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Ideal Phase in Estimating the Spatial Gradient of Magnetic Daily Variations Recorded by Magnetometer Arrays

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1. Introduction

The theory of electromagnetic induction in an horizontally-layered half-space gives the equation

$$c = Z/(\partial X/\partial x + \partial Y/\partial y) \tag{1}$$

relating surface magnetic fluctuations to the distribution of earth electrical conductivity with depth (BERDICHEVSKY *et al.*, 1969; SCHMUCKER, 1970; KUCKES, 1973). The notation in Eq. (1) is that x and y are geographic co-ordinates to the north and east respectively, while X, Y, and Z are the scalar components of the magnetic fluctuation field in the north, east, and downwards directions. The parameter c is a function of the earth electrical conductivity distribution and, to an extent usually less important, of the source-field wave-number. All of X, Y, Z, and c are frequency dependent and generally complex, having in-phase and quadrature parts.

Application of Eq. (1) has been made to a variety of data sets, (see recent publications by FAINBERG and BERDICHEVSKY, 1977; CONNERNEY and KUCKES, 1980; and JONES, 1980). For Australian data the equation has been applied particularly to daily variations recorded by magnetometer arrays (LILLEY, 1975; LILLEY and SLOANE, 1976; WOODS and LILLEY, 1979; WOODS, 1979; LILLEY *et al.*, 1981a). With daily variations the known westward movement of the source-field with the phase of the sun leads to a simplifying characteristic in Eq. (1), as the y-dependence of the term $\partial Y/\partial y$ should then be due mainly to the rotation of the earth only. Indeed if this condition is insisted upon, the best fit of observed data to an "ideal-phase" criterion can be sought.

Such a characteristic of ideal quiet daily variation was used in obtaining some of the results presented by LILLEY *et al.* (1981a). In that paper the method was mentioned in brief, and the present note now describes and discusses the principles and use of "ideal-phase" analysis of magnetic daily variations more fully.

2. Model of the Daily-Variation Source-Field

The magnetic quiet daily variation is observed around the earth to depend on season, longitudinal zone, geomagnetic latitude, geographic latitude, and local solar time (MATSUSHITA, 1966). Its main characteristics on a regional scale may be described by a model in which the daily variation is taken to be a function of geographic latitude and local solar time only. That is, a steady source current system is envisaged on the daytime side of the earth, beneath which the earth rotates. Approximations to follow below will be based on such a simple model, which also necessarily assumes an earth of radial symmetry in its electrical conductivity structure.

An implication of such a model is that the phases of all X, Y, and Z components will then vary strictly with local solar time, that is with longitude itself. In particular, there should be an increasing lag in phase of 4 min of time per degree of westward longitude; and indeed, array observations of magnetic daily variations commonly exhibit phase patterns according to this rule.

A second implication of the model is that amplitudes will be longitude-independent. That is, all stations at a given latitude will observe the same magnetic variation signal: the effects of the different station longitudes will be restricted to the phase values only, leaving the amplitude values longitude-free. In this sense the "ideal phase" model is also one of "ideal amplitude" behaviour.

3. The Ideal Phase Method

Under the assumption of ideal phase, two stations at different latitudes are then sufficient in principle to give an estimate of c. The quiet daily magnetic variations at each can be corrected in phase to the longitude of some arbitrary datum, (say that of a central point), and the derivative $\partial X/\partial x$ in Eq. (1) estimated by the approximation

$$\partial X/\partial x \approx \Delta X/\Delta x$$

where ΔX represents the difference in the (complex) X components of the two stations, which are Δx apart in north geographic distance. The mean of the Y-values for the two stations can be taken as Y, and the derivative $\partial Y/\partial y$ estimated by

$$\partial Y/\partial y \approx \Delta Y/\Delta y$$

where now the difference in Y over an eastwards distance Δy is appropriate to a phase increase of 4 min of time per increasing degree of westwards longitude. The mean of the Z-values at the two stations can be taken as Z, and then c determined using Eq. (1).

However, such an estimate of c from just two stations may be vulnerable to the effects of various perturbations, not only in the magnetic daily variation source field but also particularly in the local geology departing from strict one-dimensionality in its horizontal layering. It should therefore be an advantage to have measurements from an array of stations; indeed in all cases it should be advantageous to have observations from as many sites as possible, to check on the one-dimensionality of the local geology and to smooth out the effects of non-uniformities. Observations from an array of stations can be analysed as in the following example.

4. Example

The data for this example are taken from the 1976 magnetometer array study in central Australia. Calculations are carried out with numbers of three and four digits but this is not meant to imply that all the quoted digits are significant. The sites are shown in Fig. 1, and the records analysed are the five continuous quiet days shown in Fig. 2. The present example is for the 12 h component of the five quiet days, the values of which for the different stations are listed in Table 1. Similar analyses have also been carried out at the other harmonic periods of 24 h, 8 h, 6 h, and 4 h. In Table 1 the amplitude values are given in units of nT·h, resulting from computing a direct Fourier transform of the five days of record using Eq. (1) of LILLEY (1975b). Amplitudes of equivalent traditional Fourier series coefficients in nT may be obtained by dividing the Fourier transform amplitudes by 60, (following LILLEY, 1975b, Eq. 4). Phases quoted are relative to 1500 h 10 August 1976 U.T. and are "phase lags," in the sense that a more positive phase value corresponds to the shift of a waveform to a later real time.

Figure 3 shows the data of Table 1 corrected according to ideal phase to the datum longitude of 136°E, and then plotted against latitude. For example, station A1 on Table 1 records X phase of 0.572 cycle at longitude 140°47′E. For an ideal phase model with lag increasing 4 min of time per degree of westwards longitude, the correction to apply to a phase recorded at a station of longitude 140°47′E to reduce it to datum longitude 136°E is +19.1 min of time. For the variation component of period 12 h, this correction changes the observed phase of 0.572 cycle to 0.599 cycle, and the latter value has been plotted on the X-phase graph of Fig. 3. The amplitude values of Table 1 have in Fig. 3 been plotted directly as functions of latitude, consistent with the daily-variation model according to which observed amplitudes are longitude-independent.

Straight lines have been fitted to the graphs of Fig. 3, using a standard "least

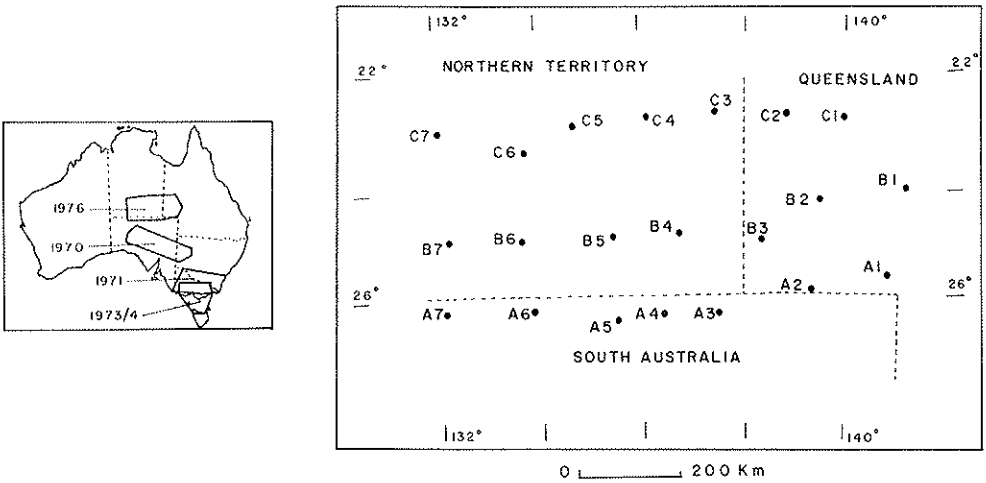

Fig. 1. Observing sites of the 1976 central Australia magnetometer array.

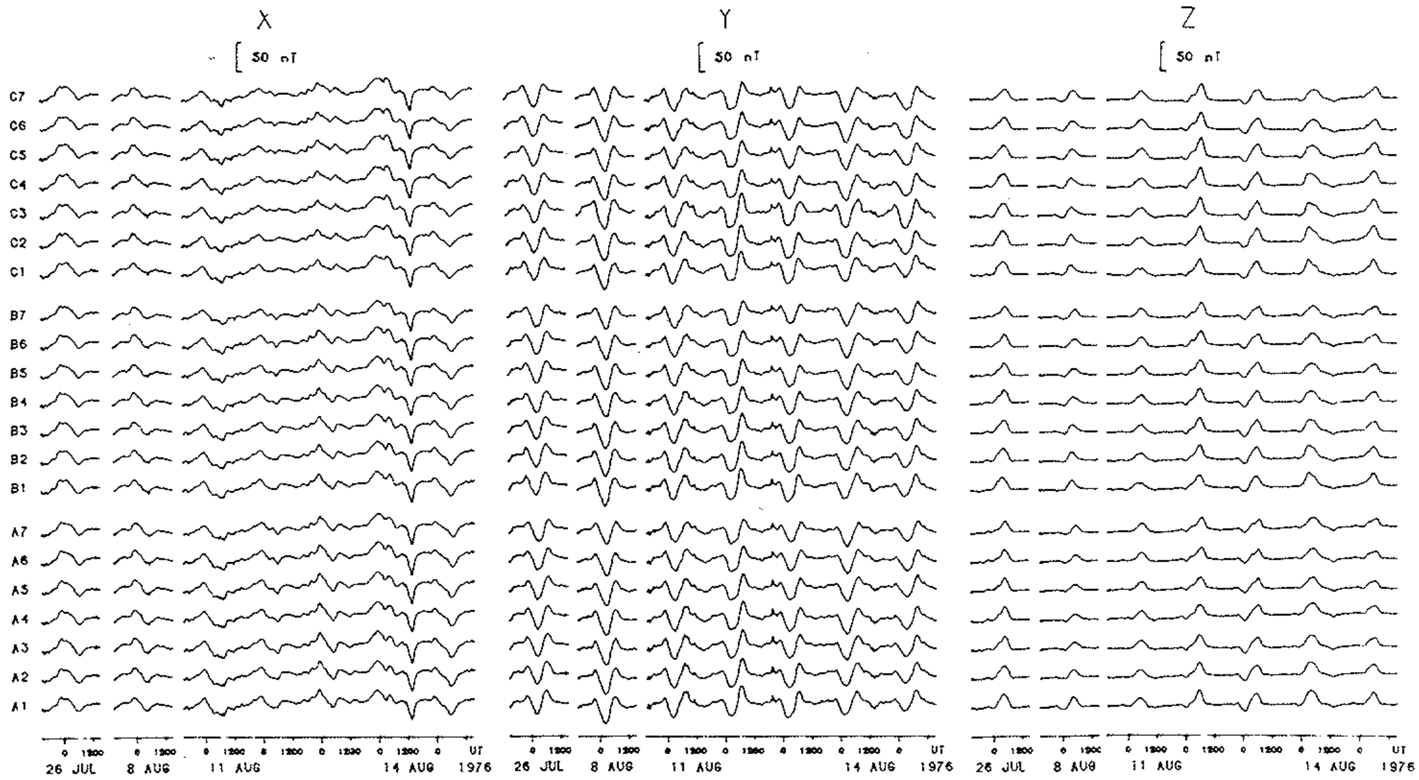

Fig. 2. Records of the magnetic variations of seven quiet days (not all consecutive) recorded by the 1976 array in central Australia. The example of the present paper is based on the five consecutive days from 11 to 15 August.

Table 1. Amplitudes and phases of the 12 h component of the 11–15 August 1976 records in Fig. 2.

Code	Station	Lat. S.	Long. E.	X amp (nT·h)	Y amp (nT·h)	Z amp (nT·h)	X phase (cycles)	Y phase (cycles)	Z phase (cycles)
A1	MLD	25°38′	140°47′	295.5	659.6	320.8	.572	.347	.147
A2	BVL	25°55′	139°20′	296.7	633.3	272.8	.579	.360	.149
A3	PLR	26°19′	137°33′	300.3	633.5	264.4	.595	.376	.163
A4	MKR	26°19′	136°26′	251.6	659.9	255.7	.607	.388	.176
A5	DHS	26°26′	135°29′	264.4	610.4	245.4	.603	.399	.175
A6	TYN	26°13′	133°52′	237.7	606.5	238.3	.621	.416	.185
A7	ERN	26°15′	132°10′	212.7	590.3	245.7	.632	.428	.198
B1	DVP	24°09′	141°06′	253.2	680.2	271.7	.570	.346	.123
B2	BDR	24°21′	139°27′	229.7	674.5	282.9	.584	.368	.140
B3	MNC	25°04′	138°20′	239.9	662.5	285.4	.596	.376	.162
B4	GSY	24°59′	136°48′	216.1	631.9	288.6	.602	.391	.160
B5	RKD	25°02′	135°27′	204.5	641.1	270.1	.591	.403	.174
B6	IDR	25°04′	133°43′	204.6	641.1	269.9	.627	.417	.172
B7	ANG	25°02′	132°16′	174.5	613.4	259.2	.642	.428	.191
C1	BOL	22°55′	139°53′	167.9	676.7	296.2	.571	.364	.129
C2	GLN	22°53′	138°49′	149.4	689.3	322.7	.558	.377	.142
C3	MQA	22°49′	137°21′	120.6	751.0	332.5	.601	.396	.154
C4	JRV	22°54′	136°08′	125.8	653.9	322.1	.611	.399	.154
C5	MRK	23°02′	134°41′	131.6	633.5	327.7	.616	.411	.169
C6	BNS	23°31′	133°50′	138.8	661.6	292.2	.635	.418	.178
C7	DRW	23°09′	132°10′	115.1	661.7	293.7	.667	.433	.187

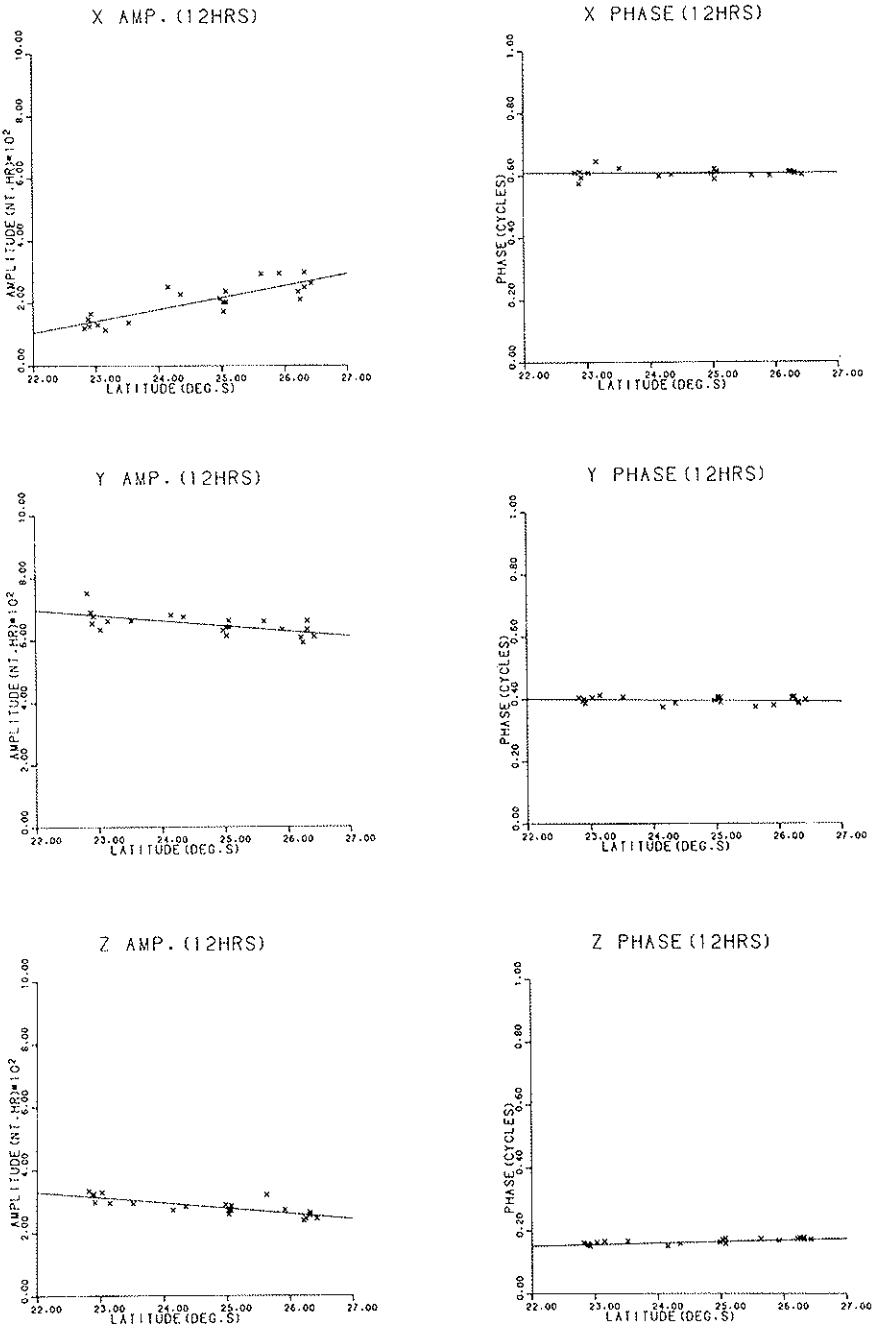

Fig. 3. The amplitude and phase data of Table 1 reduced to longitude 136°E assuming ideal phase and then plotted against latitude.

squares" procedure. The variations of the different parameters with latitude would not necessarily be expected to be linear; however straight lines are a first approximation and are basic to determining spatial derivatives, in the actual calculation of a c value as now follows.

4.1. *Details of calculation*

A central latitude of 24°30′S is chosen arbitrarily, and values of the X parameters north and south of the central point (say at 24°S and 25°S) are determined, giving values (call them X_N and X_S) a distance 111 km apart. From the lines on the X-graphs in Fig. 3 these values are

$$X_N: \text{amplitude} = 180.9\,\text{nT}\cdot\text{h}\,;\ \text{phase} = 0.607\ \text{cycle}$$
$$X_S: \text{amplitude} = 218.9\,\text{nT}\cdot\text{h}\,;\ \text{phase} = 0.606\ \text{cycle}$$

with $\Delta x = 111$ km.

At 24°30′S, the lines on the Y-graphs in Fig. 3 give a Y-amplitude value of 653.6 nT·h and a phase value of 0.395 cycle (for longitude 136°E). On the basis of ideal phase, the Y-phase 100 km west of this point would be 0.400 cycle. Hence, two values for Y are taken as

$$Y_E: \text{amplitude} = 653.6\,\text{nT}\cdot\text{h}\,;\ \text{phase} = 0.395\ \text{cycle}$$
$$Y_W: \text{amplitude} = 653.6\,\text{nT}\cdot\text{h}\,;\ \text{phase} = 0.400\ \text{cycle}$$

with $\Delta y = 100$ km.

Also at 24°30′S, the lines on the Z-graphs in Fig. 3 give

$$Z: \text{amplitude} = 286.5\,\text{nT}\cdot\text{h}\,;\ \text{phase} = 0.165\ \text{cycle}.$$

The above values, put into Eq. (1) in the form

$$c = Z/[(X_N - X_S)/\Delta x + (Y_E - Y_W)/\Delta y]$$

then give the result

$$c_{\text{real}} = 505.8\ \text{km}\,; \qquad c_{\text{imag}} = -152.0\ \text{km}\ .$$

The resolution of the method can be explored by carrying out similar calculations at the ends of the lines shown in Fig. 3, say at 23°S and 26°S latitudes. Table 2 summarizes c-

Table 2. Values of the parameter c obtained using the ideal phase method for the quiet days of 11–15 August 1976. The c values are given as pairs of numbers, the first number being the real part of c and the second number being the imaginary part.

Period:	24 h	12 h	8 h	6 h	4 h
Latitude of central point:					
23°S	785.0, 28.79	552.3, −128.4	404.0, −170.6	337.6, −119.7	284.3, −138.8
24°30′S	710.2, −78.21	505.8, −152.0	405.8, −145.7	334.5, −115.0	242.0, −128.1
26°S	654.1, −168.5	430.2, −151.7	408.7, −120.5	336.8, −118.8	191.7, −101.9

values determined in this way, and also for the other harmonics of the data. Because the straight lines of Fig. 3 may be poor representations of the latitudinal distributions at the ends of the ranges shown, the 23°S and 26°S determinations may be inferior to the 24°30′S determination. However the 23°S and 26°S determinations give some indication of the stability of the method to demonstrated changes in the parameters involved.

5. Conclusion

The basic characteristic of the quiet daily variation, that it rotates with apparent sun, may be exploited in using observed daily-variation data to give estimates of earth electrical conductivity via determinations of the response parameter c. While in principle only two stations are needed to give an estimate of c, a whole array of stations greatly improves precision.

In the example given, the "ideal phase" model has been used twice. In its first use, all phase data recorded by a magnetometer array have been reduced to some datum longitude, so that the phase and amplitude variations recorded over a two-dimentional array may be examined as functions of one parameter (latitude) only, and plotted as shown in Fig. 3.

The second use of ideal phase has been in the generation of Y-variation with longitude, needed for computation of the $\partial Y/\partial y$ term in Eq. (1).

The actual c-values listed in Table 2 form part of the results presented in LILLEY *et al.* (1981a), and the interpretation of them is the subject of a companion paper by LILLEY *et al.* (1981b).

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