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On Estimating Electrical Conductivity Using Gradient Data from Magnetometer Arrays

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1. Introduction

A major objective of electromagnetic geophysics has been the determination of subsurface electrical conductivity through the observation of surface magnetic variations. The traditional treatment of this problem on a global scale has assumed spherical symmetry in the electrical conductivity of the earth. On a continental scale it may be reasonable to assume the geometry of a horizontally-layered half-space. Magnetotelluric experiments have been performed to determine electrical conductivity profiles beneath continents, and some magnetic variation analyses have been carried out using ratios of horizontal to vertical fluctuation fields.

It has been realized that magnetometer arrays, giving information on the horizontal gradients of variation components, should also have application to the problem. This note describes an exercise intended to study the feasibility of measuring gradient data directly off maps of array results. The results show considerable scatter, but may indicate how the method can advance to the point of providing useful continental conductivity profiles. For two cases where the results appear consistent, an approximate inversion method is followed to give conductivity-depth estimates.

2. Theory

An earlier paper (LILLEY, 1975) summarized theoretical results for electromagnetic induction in a layered half-space, and gave the equation

$$\frac{H_z}{\partial H_x/\partial x+\partial H_y/\partial y}=\frac{1}{\theta_1 G_1(0)} \tag{1}$$

where H_x, H_y and H_z are the components of a magnetic variation field of time dependence $e^{i\omega t}$, x and y are two horizontal space co-ordinates (with z vertically down), and $\theta_1 G_1(0)$ is a function of the horizontal layering, of ω, and of a wave number ν characterizing the source field. The geophysical objective is to determine $\theta_1 G_1(0)$, hoping to infer from it the conductivity structure. Thus in Eq. (1) above, it is hoped to estimate the right-hand-side through observation of the left-hand-side, which will be denoted by c:

$$\text{i.e.}\quad c = H_z/(\partial H_x/\partial x + \partial H_y/\partial y)\,.$$

Magnetometer array data presented in map form allow estimates of the gradient terms, $\partial H_x/\partial x$ and $\partial H_y/\partial y$, in addition to estimates of H_z. Thus it should be possible to estimate the left-hand-side of Eq. (1) directly, and some exploratory results follow, made using magnetometer array maps already in existence.

Each calculation of c has been done in the following manner.

(i) An area of an array map relatively clear of local induction anomalies is taken, of order several hundred km in horizontal dimension.

(ii) At a distance $\Delta x/2$ north of a chosen central point, an amplitude value A_X^N and a phase value P_X^N (in radians) are read off the X amplitude and X phase array maps. Similarly, at a distance $\Delta x/2$ south of the central point, amplitude and phase values of A_X^S and P_X^S are read off.

(iii) Taking then the Y amplitude and Y phase maps, at distances $\Delta y/2$ east and west of the central point, amplitude and phase pairs

$$(A_Y^E, P_Y^E) \quad \text{and} \quad (A_Y^W, P_Y^W) \quad \text{are read off}\,.$$

(iv) Taking the Z maps, values A_Z and P_Z are read off at the central point.

(v) The complex c value is then calculated according to

$$c = \frac{A_Z \exp(-iP_Z)}{[A_X^N \exp(-iP_X^N) - A_X^S \exp(-iP_X^S)]/\Delta x + [A_Y^E \exp(-iP_Y^E) - A_Y^W \exp(-iP_Y^W)]/\Delta y} \tag{2}$$

the units of c being those of Δx and Δy, (km in this note). In Eq. (2), negative signs are introduced into exponents to allow for the fact that magnetometer array studies to date have generally used lag phases, (so that a waveform occurring at a later time has a more positive phase), whereas the $e^{i\omega t}$ commonly introduced in the theory implies lead phases, (of opposite sign to lag phases).

3. Data

Estimates of c have been made from eighteen sets of magnetometer array maps, thirteen sets being for Australia and five sets being for North America. Details concerning these sets are given in Table 1.

Table 1.

Map set	Period (minutes)	Region		Source
A	33	Western Victoria, Australia		Bennett (1972) thesis, Map 3
B	39	〃	〃	Bennett (1972) thesis, Map 4
C	46	〃	〃	Bennett (1972) thesis, Map 5
D	49	〃	〃	Bennett (1972) thesis, Map 6
E	64	〃	〃	Bennett (1972) thesis, Map 7
F	82	〃	〃	Lilley and Bennett (1972) Fig. 5
G	85	〃	〃	Lilley and Bennett (1972) Fig. 6
H	98	〃	〃	Bennett (1972) thesis, Map 10
I	128	〃	〃	Bennett (1972) thesis, Map 11
J	360	〃	〃	Bennett and Lilley (1973) Fig. 4a
K	480	〃	〃	Bennett and Lilley (1973) Fig. 5a
L	720	〃	〃	Bennett and Lilley (1973) Fig. 6a
M	1440	〃	〃	Bennett and Lilley (1973) Fig. 7a
N	67	South Dakota-Nebraska U.S.A.		Porath and Dziewonski (1971) Fig. 6
P	29	〃	〃	Porath and Dziewonski (1971) Fig. 7
Q	48	Northern Montana, U.S.A.		Camfield *et al.* (1971) Fig. 6
R	48	〃	〃	Camfield *et al.* (1971) Fig. 9
		(Note that map sets Q and R are different substorm events)		
S	480	Northern Montana, U.S.A.		Camfield and Gough (1975) Fig. 7

Three such estimates of c were made for each map set, by varying the position of the central point and the sizes of Δx and Δy. Figure 1 shows an example of one map set (F), and the data values read off it for the first estimate. For the Australian data, the third estimates were obtained from values read off after the maps had been smoothed by inspection.

4. Results

The c values obtained are plotted in Fig. 2. The estimates cover a period range of two powers of ten. Parameter $|c|$ is equivalent to the penetration depth $|P|$ of Kuckes (1973, Fig. 6), and Kuckes's published estimates (of the modulus of P only) are therefore included in Fig. 2c for comparison.

Given an observed $|c|$ value, the electrical resistivity of a uniform half-space which would have the same response can be calculated (under the assumption of small values of the wave-number ν). In analogy with magnetotelluric practice this resistivity is called the apparent resistivity, ρ_a, and it is related to $|c|$ by the equation

$$\rho_a = 0.4\pi\omega(|c|)^2$$

where ρ_a is in ohm·m, ω is in rdn/s, and c is in km. Such apparent resistivity values have been calculated for all $|c|$ estimates, and are presented in Fig. 3.

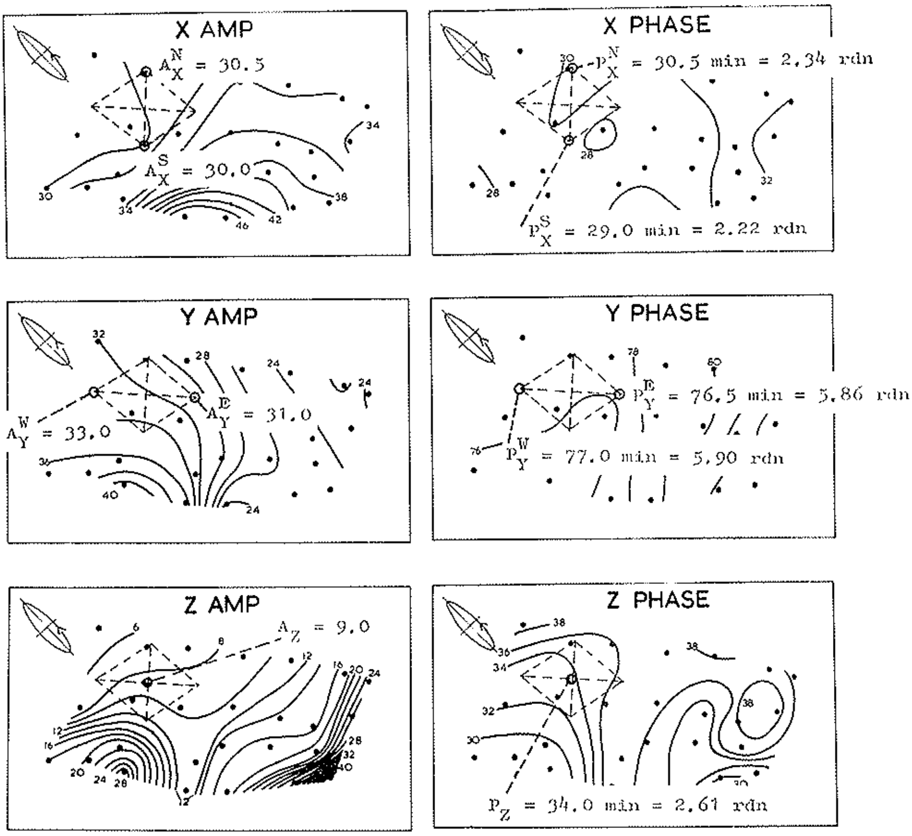


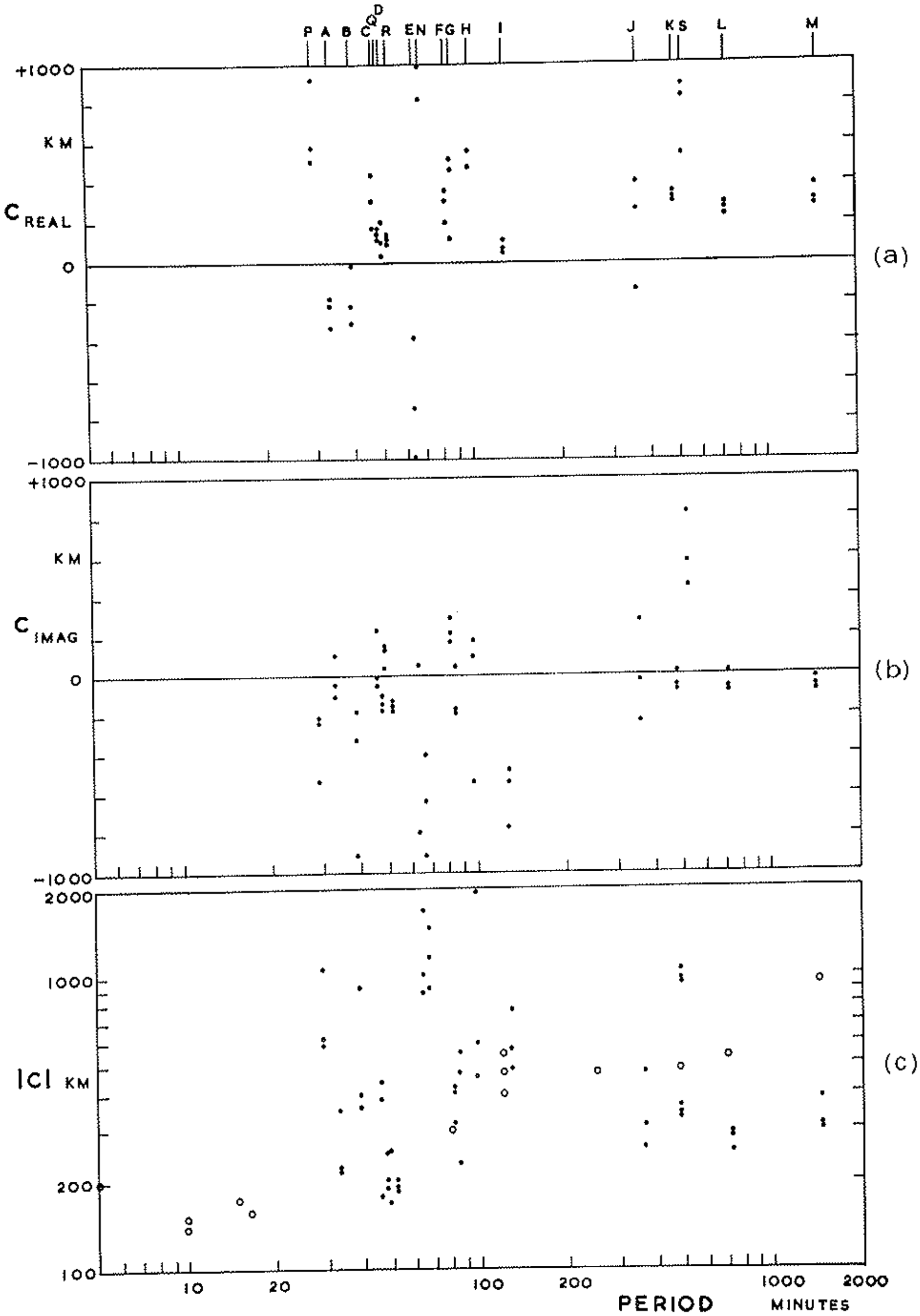
Fig. 1. Example of a set of array maps (map set F from Lilley and Bennett (1972)). Marked in is a grid used to read off data values; the values themselves are also given.

5. Discussion

5.1 Scatter of different estimates

Scatter within the different c-estimates obtained for a particular map set shows the sensitivity of Eq. (2) to slight variation in the parameters involved.

Fig. 2. The estimates of complex parameter c obtained from the magnetometer array maps listed in Table 1. The R results are plotted at 52 min rather than 48 min to avoid confusion with D and Q; similarly the S results are plotted at 520 min rather than 480 min to avoid confusion with K. Missed off Fig. 2a is an N value of 1165 km and an H value of 1957 km. Missed off Fig. 2b is an E value of -1521 km; and missed off Fig. 2c is a D value of 45 km. In the $|c|$ plot, open circles are the results from Kuckes (1973).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+1000
KM
C REAL
0
−1000
(a)
+1000
KM
C IMAG
0
−1000
(b)
2000
1000
|C| KM
200
100
(c)
10
20
100
200
1000
2000
PERIOD
MINUTES

Fig. 2.

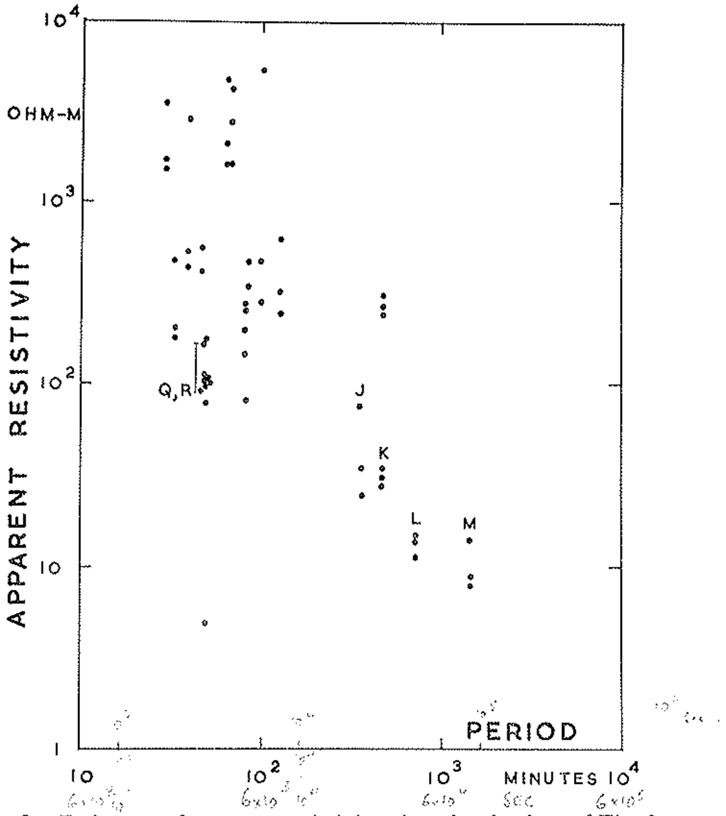

Fig. 3. Estimates of apparent resistivity given by the data of Fig. 2.

Map sets of array observations may require special smoothing before *c* estimates can be made from them.

Scatter from map set to map set is at variance with the expectation, on theoretical grounds, that the real and imaginary parts of *c* should vary smoothly with frequency. Four different factors which might cause scatter in the present estimates are as follows:

5.1.1 Source fields too remote

For magnetometer arrays in mid-latitudes, the source fields of substorms are probably in auroral latitudes, perhaps thousands of km away. As a consequence, the regional vertical field fluctuations in the array area are weak, and difficult to measure accurately. The most consistent substorm estimates in Fig. 2 are sets Q and R, which are from the array operated at highest latitudes, nearest to the auroral zones.

5.1.2 Departure from horizontal layering

For the Australian substorm data, the source fields probably occurred over the Southern Ocean. The contrast in horizontal layering between continent and ocean may greatly complicate experimental results in such a case.

5.1.3 Polarization dependence

A consequence of regional vertical fluctuation fields being weak is that locally-induced anomalous fields will contribute greater errors. It may be necessary to reduce data according to some relation like

$$H_z = AH_x + BH_y + c(\partial H_x/\partial x + \partial H_y/\partial y)$$

to determine c, where now the first two terms on the right-hand-side will account for locally-induced vertical fields; (A and B are complex constants, frequency-dependent). This approach is similar in principle to that of COCHRANE and HYNDMAN (1970), who removed a "correlated" vertical field component before examining vertical to horizontal field ratios.

5.1.4 Mixed modes in the source fields

Different modes (i.e. different ν-combinations) in the source fields of different map sets would produce some variation in the c estimates, but probably not enough to account for the scatter in Figs. 2 and 3.

5.2 Interpretation of the more consistent data

There are two groups of c estimates which are noticeably more self-consistent than the rest. The first group is from map sets Q and R, and the second set is from map sets J, K, L and M, this latter group being the daily variation data from Australia. The distinguishing feature of these groups may be that they are the ones for which the source-field currents have passed most nearly overhead.

It is of interest to invert these more consistent c estimates on the basis of the two-layer model of SCHMUCKER (1970) and KUCKES (1973), in which a poor conductor overlies a good conductor. The real part of c then gives a depth, and the imaginary part, c_i, gives a conductivity estimate, σ_s, through

$$\sigma_s = (0.8\pi\omega c_i^2)^{-1}$$

where σ_s is in S/m and c_i is in km. Sets Q and R together thus give a conductivity-depth estimate of 10^{-2} S/m at depth 140 km, and sets J, K, L and M an estimate of 10^0 S/m at depth 300 km. These results are comparable with currently accepted continental conductivity models.

It may also be noted that the Q and R results in Fig. 3 give an apparent resistivity consistent with the magnetotelluric determinations for the same area made by CANER *et al.* (1969), REDDY and RANKIN (1972), and VOZOFF and ELLIS (1966).

6. Conclusions

Considerable care is necessary if array maps at substorm periods are to give information on geologic structure through Eq. (1). A more involved procedure may develop through a different approach to the time-series analysis of array-station observations, or through improved smoothing of map sets in the spatial domain.

Improvement may also come from the careful selection of fluctuation events, choosing those with source-fields most nearly directly overhead. Daily variations in mid-latitudes are in this category, and appear to be suitable basic data.

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Note added in proof: For $c=c_r+ic_i$, WEIDELT (1972) gives general conditions $c_r \geq 0$, $c_i \leq 0$. The data used for interpretation in section *5.2* obey these conditions, which however discriminate against many of the values in Figs. 2 and 3.