at the Australian station MDR (Z component). (b) Fourier coefficient amplitudes from Camfield (1973) computed for five quiet days recorded at the Canadian station DON (Y component). (c) Fourier coefficient amplitudes from Caner (1971) for 14 days of variation recorded at the Canadian station Penticton (Z component).

4.3 *Spectra from Camfield* (1973)

Camfield (1973) and Camfield & Gough (1975) digitized variograms for $5\frac{1}{2}$ successive quiet days recorded in western North America, ultimately obtaining harmonic coefficients with a frequency resolution of 0·2 cycle/day. An example is shown in Fig. 3(b). The amplitude values every 0·2 cycle/day have been joined together to make inspection of the figure easier, but no other significance is attached to the profiles at frequency values in between the discrete harmonic points. Camfield's purpose was to determine the *Sq* variation across a particular region of the North American continent. The spectrum of Fig. 3(b) is presented here, however, to emphasize the point that if the five days are considered to be separate events, then the data could be analysed with many zeros added, to fill in the spectra between the points spaced 0·2 cycle/day apart, and possibly disclose the secondary maxima evident for $n = 5$ in Fig. 1. A possible application of data like Camfield's when the five days are analysed individually is mentioned in Section 6.3.

4.4 *Spectra from Caner* (1971)

Caner (1971, p. 7205) reports the analysis of 14 days of data recorded in Canada, digitized at half-hour intervals. Simple harmonic analysis of such data would give amplitude values at frequency intervals of 1/14 (= $7 \cdot 1 \times 10^{-2}$) cycle/day; there would be values at the exact periods of 24 hr, 12 hr, 8 hr and 6 hr, with 13 intermediate values between each harmonic mentioned. Caner's figures (one of which is reproduced as Fig. 3(c) of this paper) show, however, some 64 discrete spectral values between 0 and 3 cycle/day, implying an initial data length of some $21\frac{1}{3}$ days, or 1024 data points at half-hour intervals. Because some 'fast Fourier' computing routines operate on data sets which are 2^N points in number (where N is an integer), it seems likely that 351 zeros were added to Caner's original data before Fourier analysis, giving amplitude estimates at frequency intervals of $4 \cdot 7 \times 10^{-2}$ cycle/day.

For the reasons given in Section 2, a 14-day series may be expected to have an amplitude spectrum with very sharp peaks, and strong suppression in between them. The function $|\sin (nuS/2) . \operatorname{cosec} (uS/2)|$ for $n = 14$ and $S = 1$ day will fall to one half of its peak value for a frequency shift of $4 \cdot 34 \times 10^{-2}$ cycle/day to either side of the peak. A consequence of the zeros added to Caner's series will be that the spectral values will not be given for exactly 24 hr, 12 hr and 8 hr, but rather for the harmonics of 1024 data points, which include:

..., 24·36 hr, 23·25 hr, ..., 12·18 hr, 11·90 hr, ..., 8·12 hr, 7·99 hr, ..., etc.

Because the spectral peaks for a 14-day series may be expected to have a half-width of only some $8 \cdot 7 \times 10^{-2}$ cycle/day, amplitude values at frequency intervals of $4 \cdot 7 \times 10^{-2}$ cycle/day may well miss the peak values, and give apparent peak estimates perhaps 10–15 per cent too low.

The point demonstrated is that in obtaining amplitude spectra for the variations of a number of successive days, either the data of an exact number of days should be analysed to give basic Fourier coefficients at 24 hr, 12 hr, 8 hr and 6 hr exactly, or, if it is wished to 'fill in' the spectrum, sufficient zeros must be added to resolve the spectral peaks completely. In one sense the inclusion of extra days in a series to be analysed can work against the analyser, by making the peaks narrower and thus their maximum values harder to detect.

4.5 *Lunar periodicities*

One point requiring discussion is the question of lunar effect in the daily variation. The lunar effect is known to have a periodicity of 24 hr 50 min, though during a single day it also is controlled by solar radiation in the ionosphere, and therefore it also will be complete day by day.

It would be incorrect to attribute the 24 hr 50 min period part of the spectrum in Fig. 2(a) (say) to lunar effect, because Fig. 2(a) is not a spectrum of periodicities at all. The effect of one lunar day will indeed be present in both Fig. 2(a) and (b), but with a distribution here undetermined. The signal which the Moon causes, with a periodicity of 24 hr 50 min, may in fact itself have relatively little $T = 24$ hr 50 min amplitude, suggesting a clear distinction between a signal which recurs with a certain periodicity, and one which contains amplitude of a certain period. To detect a lunar effect in a sequence of quiet days, destructive interference between the days must be arranged so that the modulating envelope due to the 24 hr periodicity has a zero at 24 hr 50 min, like the zeros occurring at $u = 2m\pi/nS$ (where m and n are integers but m/n is a fraction) in Fig. 1.

Taking the Fourier transform of a continuous sinusoid of period 24 hr shows that its spectrum will be zero at period 24 hr 50 min if multiples of 29·80 days of record are analysed.

4.6 The night-time currents of Ashour & Price (1965)

Treating the daily variation not as a series of pulses but as a summation of endless sinusoids, Ashour & Price (1965) concluded on theoretical grounds that induced Earth currents should also flow in the night-time hemisphere. The reason is perhaps simply that a steady field causes no induction and cannot itself be induced; consequently secondary currents flowing in the Earth must oscillate about zero baseline, and a Fourier expansion of them must have no zero-frequency term. Hence the necessity for night-time currents to offset the daytime X and Z variations, the induced parts of which for a given latitude tend to be predominantly either all positive or all negative. Variations in Y, more nearly balanced positive and negative during daytime, do not require night-time offset to the same extent.

It is possible that the traditional mathematical approach of Ashour & Price and its result of night-time currents can be matched with the approach proposed in this paper by regarding currents flowing during a particular night to be the gradual decay of daytime induction, not only of the preceding day but of all preceding days. By this cumulative effect a significant night-time current flow can be built up. If, after the apparent end of a quiet day, the induced field decays from an amplitude A according to

$$f(t) = A\,\mathrm{e}^{-kt}$$

then the combined amplitude of the induced fields from n such days in succession, $f_n(t)$, will be

$$f_n(t) = A[\mathrm{e}^{-kt}+\mathrm{e}^{-k(t+1)}+\mathrm{e}^{-k(t+2)}+\ldots+\mathrm{e}^{-k(t+n-1)}]$$

where the time t is measured in days from the apparent end of the nth day. For large n $f_n(t)$ converges,

$$f_n(t) = A\,\mathrm{e}^{-kt}/(1-\mathrm{e}^{-k})$$

$$\simeq (A/k)\,\mathrm{e}^{-kt} \text{ for small } k.$$

Thus, even if A is very small, $f_n(t)$ may be significant for k sufficiently small; that is for a sufficiently slow decay of the remaining induced currents. If it is not below the level of observation, this decay should be observed every quiet night. From such night-time records it may be possible to place an upper limit, say C, on the decay occurring over say half a day. Such an upper limit would give the inequality, for small k,

$$A/2 \leqslant C.$$

When the Ashour & Price baseline shift is regarded as being due to the gradual decay of induced currents from preceding days, it is appropriate to take local midnight as the zero level for the variation of the following quiet day. Error will enter from the fact that the gradual decay of the induced currents from previous days will continue during the day in question, and thus cause a baseline shift; this will however be no greater than the baseline shift occurring during the previous night, and so may either be corrected for as such or (if the correction is insignificant) neglected.

The requirement that there be no zero-frequency induced component does incidentally suggest a first-order correction to be made to the spectra of induced fields: the subtraction of an appropriate baseline to reduce the zero-frequency amplitude to zero, if it is not already so.

5. Sources of error in computing Fourier transforms

5.1 Random errors in digitized data

A magnetic variation record, although often recorded in analogue form, is invariably in digitized form when spectrally analysed. Then, as shown in Section 4.1, an estimate of the Fourier transform of equation (1) is obtained by computing its real part $G_R(u_r)$ as

$$G_R(u_r) = \Delta t \sum_{j=0}^{n-1} x_j \cos(2\pi rj/n)$$

and its imaginary part $G_I(u_r)$ as

$$G_R(u_r) = -\Delta t \sum_{j=0}^{n-1} x_j \sin(2\pi rj/n).$$

The functions $G_R(u_r)$ and $G_I(u_r)$ are thus linear combinations of the n data points. If each data point has a normal probable error of q, the probable error E_1 of $G_R(u_r)$ is (Braddick 1963, p. 21),

$$E_1 = \Delta t[1+\cos^2 p+\cos^2 2p+\ldots+\cos^2(n-1)p]^{\frac{1}{2}} q$$

$$= \Delta t\left[\left(n+1+\sum_{j=1}^{n-1} \cos 2jp\right) \Big/ 2\right]^{\frac{1}{2}} q$$

where $p = 2\pi r/n$. Similarly, the probable error E_2 of $G_I(u_r)$ is

$$E_2 = \Delta t\left[\left(n-1-\sum_{j=1}^{n-1} \cos 2jp\right) \Big/ 2\right]^{\frac{1}{2}} q$$

E_1 and E_2 can be computed accurately for individual cases, but the following approximate estimates can be written down directly. If j in running from 1 to $n-1$ takes $\cos 2jp$ through several cycles, and $n \gg 1$, then

$$E_1 = E_2 \simeq \Delta t\,.\,(n/2)^{\frac{1}{2}}.q$$

$$= T\,.\,(2n)^{-\frac{1}{2}}.q \quad \text{for} \quad T = n\Delta t.$$

If j in running from 1 to $n-1$ takes $\cos 2jp$ only through the start of a cycle, so that $\cos 2jp \simeq 1$ for all j, then

$$E_1 \simeq \Delta t\,.\,n^{\frac{1}{2}}.q$$

$$= T\,.\,n^{-\frac{1}{2}}.q$$

and $E_2 \simeq 0$; (note in this case $G_I(u_r) \simeq 0$ also).

The first case is the common one, and the result for it is used in the next paragraph and elsewhere in this paper.

If errors in the corresponding Fourier coefficients are required, reference to equations (4) gives the probable errors in A_0 and $A_{n/2}$ (if the latter exists) to be $(2n)^{-\frac{1}{2}} q$, and in A_r and B_r generally to be $(2/n)^{\frac{1}{2}} q$. A point plotted to represent a pair of such Fourier coefficients on an harmonic dial as in Fig. 6 below will have an error circle of radius $(2/n)^{\frac{1}{2}} q$.

For data points with probable errors of 1 nT, the digitization of the data of one day at intervals of 4 hr, 1 hr, 15 min, 5 min and 1 min will give Fourier transform amplitudes with probable errors of 0·31, 0·15, 0·07, 0·04 and 0·02 nT-day, respectively, and Fourier coefficient amplitudes in plots like those of Fig. 6 with probable-error circles of radii 0·54, 0·28, 0·14, 0·08 and 0·04 nT, respectively.

5.2 *Digitizing interval and folding frequency*

To completely avoid significant aliassing trouble, magnetograms must be digitized with an interval of 1 min or less, as done by Camfield (1973). However when a digitizing interval of perhaps an hour may be otherwise sufficient (see Section 5.1), it may be practical to simply judge hourly means: error in such judgement can be taken to be random. For the purpose to be suggested in Section 6.3 it may be most profitable to put available digitizing effort into more days at longer intervals, rather than fewer days at shorter intervals.

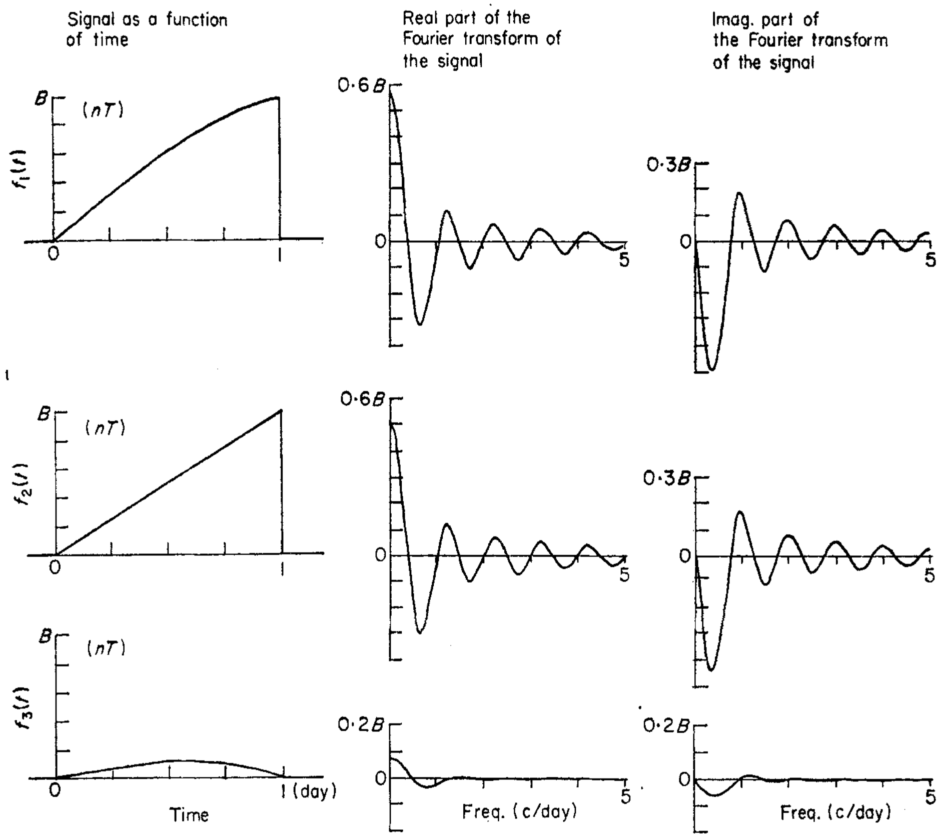

FIG. 4. The correction of a sinusoidal drift $f_1(t)$ by the subtraction of a linear trend $f_2(t)$. The error remaining is $f_3(t) = f_1(t) - f_2(t)$. The left-hand column shows signals as a function of time, and the central and right-hand columns show the real and imaginary parts of Fourier transforms of the signals.

5.3 *The assumption that each day is a complete event*

When each quiet day as recorded is taken to be a complete and distinct event, there will firstly be the errors of induced-field truncation, most serious at very long periods. For some data successive midnight values may not be the same, implying that non-daily variations have also been recorded, and that other truncation errors may be present.

Short period events, taking place at night-time and disturbing the midnight value as a result, may be rendered ineffective by taking hourly means as already described. Long period events, such as the recovery phase of a storm, may be corrected for to some extent by removing a linear trend from midnight to midnight. To examine the error resulting from the approximate nature of this correction, consider the case shown in Fig. 4.

The perturbing signal superimposed upon an ordinary quiet day is of form $f_1(t) = A \sin(2\pi t/5)$ of period 5 days. This perturbation causes a baseline shift between two successive midnights of B nT, which is removed by the common process of subtraction of a linear trend. Fig. 4 shows that the approximation of the correction is most serious at long periods, and that for a reasonable maximum value of B (say 10) the error remaining is negligible for periods of 24 hr and less.

The result would not have been so satisfactory if the perturbing signal had not been so well approximated by the linear trend. Medium period events, overlapping from day to day, also cause errors which will differ from case to case. The best safeguard against these errors seems to lie in choosing for analysis those days which have the quietest nights.

6. Procedures for reducing practical array data

6.1 *Smoothing out regional fields leaving anomalous residuals*

It is basic to magnetometer array studies when examining data from shorter period events such as substorms to separate the observed fields into regional and locally anomalous parts. At the longer periods of the daily variation and its harmonics, this exercise has been carried out by Bennett & Lilley (1973), who analysed three days together as explained in Section 4.2. Camfield (1973) presented data reduced to the stage where such a separation could be attempted by inspection.

The underlying assumption is that the primary fields will be smooth, so that short-scale perturbations in the observed combination of primary plus secondary fields must be of geological origin. This assumption is open to the consistency test, that perturbations should re-appear in the same geological locations for different magnetic variation events.

Fig. 5 shows the records of two consecutive quiet days recorded by 24 stations in southern Australia (the data upon which Fig. 2 is based are included). These two days have been analysed separately, and Fig. 6(a) and (b) show the 12-hr components of the vertical field at each station plotted on separate harmonic dials, one for each day. Fig. 6(c) shows a proposed separation into regional and anomalous parts for the fields recorded along line C of the array on the first day. Similar graphical separations can be made for all three components, on both days, but one example is sufficient to demonstrate here the method proposed.

In fact the area in question appears less anomalous at these long periods than it appeared to be at shorter periods (Gough, McElhinny & Lilley 1974), and the examples of Fig. 6 may show almost the limit of detectable anomalous effects. Similar plots for the data of Bennett & Lilley (1973) and Camfield (1973) might be expected to show anomalous and regional fields of comparable magnitude.

6.2 *Comparison with global estimates of Sq*

Much work has gone into the determination on a global scale of ' typical ' quiet days. It might be tempting to use such a global *Sq* distribution as the regional field to be subtracted from observed station values to give anomalous estimates, but such a process would introduce errors in ignoring the variation of the actual regional field from day to day.

However for the interest of comparison, the global *Sq* distribution of Matsushita (1967) is also plotted in Fig. 6(b). Possibly regional fields, smoothed from magnetometer array data like Fig. 6(c) and averaged, ultimately may contribute to the determination of detailed *Sq* models.

6.3 *A matrix of transfer functions for anomalous stations*

In analysing the response of the Earth to shorter variation events it has proved very useful to inter-relate regional and locally anomalous components with a matrix of ' transfer functions ' (Schmucker 1970, p. 20):

$$\begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} = \begin{bmatrix} h_{xx} & h_{xy} & h_{xz} \\ h_{yx} & h_{yy} & h_{yz} \\ h_{zx} & h_{zy} & h_{zz} \end{bmatrix} \begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} \tag{5}$$

X, Y and Z are variation components at a particular frequency, and the elements of

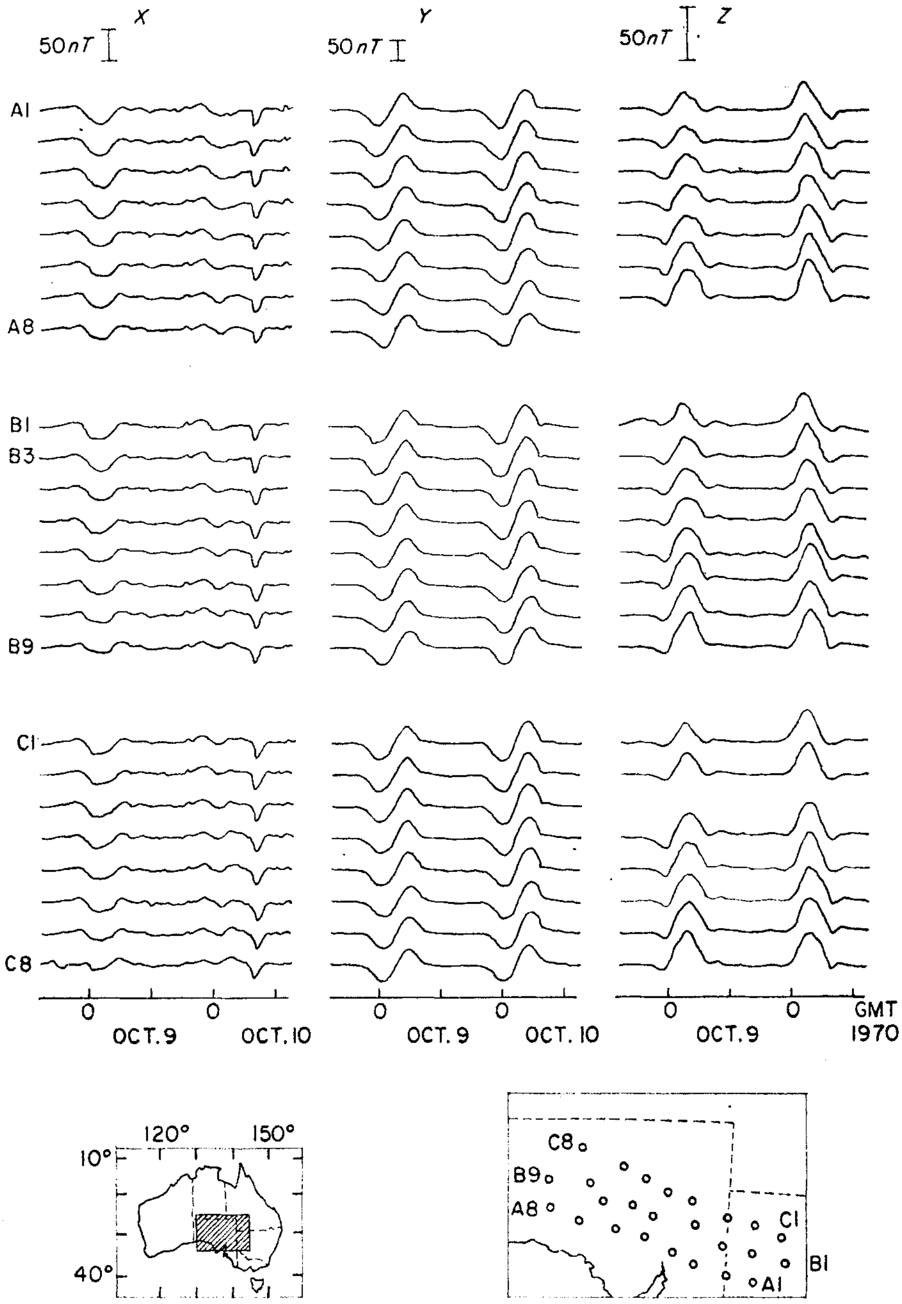

FIG. 5. Transcribed records of two quiet days recorded by a magnetometer array in southern Australia. Records are not held for station B2, or for the Z components of stations A8 and C3.

the h-matrix are complex constants, allowing the anomalous fields to have in-phase and out-of-phase parts. The subscript A refers to anomalous components at a particular station, and the subscript R to the regional components for the same station determined from inspection of the data from the whole array area.

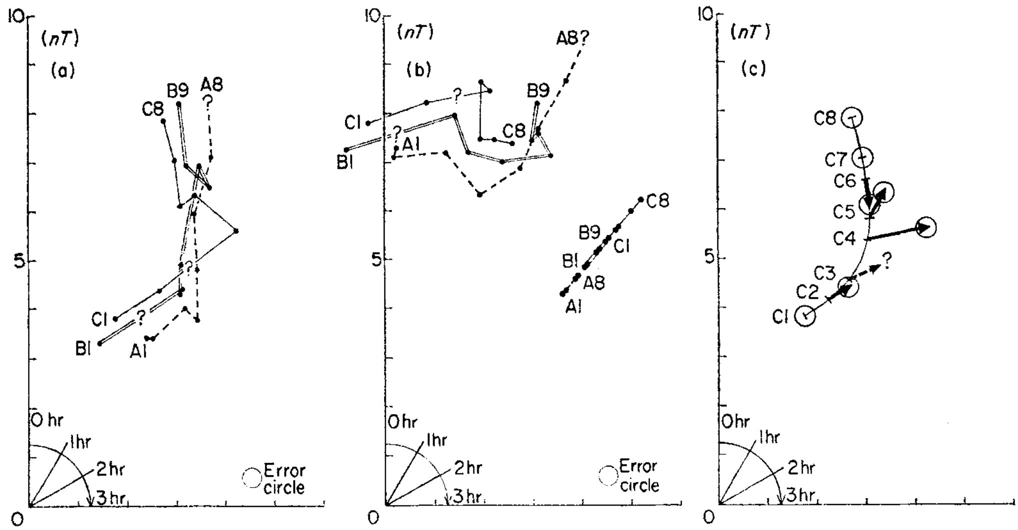

FIG. 6. Polar diagrams for the 12-hr component of daily variation, Z component. Units of amplitude are nT, and correspond to the amplitude of the 12-hr Fourier coefficient. Phase is relative to local midnight for all stations. An event with a larger and more positive phase occurred at a later real time. The two days are those of 1970 October 8–10, as shown in Fig. 5. (a) The first day. The error circle applies to all plotted points and is of radius $0{\cdot}2\ nT$, corresponding to a probable error of $1\ nT$ in all 49 data points from which the Fourier amplitudes were derived. (b) The second day. Error circle as for part (a). Also shown, along a straight line, are the *Sq* values for the appropriate stations taken from the determination of Matsushita (1967). (c) An attempt to separate anomalous and regional parts for line C on the first day. The line-C points have first been plotted, with their circles of probable error. After inspection of Fig. 6(a), a regional trend has been drawn in as the thin line on Fig. 6(c) joining points C1 and C8 (which are assumed to be not anomalous), and the appropriate positions for the intervening stations C2 to C7 have been scaled in along this line. Heavy arrows joining the position of a station on the regional line to its actual point as plotted in the error circle then show the anomalous value to be added to the regional value to give the observed value.

The degree to which such relationships also hold at long periods is of crucial importance in the interpretation of anomalous effects. A basic question in geomagnetic depth-sounding is whether long-period effects are caused by induction on a local or a global scale, and if the former whether by the horizontal or vertical component of the regional fields.

Data from individual quiet days, separated as proposed in Section 6.1 into anomalous and regional parts, may be examined to see if equation (5) holds. For daily variations the situation is more complicated than for shorter period events, as a substantial regional Z component may be present; at shorter periods, the third column of the matrix is often irrelevant because $Z_R \simeq 0$.

Solution of the matrix in equation (5) requires three independent events, or preferably more if residuals are to be obtained which will indicate how general the relationship in fact is. Limitations on the number of days of array data used to test equation (5) will come only from the number of quiet days recorded, and the resources available to digitize the data if it has been recorded in analogue form.

Variability in the events from day to day will be important. Mildly disturbed days may be most useful (as long as the disturbance is of the *SD* type and does not go on into the night), or a seasonal change if the array operation has taken place over a sufficiently long period. Variability in the lunar effect which perturbs each day is attractive, but is probably too weak to be useful.

The only reduced array data suitable for this exercise at present appear to be the five days of Camfield (1973). The data of Bennett & Lilley (1973) are more limited in covering only three days, though they could be augmented by the digitization of other quiet days recorded. The data of Fig. 5 of this paper could be similarly augmented.

6.4 *Application to horizontally-layered Earth structure*

Should the array stations show little or no anomaly (that is if polar plots like Fig. 6(a) and (b) are all smooth curves), or should it be possible nevertheless to estimate regional field distributions by smoothing out anomalies (as proposed in Fig. 6(c)), then the regional data may be useful for horizontal-layer interpretation as suggested by Schmucker (1970) and Kuckes (1973). A consistency test should be possible, that different days give the same estimates of horizontal layering.

7. Conclusions

This paper has taken an unconventional view of magnetic daily variations, treating them as a series of isolated aperiodic events rather than as a summation of endless periodic sinusoids. Should such a view be justified, the variability of different quiet days may be used to study the response of the Earth to primary inducing fields of long period. The quantitative interpretation of such response in anomalous areas will unquestionably be difficult: the aim of this paper has been to suggest a basis for the reduction of observed data from which quantitative interpretation might proceed. Particular difficulties may result from the moving-source nature of the daily variation primary field, and the fact that on some disturbed days the daily variation depending on local solar time may be perturbed by variations depending on universal time.

If the viewpoint were to be taken further, to the search for a representative *Sq* quiet day, it would favour averaging all days and then transforming the mean, rather than transforming the days as a series. The same values for the 24 hr, 12 hr, 8 hr and 6 hr harmonics will be given by both methods, but the former method will give the full spectrum for the typical individual quiet day, whereas spectra for the days analysed in series will be affected by the interference effects of Section 2.2 and Fig. 1.

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