Phases greater than 90° in MT data: Analysis using dimensionality tools

F.E.M. Lilley a,⁎, J.T. Weaver b

a Research School of Earth Sciences, Australian National University, Canberra, ACT 0200, Australia
b Department of Physics and Astronomy, University of Victoria, Victoria, BC, Canada, V8W 3P6

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A B S T R A C T

An example of magnetotelluric data is discussed which has distinctive phase values out of quadrant, and for which the Zxx element of the measured tensor is greater than the Zyy element in both real and quadrature parts. Mohr circle plots and principal value decompositions of the real and quadrature parts of the tensor clarify its understanding. An analysis of rotational invariants of the data suggests a case of galvanic distortion of a 2D structure. Results from phase tensor analysis are included, and the example is seen to be a dramatic instance of phase tensor analysis reducing an apparently 3D example to 2D characteristics.


For data whose Mohr circles do not capture the axes origin, simple conditions are derived regarding phase. These conditions govern whether or not it is formally possible for observed phases of Zxx to exceed 90°, for any rotation of the observing axes.

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1. Introduction

Central to a magnetotelluric study of Earth structure is the determination, from field measurements of the horizontal components of the electric field E and the magnetic field H as time series at a site, of values for different periods of the magnetotelluric tensor Z for that site (Weaver, 1994). In this paper, the notation will be adopted of

\[
\begin{pmatrix}
Z_{xx} & Z_{xy} \\
Z_{yx} & Z_{yy}
\end{pmatrix}
\]

for the elements of a magnetotelluric tensor, with subscripts r and q given to the real and quadrature parts of the elements, respectively. These tensor values are obtained for some particular orientation of the observing axes of E and H. If the measuring axes are rotated, the tensor values change. At the heart of this paper is firstly the graphical demonstration of how they change, using Mohr circles, and secondly the significance of quantities known as invariants, which do not change when measuring axes are rotated. The significance of invariants has been recognised for some time, see Ingham (1988), Park and Livelybrooks (1989), Fischer and Masero (1994) and Lilley (1998).

Szarka and Menvielle (1997) and Weaver et al. (2000) investigated sets of seven invariants which, together with an eighth value in the form of a geographic bearing, are needed to fully describe a complex magnetotelluric tensor of eight elements. Weaver et al. (2003) (reprinted as Weaver et al., 2006) further presented three invariants which were based on the phase tensor analysis of Caldwell et al. (2004), see also Bibby et al. (2005).

Often the interpretation of observed magnetotelluric tensors is straightforward, enabling a magnetotelluric study to proceed to completion. Sometimes however, individual sites may appear anomalous and need extra attention before their interpretation can proceed. The present paper gives a graphical analysis of an example selected for its perplexing characteristics. The calculation and display of its invariants of rotation has been found to be helpful. The practical application is that other perplexing observations may benefit from similar analysis.

The example is discussed against a wider background. The procedure for 1D inversion is invariably based on the observed 1D impedance. Similarly 2D inversion is commonly based on the TE (E-pol) and TM (B-pol) impedances. As the subject of magnetotelluric interpretation advances further into 3D inversion and modelling, the question of which parameters to invert, from a wide range of possible candidates including notably invariants as discussed in this paper, may be expected to need frequent re-visiting.

The analysis starts with principal value decomposition of the real and quadrature parts of the magnetotelluric tensor taken separately

⁎ Corresponding author.
E-mail addresses: ted.lilley@anu.edu.au (F.E.M. Lilley), jtwweaver@uvic.ca (J.T. Weaver).

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It then moves to the calculation and presentation of the invariants of rotation of Weaver et al. (2000), and to the phase tensor analysis of Caldwell et al. (2004). The recognition that just three invariants displayed as functions of period carry much important information is tested. To give a context, sets of other invariants are also displayed. Phase tensor analysis, which avoids galvanic distortion effects, reduces the 3D nature of the example to 2D form, and demonstrates that the example is strongly affected by galvanic distortion.

2. Mohr circles

Mohr circle diagrams are included in the displays, including a set for phase tensor analysis. It is instructive to examine all Mohr circles in such a case, to see whether the circles enclose or capture their origin of axes. In the present example they do not do so. Such capture, occurring for the real (or quadrature) part of the full tensor, would mean that the determinant value of that part of the tensor was negative (Lilley, 1998, p.1890). This matter also arises in the discussions of Caldwell et al. (2004, p.469) and Bibby et al. (2005, p.918).

The Mohr circles show phases greater than 90° are possible for some rotation of axes. In Section 4 below, general conditions for this phenomenon to occur are derived in terms of two angles, both calculated from the magnetotelluric tensor as observed. The two angles are invariants of rotation.

In discussing phase angles, this paper implicitly assumes that an \( \exp(+i\omega t) \) time dependence has been taken for all time-dependent quantities. Such is evidently the case for the example discussed, the EMSLAB LP04 data. When an \( \exp(-i\omega t) \) time dependence is used, phase values change sign, and quadrature Mohr circles will generally plot to the left of the vertical axis, rather than to the right (Lilley, 1998, p.1897).

3. Example from EMSLAB site LP04

3.1. Data and basic decomposition

The example is from EMSLAB long-period site LP04 (Booker and Chave, 1989). There are four figures presenting results for this site. The first (Fig. 1) shows, in its left-hand panel, apparent resistivity values calculated from the individual \( Z_{xx}, Z_{xy}, Z_{yx} \) and \( Z_{yy} \) tensor elements as observed. The graphs in the right-hand panel show corresponding phase values.

Comparing the apparent resistivity values computed for the individual tensor elements, it is seen that the \( Z_{xx} \) apparent resistivity is greater than the \( Z_{xy} \) apparent resistivity. This situation is unusual and, in the interpretation process, requires resolution. Further, the \( Z_{xy} \) phase moves out of its expected quadrant for periods above \( 10^3 \) s, though the \( Z_{yx} \) phase is in the appropriate quadrant. The \( Z_{yy} \) apparent resistivity behaviour is erratic, with value increasing markedly over the period range, and phase changing quadrant several times. Fig. 2 shows, at the top, Mohr circles for the magnetotelluric data. With increasing period, these circles show a consistent pattern. The observed points are marked by the outer ends of the radial arms, and it can be seen that with increasing period these observed points in the real circles move upwards from the horizontal axis, and then to the left to cross the vertical axis. Similar behaviour is seen in the quadrature circles.

![Fig. 1. The EMSLAB LP04 data. The units of apparent resistivity (rho Zxy as computed for the Zxy element, etc.) are ohm m, and phase angles are given in degrees.](image-url)
This behaviour explains immediately the unusual characteristics noted in Fig. 1. There where the $Z_{xy}$ phase is 90° (orange point on plot) the real Mohr circle (orange) shows its observed point to be just crossing the vertical axis. Similarly the last point (red) in the $Z_{xy}$ phase plot shows a further change in quadrant, corresponding to the observed point of the outermost (red) quadrature circle just crossing the vertical axis.

The panels below the circles in Fig. 2 show principal value decompositions of the magnetotelluric tensor, taking the real and quadrature parts of the tensor separately (Lilley, 1998). The situation is envisaged where an ideal two-dimensional tensor is measured using axes (say aligned north and east) relative to which the geologic strike has bearing $\theta_h$. The distortion of the electric field from its 2D direction is described by an angle $(\theta_e - \theta_h)$. Then rotating the magnetic axes by angle $\theta_h$ and the electric axes by angle $\theta_e$ recovers the original two-dimensional tensor, and its E-pol and B-pol values.

Using Eqs. (107)–(114) of Lilley (1998), the panels of Fig. 2 show the $\theta_e$ and $\theta_h$ values for the LP04 example, and below them, values of apparent resistivity (left) and phase (right) calculated for ‘principal’ values of the tensor, $Z_{xy}^P$ and $Z_{yx}^P$. In the calculation of the apparent resistivity and phase values, the real and quadrature parts of $Z_{xy}^P$ have first been combined in the usual way to give $Z_{xy}$ amplitudes, even though the real and quadrature parts of $Z_{xy}^P$ may have resulted from different values of $\theta_e$ and $\theta_h$. The same description applies to $Z_{yx}^P$. For

Fig. 2. Mohr circles and decomposition of the EMSLAB LP04 data, as explained in the text. The variation of period with colour in the Mohr circle diagrams is the same as for the other plots. The centre point of each circle has the same colour as the circle, and the radial arm from the centre meets the circle at the observed (i.e. before rotation) value of $(Z_{xy}, \omega)$. Rotation of the measuring axes through 180° causes all radial arms to sweep around 360°, each arm thus drawing its own circle. For the Mohr circles, each tensor element value has been scaled by multiplication by the square root of the period, to make the plot of a set of circles more compact.
the simple 2D distortion case described, such apparent resistivity and phase results are the undistorted E-pol and B-pol values (possibly interchanged).

Examining the results plotted for the LP04 example in Fig. 2, it is evident that both $\theta_e$ and $\theta_h$ are well determined and reasonably consistent for all the period range, with $\theta_e$ more constant than $\theta_h$. This behaviour is consistent with 2D structure, the variation of $\theta_h$ indicating some variation of geologic strike with period. The $Z_{xy}$ and $Z_{yx}$ apparent resistivities are well behaved, and also consistent with 2D structure. The $Z_{xy}$ and $Z_{yx}$ phases are in appropriate quadrants, and smooth.

In the Mohr circles, invariants of rotation of the measuring axes are evident. For example the centres of the circles are fixed by the observed data, and would not change even were the measuring axes to be rotated and so aligned differently. In the case of an ideal 2D structure, the E-pol and B-pol impedances are given by the two points where the circle (itself now centred on the horizontal axis) cuts the horizontal axis.

3.2. Invariants as a function of period

The third figure (Fig. 3) moves to the calculation and presentation of the invariants of rotation of Weaver et al. (2000), denoted as $I_1$ to $I_7$. Two supplementary invariants are also included, $I_0$ and $I_0$ (Weaver et al., 2003, 2006). The seven invariants ($I_1$ to $I_7$) monitor the...
dimensionality of the magnetotelluric tensor as a function of period. Specifically, \( I_1 \) and \( I_2 \) gauge the scale of the tensor: see Eq. (24) of Weaver et al. (2000). The quantities \( I_3 \) and \( I_4 \) are dimensionless, vanish for 1D, and otherwise gauge the extent of two-dimensionality: see Eqs. (26) and (27) of Weaver et al. (2000). The quantities \( I_5 \), \( I_6 \) and \( I_7 \) (also dimensionless) gauge three-dimensionality, see Eqs. (51) and (52) of Weaver et al. (2000). Of the supplementary invariants in Fig. 3, \( I_5 \) is related to \( I_1 \) and \( I_2 \). The supplementary invariant \( I_6 \) is related to \( I_5 \), in that the vanishing of \( I_5 \) implies that it is undefined.

Thus in Fig. 3 for site LP04, invariants \( I_1 \) and \( I_2 \) show a common, well-behaved smooth decrease with increasing period which is seen also in \( I_5 \). Invariants \( I_1 \) and \( I_2 \) show values zero for all their period range, a clear indication of two-dimensionality or three-dimensionality in the observed data. The values of \( I_5 \) are clearly non-zero, but \( I_6 \) and \( I_7 \) are weaker. Numerical ranges for the criteria of Weaver et al. (2000) are still a matter of experiment and discussion (Marti et al., 2005; McKay and Whaler, 2006). However, if weak \( I_5 \) and \( I_7 \) are linked to the ideal case of zero \( I_5 \) and \( I_7 \), then these criteria indicate three-dimensionality due to ‘galvanic distortion representing a pure twist, without shear, of the local electric field in a 2D region’ (Weaver et al., 2000, p.328).

The set of three independent invariants, \( J_1, J_2 \) and \( J_3 \) (Weaver et al., 2003, 2006) which can be expressed in terms of \( I_1, I_2, I_3 \) and \( I_4 \) are next considered. They are closely related to the phase tensor analysis of Caldwell et al. (2004) and summarise neatly the extent to which the magnetotelluric phase (thus a value of unity indicates a mag-

to...
completely above (or below) this axis (and the circle for the quadrature part of the tensor behaved similarly).

5. Conclusions

The observed behaviour of the magnetotelluric field at site LP04, at first inspection perplexing, is seen as a distinctive consequence of the particular orientation which the observing axes happened to have with the geologic structure. When invariants of rotation are calculated and examined, the criteria of Weaver et al. (2000) suggest galvanic distortion of a 2D region. Phase tensor analysis also reduces the apparent 3D behaviour of the example to 2D behaviour, and demonstrates that the case is one of galvanic distortion of a 2D situation. Some 3D behaviour remains at long periods, however.

More generally, when phases out of quadrant are recognised as Mohr circles which have crossed their vertical axis, it is clear that such phases are potentially of common occurrence. Whether they are realised or not in observed data then depends on the orientation of the observing axes. As derived in the previous section, a necessary and sufficient condition for the $Z_{xy}$ phase to exceed 90° for some rotation

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Fig. 4. Mohr circles for the phase-tensor decomposition of the LP04 data. The quantities $T_{11}$ and $T_{21}$ plotted to give the phase-tensor Mohr circles are dimensionless. For 1D data (points on the horizontal axis) $T_{11}$ is the trigonometric tangent of the phase value of the magnetotelluric impedance. The variation of period with colour in the Mohr circles is the same as for the other plots. The centre point of each circle has the same colour as the circle itself, and the radial arm from the centre meets the circle at the observed (i.e. before rotation) value of ($T_{11}$, $T_{21}$). Rotation of the measuring axes through 180° causes all radial arms to sweep around 360°, each arm thus drawing its own circle. Angles $\alpha$, $\beta$, $\theta$, and $\gamma$ are explained in the Appendix A.
I₂ is a measure of the 1D character of the quadrature part of a tensor. I₀ would be the distance OC in an equivalent version of Fig. 5 drawn for the quadrature part of a tensor.

I₃ is a measure of the 2D character of the real part of a tensor. With reference to Fig. 5 above, I₃ = sinλᵣ.

I₄ is a measure of the 2D character of the quadrature part of a tensor. In an equivalent version of Fig. 5 above drawn for the quadrature part of a tensor, I₄ would be sinμᵣ.

I₅ is a measure of the 3D character of the real part of a tensor. With reference to Fig. 5 above and to an equivalent figure drawn for the quadrature part of the tensor, I₅ is sin(μᵣ + λᵣ).

I₆ is a measure of the 3D character of the quadrature part of a tensor. With reference to Fig. 5 above and to an equivalent figure drawn for the quadrature part of the tensor, I₆ is sin(μᵣ − λᵣ).

I₇ is the third measure of the 3D character of a tensor. With reference to Fig. 5 above and to an equivalent figure drawn for the quadrature part of the tensor, with radial arms to the observed data points drawn for both circles, then I₇ most importantly takes into account the angle between those two radial arms. I₇ appears again below, depicted simply, as J₁J₂ (i.e. as sinγ) in the Mohr circles for a phase tensor.

I is a supplementary invariant, related to I₁ and I₂.

I₀ is a supplementary invariant, related to I₁. (If I₀ vanishes, I₇ is undefined.)

From Weaver et al. (2003), reprinted as Weaver et al. (2006):

The invariants J₁, J₂ and J₃ may be described in terms of Fig. 4 above. In that figure:

T₁₁ and T₂₂ are elements of a phase tensor after rotation.

J₁ is the horizontal distance to a circle centre from the origin of axes, and gives a basic measure for the tangent of the phase angles in the phase tensor.

J₂ is the radius of a circle, and gives a measure of the 2D character of the phase tensor.

J₃ is the vertical distance by which a circle centre is offset from the horizontal axis, and gives a measure of the 3D character of the phase tensor.

α is a supplementary measure of 2D character, comparable, in Fig. 5 above, to angle λᵣ.

β is a supplementary measure of 3D character, comparable, in Fig. 5 above, to angle μᵣ.

γ is a supplementary measure of 3D character relative to 2D character, and is given by arcsine(J₁J₂). γ/2 may be regarded as a measure of uncertainty in the strike angle θₑ.

θₑ is angle of 2D strike.

References


