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# An Application of Total-Field Magnetic Fluctuation Data to Geomagnetic Induction Studies

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The possibility is explored of using total-field fluctuation records to give information on local geomagnetic induction. Total-field fluctuation records will not usually be accompanied by local horizontal fluctuation records, and as a replacement for the latter the use of distant observatory records may be possible. Several examples are given, based on records of instruments which have operated to monitor magnetic activity for the benefit of aeromagnetic survey parties. Transfer functions are computed which appear useful as a guide in a reconnaissance study of geomagnetic induction.

The use of total-field fluctuation recordings made remote from an observatory may have several applications, such as to studies of geomagnetic induction at sea. The operation of total-field recording instruments in arrays which contain a small number of stations recording three components of magnetic fluctuations may have application to studies of continental induction.

## 1. Introduction

Studies of natural geomagnetic induction are usually based on the observation of fluctuations in the earth's magnetic field in three components, and of its surface electric field in two components.

One major tradition in such studies is to seek to correlate changes in the vertical component of the magnetic field with changes in the horizontal components of the field. That is, if the magnetic fluctuations with time $t$ in the magnetic north, magnetic east and vertically downwards directions are denoted by $h(t)$, $d(t)$ and $z(t)$ respectively, and if the Fourier transforms of these time series are denoted by $H(\omega)$, $D(\omega)$ and $Z(\omega)$, then a best fit is sought to an equation such as

$$Z(\omega)=A(\omega)\ H(\omega)+B(\omega)\ D(\omega). \tag{1}$$

The terms $A$ and $B$ are often called 'transfer functions' and together with the other terms in Eq. (1) are complex valued (with real and quadrature parts) functions of frequency. These transfer functions are generally interpreted as functions of earth electrical conductivity structure although caution may be needed to guard against their bias by source-field effects in the original $h(t)$, $d(t)$ and $z(t)$ time series.

The real parts ($A_r$ and $B_r$) of the coefficients $A$ and $B$ for a particular observing site are often represented diagrammatically by an induction arrow, drawn with component $A_r$ to the south and component $B_r$ to the west. Such an arrow is then like the induction arrow of PARKINSON (1962), and is generally modelled to point towards the higher conductivity side of any electrical conductivity contrast existing in the area of the site. Similarly the quadrature parts ($A_q$ and $B_q$) of $A$ and $B$ may also be represented diagrammatically by an arrow: though the direction of this arrow will depend on the time dependence taken when transforming to the frequency domain (LILLEY and ARORA, 1982). The common use of such arrows in electrical conductivity mapping has stimulated wide discussion on arrow determination in the literature; for recent papers see, for example, BEAMISH (1979), GOUGH and DE BEER (1980), GREGORI and LANZEROTTI (1980), JONES (1981) and WOLF (1982).

The present paper now addresses a slightly different aspet of the problem of (Parkinson) arrow determination: the use of recordings of fluctuations of the total magnetic field to yield information on the local transfer functions $A$ and $B$. Recordings of fluctuations of the total magnetic field are often made during regular magnetic surveys. For such surveys, intended to measure the spatial variation of the earth's magnetic field, magnetic fluctuations represent a possible source of error. It is thus common survey practice to monitor the magnetic field at some base station using a recording magnetometer or 'storm warning device', which thus becomes, in effect, a temporary magnetic observatory. If the survey itself is of the total magnetic field strength $\mathfrak{F}$ (as is common in airborne, seaborne and many ground surveys) then the base station usually also records fluctuations in the total magnetic field', say $f(t)$.

A major limiting factor in the use of such total-field fluctuation records is that horizontal fluctuation records do not generally exist for sites where the total field is recorded. Thus in the present paper (in Section 3 below) the substitution of distant horizontal fluctuation data, available from a permanent magnetic observatory, is investigated.

Information on the local induction functions $A$ and $B$ gained from this approach could yield much information of a reconnaissance nature about geomagnetic induction over a continent. The knowledge of such information may greatly aid the planning of future magnetic fluctuation studies intended to study the phenomena and patterns of geomagnetic induction in more detail.

## 2. Theory

At an observing site where the horizontal component of the geomagnetic field is $\mathfrak{H}$, the vertical component $\mathfrak{Z}$ and the magnetic inclination $I$, the amplitude $\mathfrak{F}$ of the total magnetic field strength will be given by

$$\mathfrak{F}=\mathfrak{H}\cos I+\mathfrak{Z}\sin I.$$

For small fluctuations $h(t)$, $d(t)$, $z(t)$ around suitable datum levels of $\mathcal{H}$, $\mathcal{D}$ and $\mathcal{Z}$, $h(t)$ and $z(t)$ will contribute to $f(t)$ according to

$$f(t)=h(t)\ \cos\ I+z(t)\ \sin\ I$$

and so by Fourier transformation

$$F(\omega)=H(\omega)\ \cos\ I+Z(\omega)\ \sin\ I \tag{2}$$

where $F(\omega)$ is the Fourier transform of $f(t)$.

If now Equation (1) is substituted into Eq. (2), the relation is obtained

$$\begin{aligned} F&=H\ \cos\ I+(AH+BD)\ \sin\ I \\ &=(A\ \sin\ I+\ \cos\ I)H+B\ \sin\ I\cdot D \end{aligned}$$

where the frequency dependence of $F$, $H$, $D$, $A$ and $B$ is understood.

Thus, seeking the best fit of $F$, $H$ and $D$ data to an equation

$$F=A_F\ H+B_F\ D \tag{3}$$

should give values of $A_F$ and $B_F$ according to

$$A_F=A\ \sin\ I+\cos\ I$$
and $B_F=B\ \sin\ I$

or equating real and quadrature components

$$A_{F_r}=A_r\ \sin\ I+\cos\ I \tag{4a}$$
$$A_{F_q}=A_q\ \sin\ I \tag{4b}$$
$$A_{F_r}=B_r\ \sin\ I \tag{4c}$$
$$B_{F_q}=B_q\ \sin\ I. \tag{4d}$$

The relationship between $A_{F_r}$ and $A_r$ according to Eq. (4a) is shown in Fig. 1. The relationships of Eqs. (4b), (4c) and (4d) are all of the same form and are shown in Fig. 2.

The inverse relationships, between $A$ and $A_F$, $B$ and $B_F$ are thus given by

$$A_r=(A_{F_r}-\cos\ I)/\sin\ I \qquad = A_{F_r}\ \mathrm{cosec}\ I - \cot\ I \tag{5a}$$
$$A_q=A_{F_q}/\sin\ I \tag{5b}$$
$$B_r=B_{F_r}/\sin\ I \tag{5c}$$
$$B_q=B_{F_q}/\sin\ I \tag{5d}$$

The relationship between $A_r$ and $A_{F_r}$ according to Eq. (5a) is shown in Fig. 1a and in Fig. 3. The relationships of Eqs (5b), (5c) and (5d) are all of the

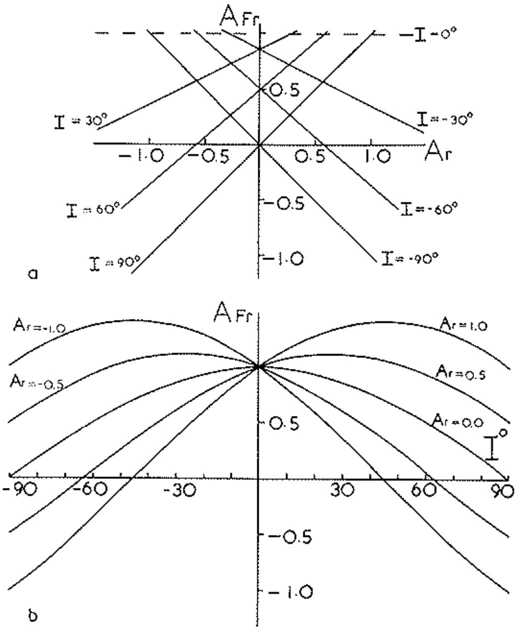

Fig. 1. Graphical expression of $A_{F_r} = A_r \sin I + \cos I$. (a) $A_{F_r}$ as a function of $A_r$ for different values of $I$. (b) $A_{F_r}$ as a function of $I$ for different values of $A_r$.

same form and are shown in Figs. 2a and 4.

## 3. Some Notes on the Theory

### *3.1 Equatorial regions*

It should firstly be noted that in deriving expressions for $A$ and $B$ in terms of $A_F$ and $B_F$ there is an implicit restriction of non-zero inclination, for clearly as I approaches zero, Equations (5a), (5b), (5c) and (5d) become unstable. In such equatorial regions $f(t)$ will, to first order, be recording $h(t)$ directly (as noted, for example, by RODEN and MASON, 1964); such information may still be valuable for magnetic fluctuation studies, but not in the tradition of examining the vertical component of the fluctuations, $z(t)$. The non-applicability of equatorial data is evident in the figures, for example in Fig. 1 (a and b) where the value of $A_{F_r}$ is unity at $I = 0°$ for all values of $A_r$.

Away from the equator, applicability of the method should be aided by the fact that magnetic inclination generally increases more rapidly than geographic latitude. Thus mid-latitude stations have relatively high magnetic inclinations and total-field fluctuations recorded at mid-latitudes contain a correspondingly large proportion of vertical-field fluctuations.

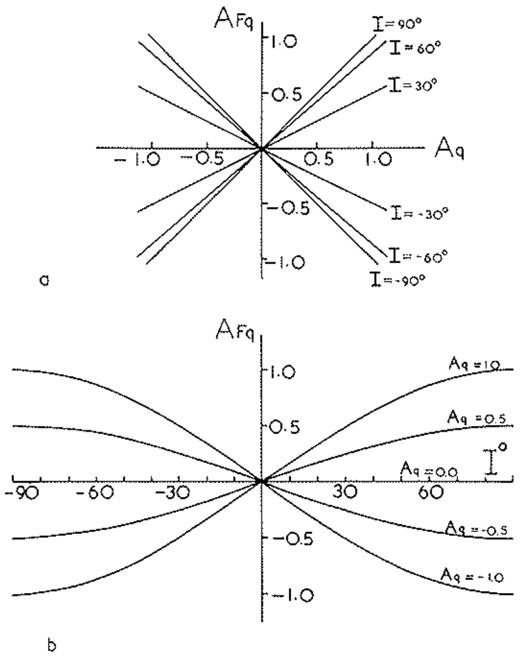

Fig. 2. Graphical experssion of $A_{Fq}=A_q \sin I$. (a) $A_{Fq}$ as a function of $A_q$ for different values of $I$. (b) $A_{Fq}$ as a function of $I$ for different values of $A_q$. Note the curves demonstrate also the graphical expression of the similar relationships $B_{Fr}=B_r \sin I$ and $B_{Fq}=B_q \sin I$.

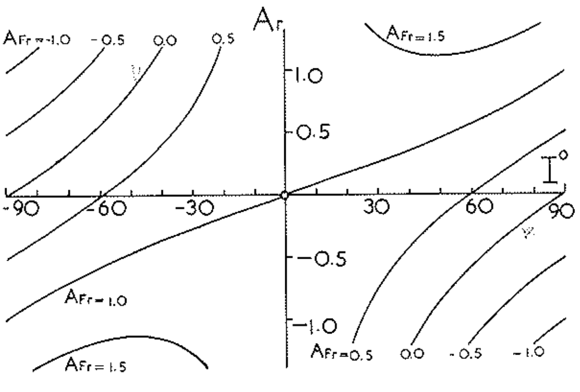

Fig. 3. Graphical expression of $A_r=(A_{Fr}-\cos I)/\sin I$. (See also Fig. 1a).

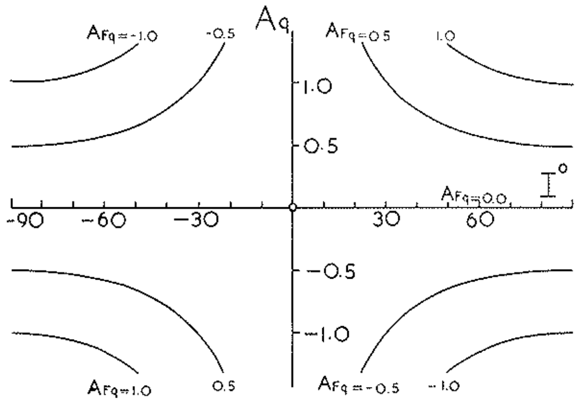

Fig. 4. Graphical expression of $A_q = AF_q/\sin I$. (See also Fig. 2a). Note the curves demonstrate also the graphical expression of the similar relationships $B_r = AF_q/\sin I$.

### *3.2 Use of horizontal fluctuations recorded at a distant observatory*

As mentioned in the introduction, the circumstances under which total fluctuations $f(t)$ have been recorded usually mean that records will not have been made of the horizontal components $h(t)$ and $d(t)$. However, such horizontal fluctuation records may be available for some distant magnetic observatory, perhaps hundreds of kilometres away. Thus a fit of observed data to Eq. (3) may be sought, using horizontal fluctuations from a distant observatory. In any such determination systematic (as well as random) differences between the local horizontal fluctuations and those at the distant observatory must be expected, and allowed for in the ultimate interpretation of any results obtained. In particular, arrows thus obtained will not be directly equivalent to regular induction arrows.

The examples which follow in this paper thus investigate the use of field-station total-field fluctuation data, and observatory horizontal-component fluctuation data. The data have been provided by the Australian Bureau of Mineral Resources.

## 4. Examples

This section presents three examples of transfer functions, determined using recordings from total-field magnetometers and from a standard magnetic observatory. The total-field magnetometers were instruments run at field stations to monitor magnetic activity for field survey parties. The examples have been chosen upon the basis of availability of records, and for varying distances of the field site from the 'base' observatory. The sites of the three examples are shown in Fig. 5.

Fig. 5. Sites of the examples in this paper. T denotes Toolangi, A denotes Albury, and L denotes Leigh Creek.

Depending on the magnetic activity recorded, either individual substorm events or full magnetic storms have been analysed. The records have been digitised at 5 min intervals, a spacing largely dictated by convenience in reading the observatory records. After digitisation, basic Fourier transformation of the records has been carried out according to the definition:

$$F(\omega)=\int_{-\infty}^{\infty} f(t)\ \exp(-i\omega t)\ dt$$

### 4.1 *Example 1: Toolangi, Victoria*

In this example the simultaneous total-field fluctuation recordings and the standard observatory recordings have been made at the same geographical location. The total-field recordings are from a proton-precession instrument which was tested during 1974 at the Toolangi Geomagnetic Observatory. Individual substorm events of typical duration two hours have been analysed.

After digitisation and Fourier transformation a best fit of some nine events to Eq. (3) has been made following a standard 'least-squares' procedure. The events have been 'band-width' averaged by including Fourier transform values for all three periods of 30, 40 and 60 min. Before their inclusion in Eq. (3) the events have first been weighted to have equal amplitude in the horizontal component of their fluctuations. That is, for each frequency, corresponding values of $F(\omega)$, $D(\omega)$, $H(\omega)$ have all been divided by the factor $(H_r^2+H_q^2+D_r^2+D_q^2)^{1/2}$ where $H_r$ and $H_q$ are the real and quadrature parts of $H(\omega)$, and $D_r$ and $D_q$ the real and quadrature parts of $D(\omega)$. The machine program used to solve Eq. (3) in its overdetermined form has also provided estimates of standard deviation errors for $A$ and $B$.

The results thus obtained for the Toolangi site, for a fluctuation band centred on period 40 min, are as follows:

$$A_{F_r}=0.23\pm0.04$$
$$A_{F_q}=0.02\pm0.04$$
$$B_{F_r}=0.03\pm0.06$$
$$B_{F_q}=0.03\pm0.06$$

where all errors quoted in the present paper are standard deviation estimates. Taking the magnetic inclination $I$ of Toolangi as $-68.5°$, application of Eqs. (5a) to (5d) produces values for $A$ and $B$ as

$$A_r=0.15\pm0.04$$
$$A_q=-0.02\pm0.04$$
$$B_r=-0.03\pm0.06$$
$$B_q=-0.03\pm0.06.$$

These results are consistent with PARKINSON'S (1962) original determination for Toolangi of a southward pointing arrow, corresponding to a tilt angle of the Parkinson plane of 10°(which in terms of the present paper would be an arrow of length 0.18 units). The result for $A_r$ is also consistent with the determination of induction arrows for south-east Australia given by BENNETT (1972) and LILLEY and BENNETT (1972), who for Toolangi quote a southward-pointing real arrow of length 0.2 units. The small value for the quadrature component $A_q$ at Toolangi determined above differs from the value ($0.18\pm0.03$ units) obtained for the longer period of 80 mins by LILLEY and BENNETT (1972). This discrepancy however is not thought to indicate a serious limitation of the present method.

### 4.2 *Example 2: Albury, New South Wales*

Albury is situated some 200 km northeast from the Toolangi Observatory, and example 2 is based upon recordings made by a total-field fluxgate instrument which recorded there during an aeromagnetic survey in 1972.

Records of a single magnetic storm of thirteen hours duration have been smoothed and digitised from total-field records from Albury, and from simultaneous horizontal fluctuation records at the Toolangi Observatory. Upon Fourier transformation independent estimates of the fluctuation amplitudes and phases should exist at fifteen periods in the band range between 30 min and 60 min. These data have been used as an 'over-determined' matrix to generate a best-fit to Eq. (3) as in example 1 above.

Results thus obtained for Albury are

$$
\begin{aligned}
A_{F_r} &= 0.15 \pm 0.05\\
A_{F_q} &= 0.01 \pm 0.05\\
B_{F_r} &= 0.14 \pm 0.04\\
B_{F_q} &= -0.03 \pm 0.04.
\end{aligned}
$$

For a magnetic inclination at Albury of $-67.5°$, application of Eqs. (5a) to (5d) produces values for $A$ and $B$ of

$$
\begin{aligned}
A_r &= 0.25 \pm 0.05\\
A_q &= -0.01 \pm 0.05\\
B_r &= -0.15 \pm 0.04\\
B_q &= 0.03 \pm 0.04.
\end{aligned}
$$

To the knowledge of the present authors no other fluctuation data have been recorded at Albury with which these present results can be compared; however Albury lies between two stations (Corowa and Corryong) of LILLEY and BENNETT (1972), and the present results are consistent with those of the 1972 paper. (In particular, the 1972 paper gives results for $A_r$ of 0.24 and 0.29, and for $B_r$ of $-0.06$ and $-0.15$, for Corowa and Corryong, respectively).

### 4.3 *Example 3: Leigh Creek, South Australia*

Leigh Creek is situated some 1000 km northwest from the Toolangi Observatory, and example 3 is based upon recordings made by a total-field fluxgate instrument which recorded at Leigh Creek during an aeromagnetic survey in the area in 1966. Records of the simultaneous horizontal field fluctuations are taken from the Toolangi Observatory. The validity of the method in this example relies on the existence of some systematic relationship (on average) between the horizontal field fluctuations at the remote field site and at the observatory. To sample as wide a range of magnetic activity as the data will allow some twenty different periods of magnetic activity, mostly substorms, have been digitised from the Leigh Creek records and from the simultaneous Toolangi records.

Fitting these events to Eq. (3) produces, for a fluctuation period of 50 min, values for $A_F$ and $B_F$ of :

$$
\begin{aligned}
A_{F_r} &= 0.25 \pm 0.03\\
A_{F_q} &= 0.01 \pm 0.03\\
B_{F_r} &= 0.10 \pm 0.02\\
B_{F_q} &= 0.0 \pm 0.02.
\end{aligned}
$$

Taking an inclination value for Leigh Creek of $-62.5°$ thus produces values for $A$ and $B$ of:

$$A_r = 0.24 \pm 0.03$$
$$A_q = -0.01 \pm 0.03$$
$$B_r = -0.11 \pm 0.02$$
$$B_q = 0.0 \pm 0.02.$$

The site of Leigh Creek falls within the area covered by the (subsequent) 1970 magnetometer array study of GOUGH *et al.* (1974). Because of possible differences in the horizontal fluctuations between Toolangi and Leigh Creek, the $A$ and $B$ transfer function values determined here are not directly equivalent to those of GOUGH *et al.* (1974). However it is worth noting that an arrow of magnitude order 0.2 to 0.3 and of direction in the southeast quadrant, as indicated in the present case, is consistent with the general pattern of the GOUGH *et al.* (1974) results. (It may also be appropriate to note here that the arrow scale omitted from Fig. 8 of the GOUGH *et al.*, 1974 paper is understood to be 3 cm to 1 arrow unit for both real and quadrature arrows, as printed).

## 5. Conclusions

It appears practical to use total-field magnetic fluctuation data, which may have been recorded by magnetic survey parties, to determine reconnaissance information for geomagnetic induction studies. Simultaneous observatory records of fluctuations in the horizontal field, though recorded some distance away, may be suitable 'base-station' data for the computation of transfer functions. In such cases it must be remembered that any 'arrows' thus formed are not directly equivalent to traditional induction arrows, but may be biassed by any nonequivalence existing between the horizontal fluctuations at the field site and at the distant observatory. The situation is similar (though on a larger scale) to the use of remote magnetic recordings to reduce local telluric observations as in the telluric-magnetotelluric method of HERMANCE and THAYER (1975; and see a recent discussion by STODT *et al.*, 1981).

As developed in the present paper, the method is not applicable to total-field fluctuation data recorded at sites of shallow magnetic inclination, for at such sites no significant part of the vertical component of fluctuations is recorded in the 'total field' component.

The development of proton-precession magnetometers has made practical the recording of total-field fluctuations in a buoy at sea, and various authors (such as HILL and MASON, 1962, RODEN and MASON, 1964, and SRIVASTAVA and FOLINSBEE, 1975) have compared oceanic buoy observations with observations made on the nearest land. Such buoy observations would presumably also be amenable to analysis by the methods of the present paper, using land observatory data. The phenomena of oceanic induction and the geomagnetic coast effect might thus be further clarified, with surface observations supplementing sea-floor measurements of magnetic fluctuations, as achieved by FILLOUX (1973 and later papers), AULD *et al.* (1979), WHITE (1979), and SEGAWA *et al.* (1982).

The use of total-field fluctuation data as in the present paper may also suggest a wider application of total-field instruments to array studies of geomagnetic induction on land, which are generally pursued with three-component recording magnetometers. Arrays of total-field instruments have been used particularly in tectonomagnetic studies (a recent report on this work is that of Johnston *et al.*, 1983), and the high resolution possible with such instruments (e.g. Ware, 1979) may prove to be of benefit in general induction work. Possibly a whole array of total-field recording instruments could be employed in general induction studies, in conjunction with a smaller number of three-component recording instruments operating at particular sites.

Such total-field instruments would not produce the horizontal spatial gradient information applicable to studies of horizontal layering (as in Woods, 1979, Lilley *et al.*, 1981, and Jones, 1982) but would contribute to the basic task of mapping the positions of continental electrical conductivity anomalies. The calculation of total-field fluctuations using observed three-component data (as in Lilley, 1982) demonstrates that in the region of a conductivity anomaly, such total-field fluctuations may detect and characterise the conductivity anomaly quite clearly.

The cooperation of the airborne and observatory sections of the Bureau of Mineral Resources in making available the data upon which the examples of this paper are based has already been mentioned and is greatly appreciated. Dr. R.M. Clark gave valuable advice on the matter of solving Eq. (3) as an overdetermined system.

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