GEOPHYSICS, VOL. 64, NO. 6 (NOVEMBER-DECEMBER 1999); P. 1720–1729, 7 FIGS.

Time-varying effects in magnetic mapping: Amphidromes, doldrums, and induction hazard

F. E. M. Lilley*, Adrian P. Hitchman*, and Lie Jun Wang‡

ABSTRACT

A magnetic amphidrome is defined as a place where changes of the magnetic field over time, as measured by a total-field magnetometer, are reduced to the point of being negligible. The reduction is caused by destructive interference between the vertical and horizontal components of the time-varying field. At an ideal amphidrome, variations with time are suppressed completely and the total-magnetic-field magnitude is steady.

Such a phenomenon may be expected to depend on the frequency content of the time variations in the vertical and horizontal components. The subject is treated first in terms of the quiet daily variation, S_q, which is studied on a global basis. It is seen that there are magnetic latitude bands, north and south of the equator, where the quiet daily variation is minimal. These zones are called the "diurnal doldrums." In addition to this global pattern, the magnetic daily variation is modified by Earth's conductivity structure locally, and S_q amphidrome behavior may be aided or obstructed locally.

The second part of the paper treats the magnetic "rapid fluctuations." A simple condition for an amphidrome is that the direction of Earth's main magnetic field be parallel to the normal of the "preferred plane" in which the small vector changes of rapid magnetic fluctuations tend to lie. Examples are given of observed data for Australia, and a numerical model of Australian electrical-conductivity structure is used to predict amphidromes regionally. Formal treatment of the preferred-plane concept involves taking the out-of-phase (or quadrature) part of the induction phenomenon into account as well, and a parameter is proposed which may be contoured to show an amphidrome minimum.

The phenomena of amphidromes are fundamental for magnetic mapping procedures. Near amphidromes, the fluctuating magnetic fields of Earth are suppressed, and their capacity for introducing error into magnetic survey data is reduced correspondingly.

The case of a "complete" or "ultimate" amphidrome, applying to both diurnal and rapid fluctuations, may be expected to be rare. None is known at present. The reason is that the diurnal doldrums, favoring S_q amphidromes, occur at low latitudes. Rapid-fluctuation amphidromes, however, are more likely to occur at mid- to high latitudes.

INTRODUCTION

Superimposed on the main part of Earth's magnetic field, which is steady during a magnetic survey, is a smaller part which fluctuates with time. The removal of this time-varying part is a major objective of survey data reduction, and it is especially important in the case of modern aeromagnetic surveys which are of high resolution (Milligan and Barton, 1997). Strategies to reduce the effects of magnetic fluctuations with time commonly influence survey operations, starting from the most basic step of ceasing data collection at times of magnetic disturbance, or "storm." Barton (1997) introduces an "aeromagnetic-risk" map to reflect the extent that temporal field changes are likely to influence survey data.

In this paper, known aspects of behavior of the time-varying part of Earth's magnetic field are reviewed, and the concept of an ideal magnetic amphidrome is introduced. The term *amphidrome* is taken by analogy with its use in physical oceanography (see Appendix A). A magnetic amphidrome is a place where fluctuations with time of the total magnetic field, as measured by a total-field magnetic survey instrument, are always nil. For a "practical" amphidrome, fluctuations are minimal. The phenomenon is caused by destructive interference

Manuscript received by the Editor December 16, 1997; revised manuscript received October 5, 1998. Published on Geophysics Online May 27, 1999.
*Research School of Earth Sciences, The Australian National University, GPO Box 4, Canberra, Australian Capital Territory 0200, Australia. E-mail: Ted.Lilley@anu.edu.au; adrian@rses.anu.edu.au.
‡Encom Technology Pty. Ltd., 118 Alfred Street, Milsons Point, New South Wales 2061, Australia. E-mail: liejun@encom.com.au.

between the signals which the horizontal and vertical magnetic components contribute to the total field.

Analogous with the physical oceanography case for tides, magnetic amphidromes will depend on frequency, notably the frequency content of the signals in the horizontal and vertical magnetic components. In this paper, two distinctive frequency ranges are considered: (a) the "quiet-day" or "diurnal" magnetic fluctuations, and (b) "rapid" fluctuations, such as occur during a magnetic storm or substorm.

For the first case, particular latitude bands, north and south of the equator, are shown to have minimum diurnal magnetic signal. These bands, termed here the *diurnal doldrums*, occur generally between geomagnetic latitudes 20° and 30°. Both in these diurnal doldrums and outside them, local induction effects at Earth's surface may be important in the creation or suppression of amphidromes at diurnal periods.

For the second case, that of rapid fluctuations, the concept of a preferred plane (Parkinson, 1962) for magnetic field changes enables an especially simple description of a magnetic amphidrome. An amphidrome will occur when the normal to the preferred plane is parallel or antiparallel to the direction of the steady total magnetic field. There is generally some frequency band over which the orientation of the preferred plane is steady. A magnetic amphidrome may be expected to exist over such a frequency band and perhaps cover variations on timescales of minutes to hours. In analyzing the rapid-fluctuation case, the question of out-of-phase or quadrature response is also important.

Examples of approximate amphidromes are presented, taken from case histories of electromagnetic induction studies for Australia. Australia is shown to be quite "amphidrome rich," especially near its southern coastline. A parameter for the quantitative assessment of amphidromes is developed, and a numerical model is used to predict values of this parameter regionally for Australia.

MODERN AEROMAGNETIC SURVEY PRACTICE

Current aeromagnetic survey practice is reviewed in Gunn (1997). Some aspects pertinent to this paper are described here.

In modern aeromagnetic mapping, an aircraft carries a magnetometer along a grid of lines covering the area to be mapped. Time variations of the magnetic field which may occur during the mapping typically are controlled by two strategies (Luyendyk, 1997).

In the first strategy, a set of "tie-lines" is flown perpendicular to the main flight lines. At crossover points where the field has been recorded twice, tie-line "misfits" are determined. The misfits are attributed to the time-varying field and other effects (such as navigation errors), and they are distributed as corrections around the grid of crossover points, and so throughout the mapping data.

In the second strategy, one or more base stations is set up to record simultaneously with the survey aircraft. A base-station record then is subtracted from the survey record, on the basis that the time-varying field is spatially uniform from base station to survey aircraft.

Both strategies may experience shortcomings. An example for the first strategy is the situation of an aircraft moving at 65 m/s while recording magnetic pulsations of amplitude 5 nT and period 100 s (Barton, 1997). The pulsation signal perturbs the aeromagnetic data while the traverse samples across a distance of about 6 km, and this extraneous contribution may be difficult to distinguish from the desired geologic signal of crustal magnetization (Milligan, 1997).

An example of a difficulty with the second strategy is the situation of an aircraft which is 100 km from its base magnetometer in a region where anomalies in Earth's electrical-conductivity structure cause the time-varying magnetic fields to change signature over distances of about 10 km. Examples of such situations for continental Australia are given by Lilley (1982), Chamalaun and Cunneen (1990), and Kusi et al. (1998). The effect may be expected to be general for magnetic surveys near coastlines because of the electrical-conductivity contrast of land and ocean. In such situations, differences of some nT for pulsations and tens of nT for substorm signals may occur between signals at the base station and at the survey aircraft.

Thus, in both the tie-line leveling strategy and the base-station subtraction strategy, spurious signals of some nT in amplitude are clearly possible. When the magnetic mapping is carried out at high resolution for expected "small-anomaly" targets such as kimberlites, mineral sands, and offshore sedimentary basin structures, spurious signals of nT amplitude are important and can become an aeromagnetic mapping "hazard."

The present paper addresses the most fundamental part of the time-varying problem, especially as it impacts the first strategy. The paper studies where, for a continent such as Australia, time-varying signals may be expected to be reduced and less hazardous because they are suppressed by electromagnetic induction effects at the Earth's surface. The amphidrome phenomenon is identified as a benchmark for minimum time-varying signals; a quantitative parameter then is developed which builds on this base.

The effects of electromagnetic induction phenomena on the second strategy clearly are related. There is a need for a method of relating different sites in a way which shows, for example, how well a base station at one place records the total-field magnetic fluctuation effects at another. The present paper also provides a first step in addressing this need.

NOTATION FOR COMPONENTS OF EARTH'S MAGNETIC FIELD

Imagine an observing site where, at some epoch, the horizontal component of the geomagnetic field is H, the vertical component is Z, and the magnetic inclination is I. The amplitude of the total-magnetic-field strength, F, at that time is given by

$$F = H \cos I + Z \sin I. \qquad (1)$$

At the same epoch, define unit vectors $\hat{\mathsf{h}}$, $\hat{\mathsf{d}}$, and $\hat{\mathsf{z}}$ in the directions of horizontal magnetic north, horizontal magnetic east, and downward, respectively. Denoting the total-magnetic-field vector by $\mathbf{F}$, equation (1) may be expressed as

$$\mathbf{F} = H\hat{\mathsf{h}} + Z\hat{\mathsf{z}}. \qquad (2)$$

Also, relative to the epoch datum levels, changes with time t of the magnetic field may be represented by time series $h(t)$, $d(t)$, and $z(t)$ in the $\hat{\mathsf{h}}$, $\hat{\mathsf{d}}$, and $\hat{\mathsf{z}}$ directions.

The full vector for the fluctuating part of the magnetic field, $\mathcal{F}(t)$, is

$$\mathcal{F}(t) = h(t)\hat{\mathrm{h}} + d(t)\hat{\mathrm{d}} + z(t)\hat{\mathrm{z}}. \qquad (3)$$

However, usually $h(t)$, $d(t)$, and $z(t)$ are all small compared with F, and then the part of the fluctuating field which is measured by a total-field magnetometer is not $\mathcal{F}(t)$ itself, but rather $\mathcal{F}(t)$ resolved in the direction of $\mathbf{F}$ (Lilley, 1991). Denoting this latter signal by $f(t)$, one may estimate it by resolving the components of $\mathcal{F}(t)$ individually in the direction of $\mathbf{F}$, and then summing, to give

$$f(t) = h(t)\cos I + z(t)\sin I. \qquad (4)$$

The signal $f(t)$ may be thought of as the scalar component of the total-field variation resolved in the direction of the steady magnetic-field vector. The principle involved—adding a small perturbation vector to a large dominant vector—is analogous to the case of a perturbation of $\mathbf{F}$ with space, discussed, for example, by Emerson et al. (1985, p. 6) and Blakely (1995, p. 178). Note that in equation (4), $d(t)$ makes no direct contribution to $f(t)$, although it may do so indirectly by inducing additional signals in $h(t)$ and $z(t)$.

Lastly, in this section, notation also is introduced for transformation to the frequency domain. The transform of $f(t)$, for example, will be denoted $\tilde{f}(\omega)$, where ω denotes angular frequency, with real and quadrature parts $\tilde{f}_r(\omega)$ and $\tilde{f}_q(\omega)$. Thus,

$$\tilde{f}(\omega) = \tilde{f}_r(\omega) + i\,\tilde{f}_q(\omega). \qquad (5)$$

DEFINITION OF A MAGNETIC AMPHIDROME

A magnetic amphidrome is a place where fluctuations with time of the total magnetic field, as measured by a total-field magnetic survey instrument, are always nil. A magnetic amphidrome is thus an ideal situation which, in practice, may be expected to be approximated at certain places. The mathematical description of the ideal is simple. Using the notation defined above, for a magnetic amphidrome to exist at a point,

$$\mathbf{F} \cdot \mathcal{F}(t) = 0 \qquad (6)$$

at that point, for all t.

Generally, the case will apply that

$$|\mathcal{F}(t)| \ll |\mathbf{F}| \qquad (7)$$

so that amphidrome conditions mean that although local variations with time may occur, they will be perpendicular to the steady field direction and hence will be undetected by a total-field instrument.

The possible existence of amphidromes will now be dealt with in two parts:

1) the magnetic quiet daily variation, S_q, sometimes termed the *diurnal*
2) the rapid fluctuations, at storm and substorm periods, typically of one hour or less

These two parts are a natural subdivision because the source fields for them are typically different. One important distinction is that for (1), a vertical component, $z(t)$, generally is present (Cambell, 1989), whereas for (2), a significant "normal" component $z(t)$ commonly is not present. Any observed $z(t)$ is the result of induction, locally or regionally, by the horizontal fluctuation components $h(t)$ and $d(t)$ (Parkinson and Hutton, 1989).

GLOBAL VIEW OF THE MAGNETIC DAILY VARIATION AND MAGNETIC DOLDRUMS

The magnetic daily variation occurs everywhere on Earth's surface and is the most consistent component of the time-varying part of Earth's magnetic field. Its amenability to harmonic analysis means that it was expressed early in terms of components of 24, 12, 8, and 6 hours (Chapman and Bartels, 1940; Matsushita, 1967, for example). The term S_q (derived from "solar quiet") is used traditionally to describe the quietest part of the magnetic daily variation, which is common from day to day.

There is a number of analyses for S_q, based on data from the worldwide network of magnetic observatories. Such analyses typically present type curves for the magnetic daily variation expressed in the traditional observatory coordinates of $h(t)$, $d(t)$, and $z(t)$, as defined above. Such analyses also commonly work in terms of local solar time and geomagnetic latitude, where the latter is relative to a best-fitting central dipole for Earth's main magnetic field (Merrill et al., 1996).

Because it is the $f(t)$ component of S_q which is important for much magnetic surveying, Figure 1 in this paper presents type curves for the global magnetic daily variation, including the total-field component, $f(t)$. The data set used to produce Figure 1 is from an analysis of records from the global magnetic observatory network for the International Year of the Quiet Sun (IQSY). The analysis is by Campbell et al. (1989). Figure 1 is a simplification of the full global picture and does not take into account, for example, variations which occur with the seasons of the year. A more complete description of the global variation of $f(t)$, based on Campbell's analysis, is presented by Hitchman et al. (1998).

Addressing some important characteristics of Figure 1, it can be seen that $f(t)$ follows $h(t)$ in equatorial regions, where I is near zero. At high northern latitudes, $f(t)$ increasingly follows

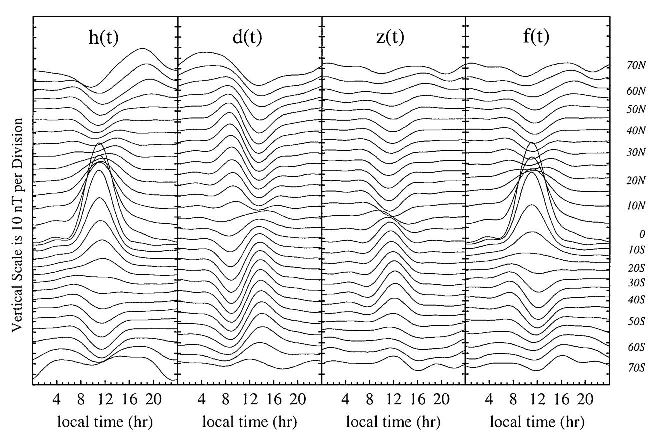

FIG. 1. Profiles for typical S_q magnetic variation during a quiet equinoctial day, in the $h(t)$, $d(t)$, $z(t)$, and $f(t)$ magnetic components. Global means are shown for different values of geomagnetic latitude. Time is local solar time.

$z(t)$ (being exactly $z(t)$ at the dip pole, where $I = 90°$). At high southern latitudes, $f(t)$ has the characteristics of $-z(t)$.

The plots may be checked against published $f(t)$ profiles. For example, Le Borgne and Le Mouël (1975) present, in their Figure 3, an $f(t)$ profile for southern Spain which accords well with the appropriate geomagnetic latitude (40°N) in Figure 1 of this article. Similarly, later in the present paper (in Figure 4), a range of quiet-day signals will be presented for inland Australia which accords with those for the appropriate latitude in Figure 1.

Figure 1 shows bands of latitude, approximately between 20° and 30° (geomagnetic) both north and south, where the $f(t)$ signal, as characterized globally for the quiet daily variation, has minimum range. Figure 2 shows these minima as a function of geomagnetic latitude and their occurrence on either side of the strong signal at the equator. The distribution of these minimum bands over the Earth's surface is shown in Figure 3.

These bands are termed here the magnetic doldrums. They should be regarded as indicative of global behavior only. As discussed in the next section, the Earth's (including oceanic) electrical-conductivity structure may have important induction effects and modify local $f(t)$ behavior. Otherwise, the magnetic doldrum latitudes should favor any total-field magnetic surveying, whether by air, sea, or land. The reason is simply that the diurnal variation may be expected to be minimal there.

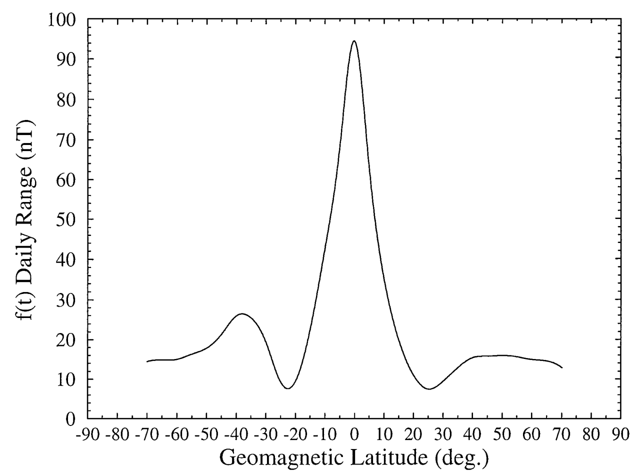

FIG. 2. Plot of daily range of $f(t)$ versus geomagnetic latitude for the quiet-day distribution shown in Figure 1. The occurrence of the diurnal doldrums between 20° and 30°, north and south, is evident.

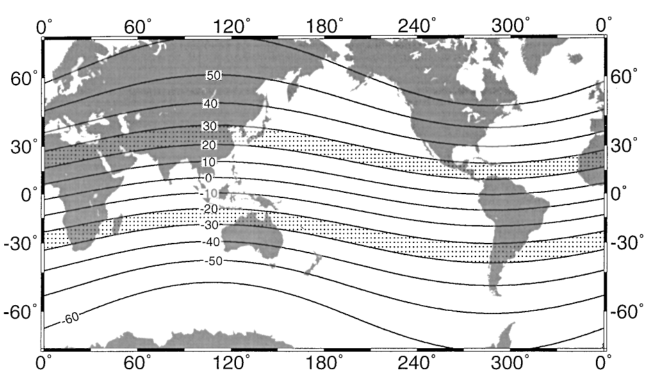

FIG. 3. Map of the world showing contours of geomagnetic latitude. The regions of the diurnal doldrums, between 20° and 30° geomagnetic latitude, north and south, are stippled.

REGIONAL AND LOCAL EFFECTS IN S_q—AN EXAMPLE

The global pattern for $f(t)$ in Figure 1, showing variation with latitude, may be affected by regional and local induction

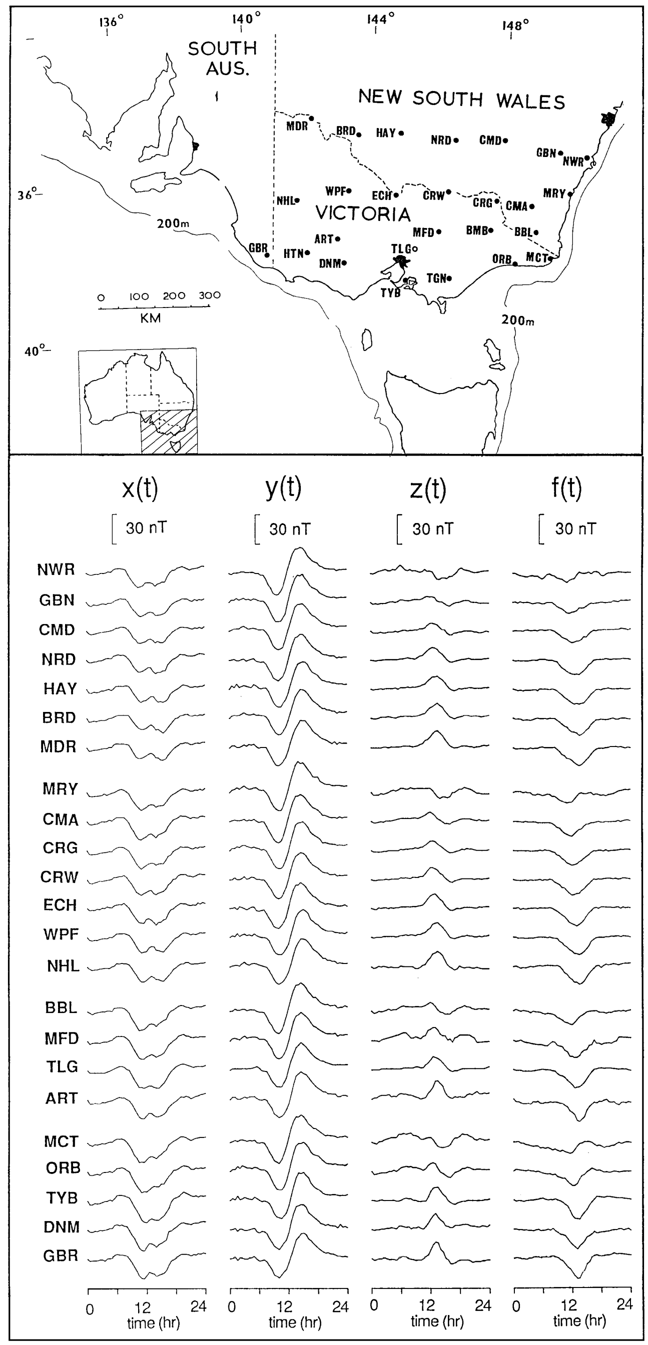

FIG. 4. Example of a magnetic quiet day recorded for southeast Australia, with much-reduced $f(t)$ signal at coastal stations NWR (Nowra) and MRY (Moruya) because of destructive interference of coast effect S_q with normal signal (as in Figure 1). The time series $x(t)$ and $y(t)$ are similar to $h(t)$ and $d(t)$ but are recorded relative to geographic, rather than geomagnetic, horizontal axes.

in conductive structures of the Earth and oceans. For example, such effects may increase $f(t)$ above the signal expected in the diurnal doldrums. Alternatively, either inside or outside the doldrums, the effects may produce an amphidrome by destructive interference.

An example of this latter case for southeast Australia can be seen in the data shown in Figure 4. These data were recorded by an early array study in Australia (Bennett, 1972; Lilly and Bennett, 1972). Originally, the data were reported and analyzed in the traditional components of north, east, and downward (Bennett and Lilley, 1973). Some years later, in connection with the magnetic survey study of Lilley (1982), $f(t)$ profiles were computed using equation (4) above. Figure 4 of the present paper is taken from Lilley (1982).

Southeast Australia presents a case history which is significant for several reasons. First, referring to the station farthest inland, MDR (Mildura), the $f(t)$ profile for this station accords with the curves in Figure 1, for geomagnetic latitude 43° south. Second, approaching the east coast of Australia from MDR, the $f(t)$ profiles reduce in strength until, at NWR (Nowra), the ideal of an amphidrome for this quiet day is obtained approximately.

In explanation of this second phenomenon, Bennett and Lilley (1973) show that the S_q pattern over the whole of southeast Australia is such as would be caused by a traditional coast effect (where a vertical signal is caused by an onshore horizontal signal) having been added vectorially to the appropriate global S_q behavior (as shown in Figure 1). For this purpose, Bennett and Lilley (1973) used the global analysis of S_q presented in Matsushita (1967). This explanation later became important in their interpretation that the coast effect required an electrical-conductivity contrast which was deeper and in addition to that provided by the continent/ocean boundary. Lilley and Parker (1976) established that a similar modification of the global pattern of S_q occurred at the west Australian coast, and Lilley (1979) showed the same to apply to the east and west coasts of North America.

Thus, the coast effect may be expected to cause major differences from the patterns of Figure 1. Strong $f(t)$ signals may be produced at doldrum latitudes, and amphidromes may be produced away from doldrums (as is the case for Nowra above). Further, strong continental conductivity anomalies may be apparent at S_q frequencies, as shown, for example, in the North American Central Plains anomaly analyzed by Camfield and Gough (1975).

The continental anomalies are more of a hazard for aeromagnetic pursuits because they do not have an obvious coastline to draw attention to their presence. The hazard in such cases arises particularly if the base station and survey instrument are in markedly different proximity to an amphidrome. In the example in Figure 4 above, a base station at NWR would indicate little S_q signal, whereas an instrument some distance inland would record a signal significantly stronger.

AN AMPHIDROME CONDITION FOR RAPID FLUCTUATIONS IN TERMS OF THE PREFERRED PLANE

In a paper which contributed to the foundations of modern electromagnetic induction studies, Parkinson (1959) noticed that at magnetic observatories in the vicinity of a conductivity contrast, the rapid fluctuations of the magnetic field, taken as successive small changes, tended to lie in some well-defined plane, generally different for each station. This plane was referred to as the "preferred plane," and Rikitake and Honkura (1985) note also its early description as the "Rikitake-Yokoyama" plane. In preparation for the discussion of such a plane, we now adopt the notation that distances in the $\hat{\mathrm{h}}$, $\hat{\mathrm{d}}$, and $\hat{\mathrm{z}}$ coordinate frame have components h, d, and z, respectively.

If simultaneous small changes from some datum level in $h(t)$, $d(t)$, and $z(t)$ are represented by δh_i, δd_i, and δz_i, respectively, so that the vector expression for the magnetic field change is

$$\delta \mathcal{F}_i = \delta h_i \hat{\mathrm{h}} + \delta d_i \hat{\mathrm{d}} + \delta z_i \hat{\mathrm{z}}, \tag{8}$$

then the relationship observed is

$$\delta z_i = A \delta h_i + B \delta d_i + \epsilon_i \tag{9}$$

where both A and B are real constants and ϵ_i is an error indicating the misfit of any particular data set to the equation of best fit.

The vector for the small field change may be written

$$\delta \mathcal{F}_i = \delta h_i \hat{\mathrm{h}} + \delta d_i \hat{\mathrm{d}} + (A \delta h_i + B \delta d_i + \epsilon_i)\hat{\mathrm{z}}, \tag{10}$$

and such small field changes may be described as lying preferentially in a plane in space given by

$$A\mathrm{h} + B\mathrm{d} - \mathrm{z} = 0 \tag{11}$$

This plane is known as the preferred plane. The normal to the preferred plane, **n**, is defined by direction cosines n_1, n_2, and n_3, given by

$$n_1 = -A/\sqrt{A^2 + B^2 + 1}, \tag{12}$$

$$n_2 = -B/\sqrt{A^2 + B^2 + 1}, \tag{13}$$

$$n_3 = 1/\sqrt{A^2 + B^2 + 1}. \tag{14}$$

The projection of the downward normal onto the horizontal plane, the construction which gives the "Parkinson arrow," is of length ℓ and bearing P where

$$\ell = \sqrt{A^2 + B^2}/\sqrt{A^2 + B^2 + 1} \tag{15}$$

and

$$P = \arctan(B/A) + \pi. \tag{16}$$

The π term is added to reverse the direction of the arrow. Such reversed arrows commonly point toward the higher side of an electrical-conductivity contrast, if the conductivity structure is inhomogeneous (Hobbs, 1992). It is also common to represent such data as an arrow of length

$$\ell' = \sqrt{A^2 + B^2}, \tag{17}$$

consistent with Parkinson's arrow being a forerunner to the "tipper" of magnetotellurics (Vozoff, 1991; Hobbs, 1992, for example).

Before discussing further the mathematical representation of the phenomenon, it is useful to see that the preferred plane and its normal give an immediate condition for the existence of a magnetic amphidrome. An amphidrome will occur for rapid fluctuations when the normal to the plane is either parallel or antiparallel to the total-magnetic-field vector, **F**. Then all rapid-fluctuation changes of the field will be orthogonal to **F**, and equation (6) will apply. In these circumstances, the Parkinson arrow will point due magnetic north or south and will be of a particular length related to the magnetic inclination at that place.

The concept of the preferred plane has proved useful in forming a visual picture of the induction process, especially in the case of a discrete body of very high electrical conductivity. At the surface of such a body, changing magnetic fields must be tangential to the surface, and so the tangential plane is the preferred plane.

The success of equation (9) in representing much observed data is taken to indicate that $z(t)$ is induced locally by $h(t)$ and $d(t)$, probably at some departure of the local geology from the basic case of horizontal (1-D) layering (Parkinson, 1983; Weaver, 1994). That the situation does not apply in this simple way at the longer periods of S_q is evident immediately from examination of Figure 1. The reason is that at the longer periods of S_q, the horizontal wavelength of the external source field becomes important, and some of the external vertical field can penetrate Earth's surface.

However, for periods of one hour or less, equation (9) has been shown to be a satisfactory representation of a great deal of observational data. Thus, given equation (9) and requiring equation (6) to be satisfied gives (see Figure 5)

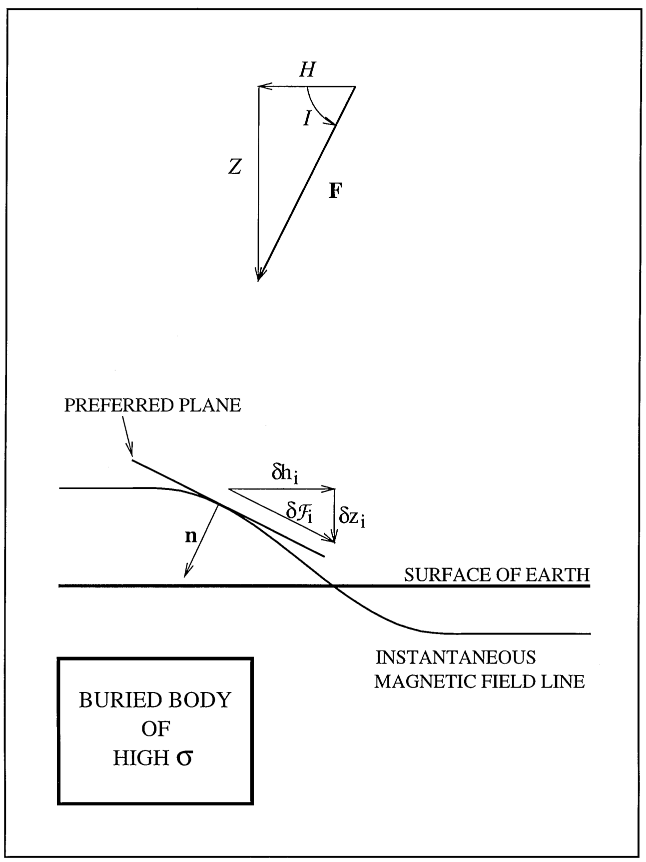

FIG. 5. Diagram showing, near a buried body of high electrical conductivity, (1) a plane (known as the "preferred plane") in which small and rapid vector magnetic changes tend to lie, and (2) the normal **n** to the preferred plane, illustrating the case of amphidrome creation, when this normal is parallel or antiparallel to the total-field vector **F**.

$$\mathbf{F} \times \mathbf{n} = \mathbf{0}, \tag{18}$$

$$B = 0, \tag{19}$$

and

$$A = -\cot I \tag{20}$$

as required conditions for an amphidrome. Hence, with the complex nature of equation (9) still to be taken into account, equations (19) and (20) give a simple condition for the existence of a magnetic amphidrome. Lists of published data may be scanned, seeking places where the (A, B) pair of values approach $(-\cot I, 0)$.

Note that these amphidromes become increasingly less likely at lower magnetic latitudes. Because of the characteristics of $\cot I$, a low numerical value for I requires $|A|$ to be increasingly strong. However, A is determined primarily by Earth's electrical-conductivity structure and rarely is greater than unity. Unity (for A) corresponds to I of 45°, so magnetic amphidromes resulting from this cause are most likely to occur on the poleward sides of $I = \pm 45°$. Using the dipole relationship (Merrill et al., 1996) between inclination I and geomagnetic latitude ϕ,

$$\tan I = 2 \tan \phi \tag{21}$$

indicates that regions on the poleward sides of $I = \pm 45°$ correspond to regions of geomagnetic latitude on the poleward sides of

$$\phi = \pm 27°. \tag{22}$$

At high I, of course, amphidromes are created by weak A with increasing ease. At the limit of

$$I = \pm 90° \tag{23}$$

at the magnetic poles, traditional 1-D electromagnetic induction phenomena, involving negligible vertical fields, would all be amphidromic. However, source fields at high dip latitudes commonly have an external $z(t)$ component, which is important. Hence, equation (9) no longer holds generally, and the problem of removal of time-varying fields from total-field magnetic surveys at high latitudes remains. This problem was recognized early by Morley (1953) and by Whitham and Niblett (1961).

AN EXAMPLE OF AN AMPHIDROME IN RAPID FLUCTUATIONS

A check of (A, B) values for Australian stations, seeking cases where the $(-\cot I, 0)$ conditions were met approximately, drew attention to a listing in Lilley and Bennett (1972) for the 1971 array study of southeast Australia, already quoted for the S_q example given above. In this listing, which is for fluctuations of period 80 minutes, several sites had (A, B) values matching amphidrome conditions, such as WPF (Wycheproof), ART (Ararat), and TGN (Taralgon). For WPF, for example, the real parts of the (A, B) values are (0.35, −0.03), and the I value is −65° with

$$\cot I = -0.35. \tag{24}$$

As for S_q, example profiles of rapid fluctuations in the total field $f(t)$ at the 1971 array sites first were plotted about 10 years later and presented by Lilley (1982). Figure 6 presents the appropriate data from that paper and shows total-field fluctuations for a typical magnetic substorm. In this figure the total-field signals for WPF, ART, and TGN now can be checked and noted for their much-subdued character. Approximate amphidrome behavior at periods of about an hour is thus evident.

Especially at ART and TGN, however, some higher frequency signal remains in the $f(t)$ profiles. Such higher frequency in $f(t)$ is consistent with the observations of Milligan et al. (1993) and Milligan (1995) of magnetic fluctuations in central Victoria, especially at micropulsation frequencies. The results of these authors show Parkinson arrows which change direction as period reduces from 10 minutes to 1 minute, causing the approximate amphidromic conditions to be lost.

Note also in Figure 6 the coast effect now enhancing the $f(t)$ signal at the coastal stations of eastern Australia, in contrast to the destructive interference which occurred in the S_q signal for the same region.

THE COMPLEX NATURE OF THE FULL PARKINSON RELATION

The phenomenon of electromagnetic induction in the Earth is frequency dependent, so that equation (9) commonly is transformed to the frequency domain and expressed as

$$\tilde{z}(\omega) = A(\omega)\tilde{h}(\omega) + B(\omega)\tilde{d}(\omega) \tag{25}$$

where the geology-dependent coefficients A and B are recognized as local functions of frequency (Everett and Hyndman, 1967; Schmucker, 1970; Parkinson and Jones, 1979, for example). In fact, the geology dependence of A and B and the nature of electromagnetic induction phenomena together mean that generally, A and B do not change rapidly with frequency, so that the same preferred plane may hold for magnetic changes ranging in period from minutes to hours.

The complex nature of the quantities appearing in equation (25) introduces an extra condition for perfect amphidrome behavior and makes it more difficult to obtain. Expressing $A(\omega)$ as

$$A(\omega) = A_r(\omega) + i A_q(\omega) \tag{26}$$

and $B(\omega)$ similarly, for ideal behavior, A_q must be zero (together with B_r and B_q). Otherwise, nonzero A_q produces a z signal out of phase with the h signal. Thus, when h is zero, z_q will be nonzero, and the particular circumstances of Figure 5 will be violated by the presence of a vertical field change. Except at the magnetic equator, this quadrature vertical field change will contribute a component to the total field.

It must be expected that such quadrature responses consistently contribute noise or scatter to the practical determination of preferred planes. Equations according to

$$A_r(\omega)\mathrm{h} + B_r(\omega)\mathrm{d} - \mathrm{z} = 0 \tag{27}$$

otherwise define planes exactly.

Quadrature induction response thus acts against ideal amphidrome behavior. Often, however, the real parts of the induction transfer functions $A(\omega)$ and $B(\omega)$ are greater than the quadrature parts, and the imperfection in amphidrome behavior is suppressed. Such is the case in the example of WPF

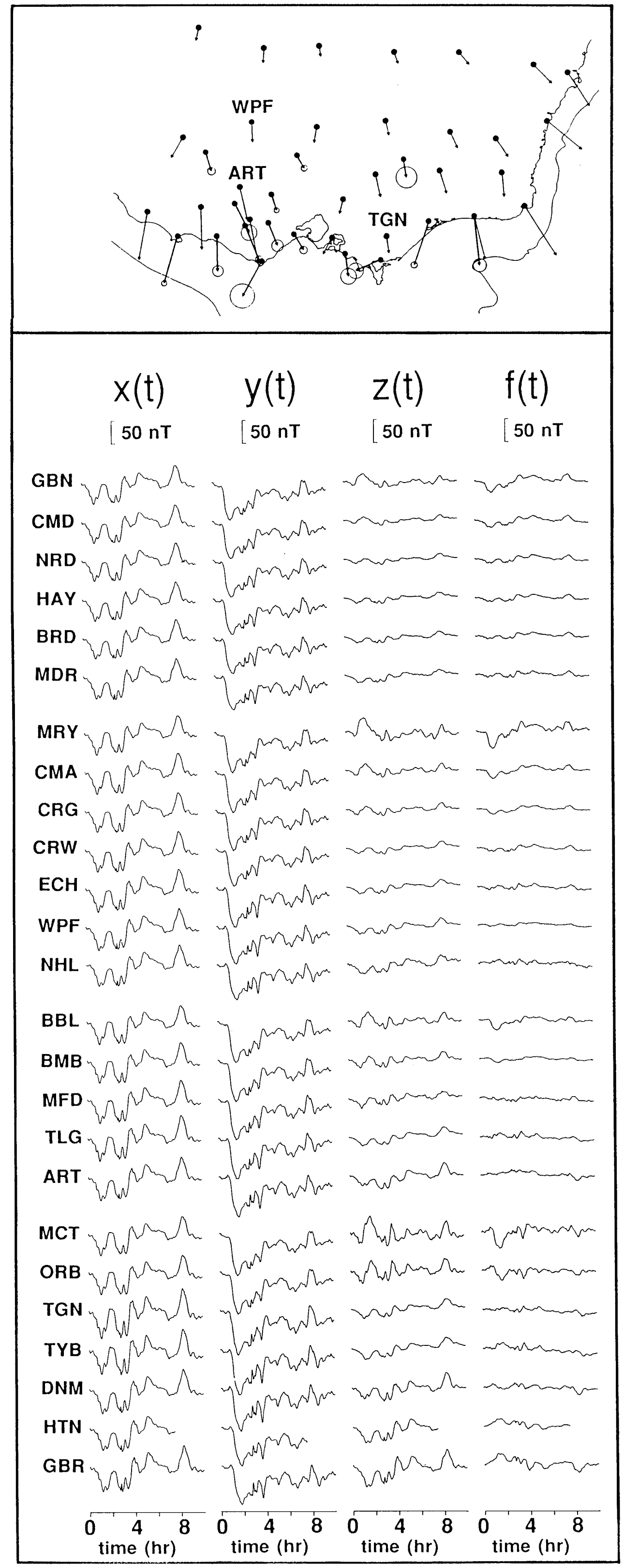

FIG. 6. Example of approximate amphidrome conditions for rapid fluctuations at some sites (WPF, ART, and TGN) in southeast Australia. The time series $x(t)$ and $y(t)$ are similar to $h(t)$ and $d(t)$ but are recorded relative to geographic axes. Real (in-phase) Parkinson arrows are shown in the top panel, with stations WPF, ART, and TGN marked. The arrow-length scale is given by the arrow for WPF being 0.36 units long [according to equation (17)].

above. From Lilley and Bennett (1972), the quadrature values of (A, B) for WPF are (0.08, 0.02), consistent with the apparent good behavior of WPF as amphidromic, in Figure 6.

AN AMPHIDROME PARAMETER

The identification of a null case, when the normal to the preferred plane is parallel to the total-field direction so that equation (6) is obeyed, suggests a quantitative "amphidrome parameter," which will be zero in the ideal amphidromic case. The parameter as developed below takes into acount both the real and the quadrature inductive responses experienced at Earth's surface.

Imagine a variation in the horizontal plane, linearly polarized, of unit magnitude. Let this variation have components h_1 and d_1, such that

$$h_1^2 + d_1^2 = 1, \tag{28}$$

and let the azimuth of the variation be θ, so that

$$h_1 = \cos\theta \tag{29}$$

and

$$d_1 = \sin\theta. \tag{30}$$

In the vertical field, the real (in-phase) response to this variation will be

$$z_r = A_r h_1 + B_r d_1, \tag{31}$$

and the component observed by a total-field magnetometer will be

$$f_r = h_1 \cos I + (A_r h_1 + B_r d_1)\sin I. \tag{32}$$

The quadrature response to the horizontal field variation of unit amplitude will be

$$z_q = A_q h_1 + B_q d_1, \tag{33}$$

and the component of this response observed by a total-field magnetometer will be

$$f_q = (A_q h_1 + B_q d_1)\sin I. \tag{34}$$

Thus, the amplitude of the effect in the total field caused by a horizontal variation of unit amplitude is

$$f = |\sin I|\left\{[(\cot I + A_r)h_1 + B_r d_1]^2 + [A_q h_1 + B_q d_1]^2\right\}^{\frac{1}{2}}. \tag{35}$$

Differentiating this expression with respect to h_1 shows that the azimuths, θ, of the horizontal variation which cause maximum and minimum effects in the total field are given by

$$\tan 2\theta = \frac{2[B_r(\cot I + A_r) + A_q B_q]}{(\cot I + A_r)^2 + A_q^2 - B_r^2 - B_q^2} \tag{36}$$

which, for a known set of A_r, B_r, A_q, B_q, and I values, allows f in equation (35) to be calculated.

This magnitude of f is a measure of how strongly time-varying signals which obey equation (25) may perturb total-field magnetic mapping measurements. It is the maximum signal in the total field possible at a particular site, for a unit horizontal-field variation of the most favourable azimuth occurring there. However, because the values of f according to equation (35) typically will be less than unity, it may be useful to multiply f by an arbitrary scaling factor of 10 to obtain a practical amphidrome parameter, β:

$$\beta = 10 f, \tag{37}$$

i.e.,

$$\beta = 10|\sin I|\left\{[(\cot I + A_r)h_1 + B_r d_1]^2 + [A_q h_1 + B_q d_1]^2\right\}^{\frac{1}{2}} \tag{38}$$

where h_1 and d_1 are calculated from equations (29), (30), and (36). Values taken by β typically will range over several units. Equation (36) has an ambiguity of $\pi/2$ in θ, corresponding to the maximum and minimum values of β. It may be most direct, in any particular case, to calculate values for β using both values of θ and take the larger as the maximum.

PREDICTION OF AMPHIDROME AREAS FOR AUSTRALIA

Recently, Wang et al. (1997) have developed a thin-sheet conductivity model for Australia in its ocean setting, which successfully reproduces the general Parkinson-arrow response for the continent, for rapid fluctuations of period typically one hour. A recent version of that model is applied here, and values of the amphidrome parameter β are computed for each of its cells, according to equation (38). The model response gives values of A_r, B_r, A_q, and B_q, and values of I are taken from McEwin (1993). Contouring the β values obtained shows regions of minima, where amphidrome conditions are approached.

The results, contoured using the methods of Wessel and Smith (1991), are plotted in Figure 7. Several points are worthy of notice. The main amphidromic features, spatially, lie in a band across southern Australia. Accepting a β value of less than two units as indicating practical amphidromic conditions, an inland part of this band is in central Victoria, which includes stations WPF, ART, and TGN, as discussed in relation to Figure 6. At the Great Australian Bight in the central part of the southern coastline the band moves to the coastline and offshore. On the westward side of the continent, the band is onshore again. It includes a region which W. D. Parkinson (private communication) recently predicted would be amphidromic, on the basis of early observations (Parkinson, 1962) in the area at Esperance and Kalgoorlie. The computer animations of Whellams (1996b; 1996a), based on the observations of Chamalaun and Barton (1993), enable further confirmation of the existence of this amphidrome band across southern Australia.

The amphidromic nature of this band is broken where it crosses known conductivity anomalies, notably in South Australia. In this connection, the scale of the grid of the model should be noted (each cell is 100 km × 100 km). This grid scale will not resolve conductivity anomalies of smaller scale, and thus the information contained in Figure 7 is of a regional nature. It does not include, for example, the Eyre Peninsula conductivity anomaly of White and Milligan (1984) and Kusi et al. (1998).

The seaward extent of the southern coastal band is a new prediction, not yet confirmed by observational data. It is, however, clearly relevant to magnetic surveys made offshore. It is hoped to confirm this seaward amphidrome in the near future, possibly by observations made by a magnetometer in a floating buoy.

An important general point shown by Figure 7 is the variation of β with latitude and its steady increase toward the equator. This result is caused by the domination, in equation (38), of the term which reflects the influence of the unit horizontal-field fluctuation alone, when resolved in the total-field direction. It is the term which remains in equation (38) if all A_r, B_r, A_q, and B_q are zero. This term provides a background effect on which all local induction effects are superimposed.

The regional nature of Figure 7 for Australia should be stressed. Where detailed knowledge is held of Parkinson-arrow patterns for the continent, more detailed patterns of β distribution can be computed and contoured on a finer scale. This task is being addressed, as is the question of effects at higher frequency.

However, from Figure 7, it is clear that for some areas, a coastal base station will not be effective for an aircraft operating offshore over the continental shelf—β parameter differences between coastline and shelf of 10% and 20% are common. In contrast, central Australia has areas (away from conductivity anomalies) where the smooth form of the β map indicates that a base station, which has the same β value and geomagnetic latitude as the survey aircraft, will be effective for the data-reduction process of base-station subtraction.

CONCLUSIONS

The possible existence of places where a total-field magnetometer would record nil changes with time has been recognized, and these places have been termed *magnetic amphidromes*. For the diurnal variation, global bands of minimum S_q are very generally between geomagnetic latitudes 20° and 30°, and these are termed *diurnal doldrums*.

For the case of rapid fluctuations, an amphidrome will occur when the normal to the preferred plane is parallel or antiparallel to the steady total-field vector.

Examples have been taken from Australian case histories and predictions have been made for Australia generally, on the basis of electromagnetic induction in a thin-sheet model for the continent and its surrounding seas. The southern coastline especially has major amphidromic areas, both inland and offshore.

Similar amphidromes may occur at other east-west-trending coastlines. It is possible, for example, that coastal amphidromes occur along the Mediterranean coast of Africa. Amphidrome phenomena depend on a combination of Earth's electrical-conductivity structure, main magnetic field direction, and the geometry of the source fields (external to Earth) which cause magnetic changes with time. Although all these factors may change over geologic timescales, their change will generally be little on a scale of, say, ten years. The least constant factor is likely to be source-field geometry, which may add imperfection to the behavior of an amphidrome from day to day.

It is unlikely that a "complete" (or "ultimate") amphidrome, where neither diurnal nor rapid fluctuation signals appear in F, will occur over a wide region. No such complete amphidromes are known to the authors at present. The doldrums are regions most favored generally for subdued S_q behavior, but the low magnetic inclination of these regions would require uncommonly high values for A [see equation (20)] for a rapid-fluctuations amphidrome to be present. A complete amphidrome could occur, however, if a local induction effect (perhaps a coast effect) reduced S_q from that shown in Figure 1 and the local values of (A, B) obeyed equations (19) and (20).

Finally, the relevance of the subject matter of the present paper to aeromagnetic mapping is emphasized. This method of geophysical exploration is undergoing a vigorous phase at present, both in Australia and worldwide. The search for increased resolution depends more and more on the removal of the time-varying part of the geomagnetic field. Understanding where such time fluctuations naturally are reduced or enhanced is, therefore, of basic importance.

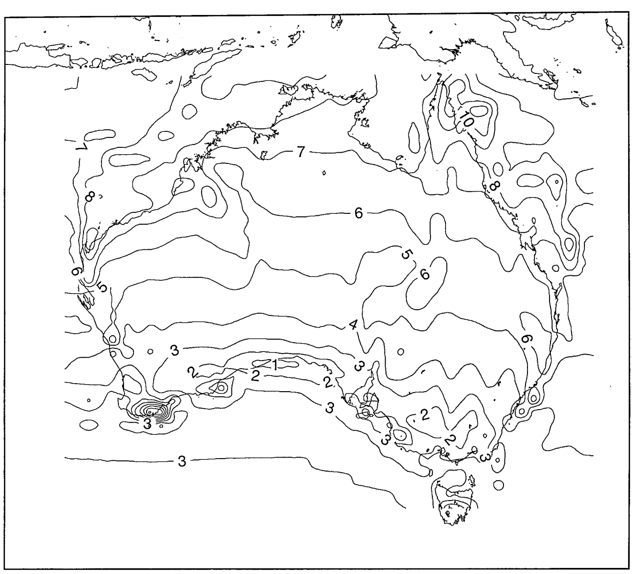

FIG. 7. Prediction of regional amphidromes for the Australian region, for magnetic fluctuations with period one hour. Contours are in units of the amphidrome parameter, β. Note the band of low β values across the southern part of the continent.

ACKNOWLEDGMENTS

We thank many people for discussing the ideas central to this paper. W. D. Parkinson made suggestions which helped develop its theory. C. E. Barton and P. R. Milligan have discussed much of the subject matter with us, as has J. T. Weaver, during a visit to Australian National University in 1996. W. H. Campbell extended substantial help in providing computer programs enabling the production of Figure 1. We have benefited from discussions with members of the Australian aeromagnetic surveying community for a number of years. The animations of magnetic fluctuations for Australia, as recorded by the AWAGS experiment and presented in the Ph.D. thesis of J. M. Whellams, have been most valuable for testing our predictions of amphidrome conditions across the southern part of Australia.

We thank Associate Editor Shanti Rajagopalan and two referees for valuable criticisms and suggestions.

REFERENCES

Barton, C. E., 1997, An "aeromagnetic risk map" of Australia, *in* Milligan, P. R., and Barton, C. E., Eds., Transient and induced variations in aeromagnetics: Austral. Geol. Surv. Org. Record, **1997/27**, 27–29.

Bennett, D. J., 1972, Geomagnetic depth-sounding studies in southeastern Australia: Ph.D. thesis, Australian National Univ.

Bennett, D. J., and Lilley, F. E. M., 1973, An array study of daily magnetic variations in southeast Australia: J. Geomag. Geoelectr., **25**, 39–62.

Blakely, R. J., 1995, Potential theory in gravity and magnetic applications: Cambridge Univ. Press.

Camfield, P. A., and Gough, D. I., 1975, Anomalies in daily variation magnetic fields and structure under northwestern United States and southwestern Canada: Geophys. J. Roy. Astr. Soc., **41**, 193–218.

Campbell, W. H., 1989, The regular geomagnetic-field variations during quiet solar conditions, *in* Jacobs, J. A., Ed., Geomagnetism: Academic Press, **3**, 385–460.

Campbell, W. H., Schiffmacher, E. R., and Kroehl, H. W., 1989, Global quiet day field variation model WDCA/SQ1: EOS, **70**, 66–74.

Chamalaun, F. H., and Barton, C. E., 1993, The large-scale electrical conductivity structure of Australia: J. Geomag. Geoelectr., **45**, 1209–1212.

Chamalaun, F. H., and Cunneen, P., 1990, The Canning Basin geomagnetic induction anomaly: Austral. J. Earth Sci., **37**, 401–408.

Chapman, S., and Bartels, J., 1940, Geomagnetism: Oxford Univ. Press.

Emerson, D. W., Clark, D. A., and Saul, S. J., 1985, Magnetic exploration models incorporating remanence, demagnetization and anisotropy: HP41C handheld computer algorithms: Expl. Geophys., **16**, 1–122.

Everett, J. E., and Hyndman, R. D., 1967, Geomagnetic variations and electrical conductivity in south-western Australia: Phys. Earth Plan. Int., **1**, 24–34.

Gunn, P. J., Ed., 1997, Airborne magnetic and radiometric surveys: AGSO J. Austral. Geol. Geophys., **17**.

Hitchman, A. P., Lilley, F. E. M., and Campbell, W. H., 1998, The quiet daily variation in the total magnetic field: global curves: Geophys. Res. Lett., **25**, 2007–2010.

Hobbs, B. A., 1992, Terminology and symbols for use in studies of electromagnetic induction in the earth: Surv. Geophys., **13**, 489–515.

Kusi, R., White, A., Heinson, G., and Milligan, P., 1998, Electromagnetic induction studies in the Eyre Peninsula, South Australia: Geophys. J. Internat., **132**, 687–700.

Le Borgne, E., and Le Mouël, J. L., 1975, A conductivity anomaly in the western Mediterranean: Geophys. J. Roy. Astr. Soc., **45**, 939–955.

Lilley, F. E. M., 1979, Magnetic daily variations compared between the east and west coasts of Canada: Can. J. Earth Sci., **16**, 585 – 592.

Lilley, F. E. M., 1982, Geomagnetic field fluctuations over Australia in relation to magnetic surveys: Bull., Austral. Soc. Expl. Geophys., **13**, 68–76.

Lilley, F. E. M., 1991, Electrical conductivity anomalies in the Australian lithosphere: Effects on magnetic gradiometer surveys: Expl. Geophys., **22**, 243–246.

Lilley, F. E. M., and Bennett, D. J., 1972, An array experiment with magnetic variometers near the coasts of southeast Australia: Geophys. J. Roy. Astr. Soc., **29**, 49–64.

Lilley, F. E. M., and Parker, R. L., 1976, Magnetic daily variations compared between the east and west coasts of Australia: Geophys. J. Roy. Astr. Soc., **44**, 719–724.

Luyendyk, A. P. J., 1997, Processing of airborne magnetic data: J. Austral. Geol. and Geophys., **17**, 31–38.

Matsushita, S., 1967, Solar quiet and lunar daily variation fields, *in* Matsushita, S., and Campbell, W. H., Eds., Physics of Geomagnetic Phenomena: Academic Press, 302–424.

McEwin, A. J., 1993, Epoch 1990.0—Epoch charts of the geomagnetic field in the Australian region: Austral. Geol. Surv. Org.

Merrill, R. T., McElhinny, M. W., and McFadden, P. L., 1996, The magnetic field of the Earth: Paleomagnetism, the core, and the deep Mantle: Academic Press.

Milligan, P. R., 1995, Short-period geomagnetic variations recorded concurrently with an aeromagnetic survey across the Bendigo area, Victoria: Expl. Geophys., **26**, 527–534.

Milligan, P. R., 1997, Micropulsations from an airborne magnetic survey perspective, *in* Milligan, P. R., and Barton, C. E., Eds., Transient and induced variations in aeromagnetics: Austral. Geol. Surv. Org., Record, **1997/27**, 8–10.

Milligan, P. R., and Barton, C. E., Eds., 1997, Transient and induced variations in aeromagnetics: Austral. Geol. Surv. Org. Record, **1997/27**.

Milligan, P. R., White, A., Heinson, G., and Brodie, R., 1993, Micropulsation and induction array study near Ballarat, Victoria: Expl. Geophys., **24**, 117–122.

Morley, L. W., 1953, The areal distribution of geomagnetic activity as an aeromagnetic survey problem near the auroral zone: Trans., Am. Geophys. Union, **34**, 836–840.

Parkinson, W. D., 1959, Directions of rapid geomagnetic fluctuations: Geophys. J. Roy. Astr. Soc., **2**, 1–14.

Parkinson, W. D., 1962, The influence of continents and oceans on geomagnetic variations: Geophys. J. Roy. Astr. Soc., **6**, 441–449.

Parkinson, W. D., 1983, Introduction to Geomagnetism: Scottish Academic Press.

Parkinson, W. D., and Hutton, V. R. S., 1989, The electrical conductivity of the earth, *in* Jacobs, J. A., Ed., Geomagnetism: Academic Press, **3**, 261–321.

Parkinson, W. D., and Jones, F. W., 1979, The geomagnetic coast effect: Rev. Geophys. Space Phys., **17**, 1999–2015.

Pond, S., and Pickard, G., 1983, Introductory dynamical oceanography, 2nd ed.: Pergamon Press.

Rikitake, T., and Honkura, Y., 1985, Solid earth geomagnetism: Terra Sci. Publ. Co.

Schmucker, U., 1970, Anomalies in geomagnetic variations in the south-western United States: Bull. Scripps Inst. Oceanog., **13**.

Vozoff, K., 1991, The magnetotelluric method, *in* Nabighian, M. N., Ed., Electromagnetic methods in applied geophysics: Soc. Expl. Geophys., **2**, 641–711.

Wang, L., Lilley, F. E. M., and Chamalaun, F. H., 1997, Large-scale electrical conductivity structure of Australia from magnetometer arrays: Expl. Geophys., **28**, 150–155.

Weaver, J. T., 1994, Mathematical methods for geoelectromagnetic induction: Research Studies Press.

Wessel, P., and Smith, W. H. F., 1991, Free software helps map and display data: EOS, **72**, 441.

Whellams, J. M., 1966a, Geomagnetic fluctuation animations: appendix to Ph.D. thesis, Flinders Univ. of South Australia, available at http://gaea.es.flinders.edu.au/~mgjmw/animlink/appen-a.html.

——— 1996b, Spatial inhomogeneity of geomagnetic fluctuation fields and their influence on high resolution aeromagnetic surveys: Ph.D. thesis, Flinders Univ. of South Australia.

White, A., and Milligan, P. R., 1984, A crustal conductor on Eyre Peninsula, South Australia: Nature, **310**, 219–222.

Whitham, K., and Niblett, E. R., 1961, The diurnal problem in aeromagnetic surveying in Canada: Geophysics, **26**, 211–228.

APPENDIX A

NOTE ON THE TERM *AMPHIDROME*

A printing of the *Shorter Oxford English Dictionary* dated 1950 describes *amphidromic* as "pertaining to the Amphidromia (an Attic festival at the naming of a child, when friends carried it round the hearth, and then named it)."

The term has been adopted in oceanography as a place in the ocean where the tide of a particular frequency has a "null point," or "node"; i.e., rise and fall of tide are nil (Pond and Pickard, 1983). Looking radially across the surface of the ocean from such a tidal amphidrome, one can see that tidal range increases and phase values rotate about the point. Thus, tides circulate around the nodal point, reminiscent of the amphidromic festival.

A magnetic amphidrome is similar. The amphidrome may be at a single point or spread out over a region (for example, a band along the coast of southern Australia, as shown in Figure 7). Phase will not generally circulate around a magnetic amphidrome as systematically as does tidal phase around an oceanic amphidrome; however, generally, magnetic disturbance phase around a magnetic amphidrome certainly will change with time.

The frequency dependence of oceanographic amphidromes and their ideal nature relative to the practical noise in the ocean tides are characteristics expected to be shown also by magnetic amphidromes.