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On Paleomagnetic Data and Dynamo Theory

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Sufficient paleomagnetic data of wide distribution now exist for their use in the determination of terms of higher degree than dipole in the past geomagnetic field. For times older than 5 Myr it is necessary to carry out global reconstructions based upon information from sea-floor spreading. Best fits of second and third degree zonal terms (quadrupole and octupole), normalised by the dipole term, may then be obtained for spherical harmonic descriptions of the past geomagnetic field.

Such paleomagnetic results are examined for their significance for dynamo theory, using the analysis developed in terms of spherical harmonics by BULLARD and GELLMAN (1954), and applied to mean field electrodynamics by ROBERTS and STIX (1972). The paleomagnetic results provide data for ratios of the values at the core-mantle boundary of the radial functions which describe the zonal dipole, quadrupole and octupole geomagnetic components. The variations with time of these ratios over the last 200 Myr indicate an evolution of the geodynamo, and show that the octupole geomagnetic component may be linked to the dipole geomagnetic component in a more steady manner than is the quadrupole geomagnetic component. One possible interpretation of such an observation is that it indicates different patterns of behaviour between quadrupole and dipole-octupole fluid flow components in the earth's core.

1. Introduction

In the continuing search to determine the dynamo process in the earth's core which gives rise to the geomagnetic field, the observed behaviour of this field over time provides most valuable information. On the historical time-scale of the last several hundred years, the geomagnetic field has been recorded by observatories (ALLDREDGE, 1967; PARKINSON, 1983). On a time-scale of thousand of years the geomagnetic field has been recorded in sources such as lake sediments (CREER *et al.*, 1983); and on a time-scale of tens of thousands of years and longer, by traditional paleomagnetic data (MERRILL and MCELHINNY, 1983). Amongst the many contributions of paleomagnetic data to the earth sciences, the most important has perhaps been the demonstration of continental drift (IRVING, 1964; MCELINNY, 1973), which depended fundamentally upon the dipole nature of the geomagnetic field. The history

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of reversals of this dipole field (JACOBS, 1984) has in turn made possible a whole range of contributions, such as the precise quantitative explanation of the magnetic stripe patterns of the oceans in terms of sea-floor spreading.

The present paper discusses results which seek information from paleomagnetic data regarding the past geomagnetic field beyond the dipole term. Such information is of a time-averaged and space-averaged nature. That is, the results are averaged over certain spans of geological time, due firstly to the imprecise dating of many paleomagnetic results, and secondly to the necessity of spanning a finite time to enable sufficient numbers of paleomagnetic results to be included so that the results obtained from them are sufficiently accurate. The average over space is for a similar reason, to obtain data in numbers sufficient to give an accurate result. While such averaging of the paleomagnetic data used over time and space has been inevitable, there may be some advantages in the process, in that one may hope then to be examining long-lasting and basic aspects of the geodynamo rather than perhaps shorter-term and transient effects.

The space-averaging of the data referred to, in which paleomagnetic results are grouped into strips of geographic latitude, can be carried out only when global reconstructions are made of continents in their correct positions of latitude for the geologic time appropriate to the paleomagnetic result. The work referred to below of LEE (1983) and LEE *et al.* (1986) achieved such reconstructions by using sea-floor spreading data (JURDY, 1974; and private communications of 1979) to move the other continents into their correct positions relative to Africa.

The purpose of the present paper is to explore some of the possibilities for dynamo theory which are suggested by the recent paleomagnetic results. The quadrupole and octupole information discussed, smoothed over time, provides a record of dynamo history which though not complete may nevertheless be of value in exhibiting characteristics which are diagnostic of certain dynamo behaviour.

The paleomagnetic results are in spherical harmonics, and lead naturally to the spherical harmonic analysis of dynamo theory by BULLARD and GELLMAN (1954). Some relevant aspects of dynamo theory are first reviewed, both for Bullard-Gellman analysis and mean-field electrodynamics.

2. Some Points on Dynamo Analysis

2.1 *Bullard-Gellman formalism*

The essence of the BULLARD and GELLMAN (1954) analysis of homogeneous dynamo action in spherical co-ordinates (r, θ, ϕ) is first to recognize that a divergenceless vector field (be it of magnetic flux or fluid flow) may be regarded as consisting of poloidal (denoted $\underset{\sim}{S}$) and toroidal (denoted $\underset{\sim}{T}$) vector components, each derived in a standard way from some scalar function of radius only. A toroidal vector $\underset{\sim}{T}$ has components

$$T_r = 0, \quad T_\theta = \frac{T(r)}{r\sin\theta}\frac{\partial Y}{\partial\phi}, \quad T_\phi = -\frac{T(r)}{r}\frac{\partial Y}{\partial\theta}$$

and a polodial vector $\underset{\sim}{S}$ has components

$$S_r = \frac{n(n+1)}{r^2} S(r) Y, \quad S_\theta = \frac{1}{r} \frac{\partial S(r)}{\partial r} \frac{\partial Y}{\partial \theta}, \quad S_\phi = \frac{1}{r \sin\theta} \frac{\partial S(r)}{\partial r} \frac{\partial Y}{\partial \phi}$$

where Y is the surface harmonic $P_n^m (\cos \theta) {}^{\sin}_{\cos} \; m\phi$ of degree n and order m. $T(r)$ and $S(r)$ are the scalar functions from which the vector components are derived, and may be more fully written as $T_n^m(r)$ and $S_n^m(r)$.

There are several important boundary conditions at the surface of the sphere in which the dynamo action takes place. Outside this sphere it must be possible to express the magnetic field as the gradient of a scalar potential only. BULLARD and GELLMAN (1954) express this external field as

$$\underset{\sim}{B}_e = \sum_\beta c_\beta \text{ grad } (Y_\beta / r^{\beta+1}) \tag{1}$$

where β denotes a particular surface harmonic (and where appropriate the degree of that harmonic) and c_β denotes the coefficient for that harmonic (generally c_β may be a function of time). The boundary conditions which then apply at the boundary of the sphere ($r=1$) are:

$$T_\beta = 0$$

$$\frac{\mathrm{d}S_\beta}{\mathrm{d}r} + \beta \; S_\beta = 0$$

and

$$c_\beta = -\beta S_\beta \, .$$

Dynamo action is portrayed as some component of the fluid flow (say $\underset{\sim}{s}_\alpha$ or $\underset{\sim}{t}_\alpha$, using now lower case letters to distinguish fluid flow components from magnetic field components) acting on some existing component of the magnetic field (say $\underset{\sim}{S}_\beta$ or $\underset{\sim}{T}_\beta$) to produce, by electromagnetic induction, some (generally) different component of the magnetic field (say $\underset{\sim}{S}_\gamma$ or $\underset{\sim}{T}_\gamma$). Such an interaction is then denoted as $(\underset{\sim}{s}_\alpha \underset{\sim}{S}_\beta \underset{\sim}{T}_\gamma)$ or $(\underset{\sim}{t}_\alpha \underset{\sim}{S}_\beta \underset{\sim}{T}_\gamma)$ etc.

An important part of the Bullard-Gellman analysis is the observation that the properties of spherical harmonics are such that large numbers of interactions have null effect. That is, there are many interactions of form say $(\underset{\sim}{s}_\alpha \underset{\sim}{S}_\beta \underset{\sim}{S}_\gamma)$ for which no production is possible of $\underset{\sim}{S}_\gamma$ by the action of $\underset{\sim}{s}_\alpha$ on $\underset{\sim}{S}_\beta$. The interactions thus forbidden are specified by the Bullard and Gellman selection rules, and this phenomenon of null or forbidden interactions greatly simplifies the mathematical representation of dynamo action. The whole class of $(\underset{\sim}{t}_\alpha \underset{\sim}{T}_\beta \underset{\sim}{S}_\gamma)$ interactions are null.

One consequence of the forbidden interactions is the recognition of the possibility of separate and independent chains of dynamo action since, for some fluid flow patterns, some groups of magnetic field components will never interact with others. For example, Table 1 shows the interactions possible directly between the

Table 1. The possible interactions allowed directly between the zonal magnetic field components $\underset{\sim}{S}_1$, $\underset{\sim}{S}_2$ and $\underset{\sim}{S}_3$, according to the selection rules of BULLARD and GELLMAN (1954).

A. *Interactions of type* $(\underset{\sim}{s}_\alpha \underset{\sim}{S}_\beta \underset{\sim}{S}_\gamma)$		allowed
(i)	$(\underset{\sim}{s}_\alpha \underset{\sim}{S}_1 \underset{\sim}{S}_2)$	$(\underset{\sim}{s}_1 \underset{\sim}{S}_1 \underset{\sim}{S}_2)$
		$(\underset{\sim}{s}_3 \underset{\sim}{S}_1 \underset{\sim}{S}_2)$
(ii)	$(\underset{\sim}{s}_\alpha \underset{\sim}{S}_1 \underset{\sim}{S}_3)$	$(\underset{\sim}{s}_2 \underset{\sim}{S}_1 \underset{\sim}{S}_3)$
		$(\underset{\sim}{s}_4 \underset{\sim}{S}_1 \underset{\sim}{S}_3)$
(iii)	$(\underset{\sim}{s}_\alpha \underset{\sim}{S}_2 \underset{\sim}{S}_3)$	$(\underset{\sim}{s}_1 \underset{\sim}{S}_2 \underset{\sim}{S}_3)$
		$(\underset{\sim}{s}_3 \underset{\sim}{S}_2 \underset{\sim}{S}_3)$
B. *Interactions of type* $(\underset{\sim}{t}_\alpha \underset{\sim}{S}_\beta \underset{\sim}{S}_\gamma)$		
All prohibited.		

Note: all the allowed interactions listed are also reversible.

zonal magnetic field components $\underset{\sim}{S}_1$, $\underset{\sim}{S}_2$ and $\underset{\sim}{S}_3$, and demonstrates that the $\underset{\sim}{S}_2$ component is linked directly to the $\underset{\sim}{S}_1$ and $\underset{\sim}{S}_3$ components by $\underset{\sim}{s}_1$ and $\underset{\sim}{s}_3$ components of fluid flow, but not by an $\underset{\sim}{s}_2$ component of fluid flow. Thus, under the action of (for example) an $\underset{\sim}{s}_2$ component of flow alone, the $\underset{\sim}{S}_1$ (dipole) and the $\underset{\sim}{S}_2$ (quadrupole) components of magnetic field are independent, though the $\underset{\sim}{S}_1$ component is linked to the $\underset{\sim}{S}_3$ (octupole) component. This phenomenon raises the possibility of the existence of separate dipole and quadrupole dynamo mechanisms, which will be referred to again below.

Because data for reversed polarity are an important part of the paleomagnetic results, it is appropriate to note that for Bullard-Gellman interactions (as for dynamo theories generally) there is no necessary or preferred polarity of magnetic field. An interaction $(\underset{\sim}{s}_\alpha \underset{\sim}{S}_\beta \underset{\sim}{S}_\gamma)$ will have the same strength as $(\underset{\sim}{s}_\alpha \underset{\sim}{S}_\delta \underset{\sim}{S}_\varepsilon)$ where

$$\underset{\sim}{S}_\delta = -\underset{\sim}{S}_\beta$$

and

$$\underset{\sim}{S}_\varepsilon = -\underset{\sim}{S}_\gamma$$

that is, for magnetic components both of reversed polarity.

The same rule does not hold for reversing the velocity field; so that while the magnetic field reversals may be accepted with equanimity, no such freedom is allowed with velocity field reversals.

2.2 *Mean field electrodynamics*

Bullard-Gellman analysis is most effective in describing simple large-scale dynamo flow, in which both magnetic field and fluid velocity can be expressed in terms of spherical harmonics of low degree. In dynamo generation by mean field electrodynamics, small scale turbulent fluctuations of magnetic field and fluid flow occur about large scale 'mean-field' values.

If the geodynamo is powered by some mechanism of mean field electrodynamics, paleomagnetic results should give information on the magnetic field mean value. Here also Bullard-Gellman formalism has application. ROBERTS and STIX (1972) show that the dynamo source term (the α-effect) for mean field electrodynamics can be expanded to give similar interactions to those described in Section 2.1 above.

ROBERTS and STIX (1972) derive one result of special interest to the present paper. They choose, in a sphere, a particular velocity mean-field $\underset{\sim}{u}$ which has the symmetry of even parity, with respect to the equatorial plane,

$$\underset{\sim}{u} = \underset{\sim}{t}_1 + \underset{\sim}{s}_2 + \underset{\sim}{t}_3 + \dots + \underset{\sim}{s}_{2n} + \underset{\sim}{t}_{2n+1} + \dots$$

and an α-effect of odd parity, which is physically reasonable for a source arising from Coriolis force in a rotating sphere. Then the magnetic field solutions for the mean-field dynamo action divide into two families: a 'dipole family',

$$\underset{\sim}{S} = \underset{\sim}{S}^m_{m+1} + \underset{\sim}{S}^m_{m+3} + \dots + \underset{\sim}{S}^m_{m+2k+1} + \dots$$

$$\underset{\sim}{T} = \underset{\sim}{T}^m_m + \underset{\sim}{T}^m_{m+2} + \dots + \underset{\sim}{T}^m_{m+2k} + \dots$$

and a 'quadrupole family',

$$\underset{\sim}{S} = \underset{\sim}{S}^m_m + \underset{\sim}{S}^m_{m+2} + \dots + \underset{\sim}{S}^m_{m+2k} + \dots$$

$$\underset{\sim}{T} = \underset{\sim}{T}^m_{m+1} + \underset{\sim}{T}^m_{m+3} + \dots + \underset{\sim}{T}^m_{m+2k+1} + \dots$$

for each m.

The separate dipole family and quadrupole family solutions allow the possibility of separate and independent dipole and quadrupole magnetic field generation, analogous to that raised above for a Bullard-Gellman flow which keeps separate the $\underset{\sim}{S}_1$ and $\underset{\sim}{S}_2$ magnetic components. This possibility is an important question in the dynamo generation of the geomagnetic field, and one which paleomagnetic results can address.

3. The $G2$ and $G3$ Coefficients

Because paleomagnetic intensity data are fewer in number and often less well-determined than paleo-inclination data, the analysis procedures followed to give the paleomagnetic results are usually based on paleo-inclination data only. Therefore, determinations of higher-degree spherical harmonic terms are made relative to the strength of the dipole term, and the absolute strengths of all terms are in fact unknown. These normalised terms, $G2$ and $G3$, are defined as follows.

In a traditional spherical harmonic analysis of the earth's magnetic field (as first carried out by Gauss in 1839) the magnetic scalar potential V at the surface is given by

$$V = a \sum_{n=1}^{\infty} (a/r)^{n+1} \sum_{m=0}^{n} (g_n^m \cos m\phi + h_n^m \sin m\phi) \; P_n^m(\cos\theta) \qquad (2)$$

where a denotes the earth's mean radius, r denotes the distance of an observation site from the geocentre, θ and ϕ denote the geographic colatitude and longitude respectively of an observation site, and g_n^m and h_n^m denote spherical harmonic coefficients (Gauss coefficients) of degree n and order m. The function P_n^m $(\cos\theta)$ denotes a partially normalised Legendre polynomial (or Schmidt function); and commonly the following are used:

$$P_1^0 (\cos\theta) = \cos\theta,$$
$$P_2^0 (\cos\theta) = 0.5\,(3\cos^2\theta - 1),$$
$$P_3^0 (\cos\theta) = 0.5\,(5\cos^3 - 3\cos\theta).$$

For the azimuthal symmetry which applies when data are taken around strips of geographic latitude, m=0 and zonal terms g_1^0, g_2^0, and g_3^0 etc. arise in the analysis of the paleo-inclination data. The g_1^0 term represents the dipole component, so because the other terms are normalised with respect to this term, the parameters actually determined are $G2$ and $G3$, where

$$G2 = g_2^0/g_1^0$$

and

$$G3 = g_3^0/g_1^0.$$

4. Principles of Interpretation

It is first relevant to note that the $G2$ and $G3$ Gauss coefficients are connected with the scalar radial generating functions of Bullard-Gellman analysis, through the boundary conditions described in Section 2.1 above. The condition important here is that

$$c_\beta = -\beta S_\beta$$

and Equation (1) must be resolved with Eq. (2) for expressing the same field external to the dynamo.

Inspection of Eq. (1) and (2) shows that resolution is achieved if the radial distance in Eq. (2) is normalised relative to the core radius (the assumed surface of the sphere containing the dynamo action) instead of to the radius of the earth. This different normalisation will then have the effect of requiring that all the coefficents g_n^m and h_n^m are multiplied by a factor $(a/A)^{n+2}$ where A is the core radius. Taking a=6371 km and A=3470 km (GARLAND, 1979) gives (a/A)=1.84.

Denote the values of $G2$ and $G3$ re-normalised to the core-mantle boundary as $G2^*$ and $G3^*$. Then

$$G2^* = 1.84\ G2$$

and

$$\begin{aligned} G3^* &= (1.84)^2\ G3 \\ &= 3.39\ G3. \end{aligned}$$

For coefficients c_β as defined above

$$\begin{aligned} c_1^0 &= -S_1^0(1) \\ c_2^0 &= -2S_2^0(1) \\ c_3^0 &= -3S_3^0(1) \end{aligned}$$

(from now on zero superscripts for zero order will be omitted for simplicity) so that

$$\begin{aligned} G2^* &= c_2/c_1 \\ &= 2S_2(1)/S_1(1) \end{aligned}$$

and

$$\begin{aligned} G3^* &= c_3/c_1 \\ &= 3S_3(1)/S_1(1). \end{aligned}$$

Thus the numerical values of $G2$ and $G3$ obtained from the paleomagnetic data can be related to values, at the core mantle boundary, of radial functions for the magnetic field.

In interpreting such $G2$ and $G3$ values, it is therefore possible to think in terms of ratios of values at the core-mantle boundary, either of major field or 'mean field' magnetic components. Assuming such boundary values are indicative of the main dynamo process, extra information of a diagnostic kind may then come from a scrutiny of the behaviour of these values over geologic time.

An initial and important point to check is the behaviour of the geomagnetic field upon reversing. It is evident from the definitions of $G2$ and $G3$ above that if a geomagnetic reversal is complete and involves all geomagnetic components, then $G2$ and $G3$ values will be unchanged through reversals, as all of g_1, g_2 and g_3 will change sign. If however a reversal occurred during which the dipole term changed sign but the quadrupole term remained unchanged, then the sign of $G2$ would change, and indeed a mean $G2$ value formed from both normal and reversed data would, in a simple case, be zero. Consistency of sign in $G2$, and in $G3$, is thus an important indicator of complete geomagnetic reversals, in which all components change together.

5. Speculative Discussion of Some Results for the Time-Dependence of $G2$ and $G3$

Attention will now focus on the $G2$ and $G3$ results of LEE *et al.* (1986), shown in Fig. 1. The first remark to make is that the results shown are for normal and reversed

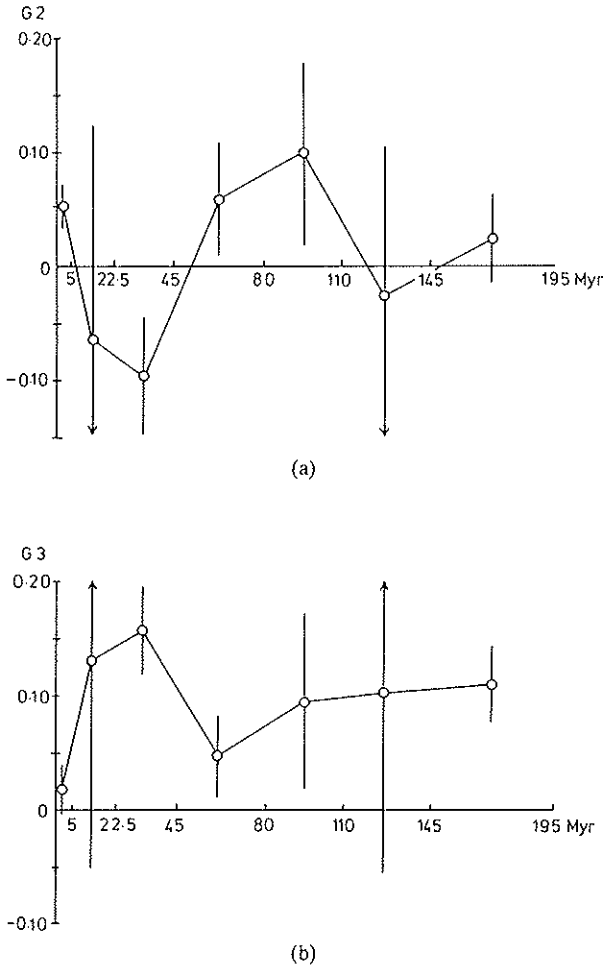

Fig. 1. Behaviour of the geomagnetic field over the last 195 Myr from LEE *et al.* (1986). (a) The $G2$ coefficient. (b) The $G3$ coefficient. The vertical bars represent 95% confidence intervals for the plotted points. The points are plotted in the middle of the time spans which they represent. These time spans are: present–5, 5–22.5, 22.5–45, 45–80, 80–110, 110–145, and 145–195 Myr.

polarity paleomagnetic data combined, and the work of LEE (1983) found similar results for normal and reversed polarity data taken separately. As discussed in the previous section, such consistency must mean that g_2^0 and g_3^0 have generally reversed together with g_1^0, indicating that on the time-scales of geomagnetic reversals all three coefficients g_1, g_2 and g_3 are linked. The present speculative interpretation of this result is that such reversals are essentially magnetic phenomena, with magnetic field reversing but (mean) velocity field remaining unchanged.

However on a longer time-scale, a different phenomenon may be evident in Fig. 1. The $G3$ value is consistently of the same sign (positive) indicating a consistent g_3 to g_1 ratio; whereas $G2$ changes sign over the 200 Myr covered by the span of the figure, indicating that the g_2 to g_1 ratio changes sign.

One interpretation which can be made of this behaviour is that on the time-scale of order 50 Myr, Figure 1(a) indicates that the relationship of the quadrupole magnetic field to the dipole magnetic field may change, while Figure 1(b) indicates that the relationship of the octupole magnetic field to the dipole magnetic field remains more fixed. Such behaviour may be accounted for as a velocity phenomenon, in which a quadrupole component of flow linking the dipole and octupole magnetic field components remains steady, while a dipole-octupole component of flow linking the dipole and quadrupole magnetic field components changes over time-scales of 50 Myr. OLSON (1981) notes 10 Myr or greater as a time-scale over which dipole moment variability may reflect important trends in the core's energy balance.

6. Conclusions

Due to the complicated nature of dynamo theory, observations of the actual geomagnetic field over long periods of time have much value in restricting dynamo models which may be set up for the core of the earth. In the present paper some recent paleomagnetic results have been interpreted with this intention.

The results of the exercise are both qualitative and quantitative. Qualitative, in the consistency or inconsistency with time of the signs of the $G2$ and $G3$ parameters determined, and quantitative in the values found for $G2$ and $G3$, which translate to ratios of the values at the core-mantle boundary for the radial functions expressing magnetic field strength in the core.

The qualitative results are interpreted as a long-time evolution in the geodynamo, and possibly indicate that in the long-term there is only a weak interdependence between the dipole and quadrupole families of magnetic field components, the generation of which may involve different quadrupole and dipole-octupole components of fluid flow.

Further refinement of the history of the geomagnetic field should allow further development of the ideas expressed in this paper, which at present must be regarded as preliminary.

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