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Electrical conductivity from Australian magnetometer arrays using spatial gradient data

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Quiet daily magnetic variations recorded by magnetometer arrays in Australia are analysed to obtain electromagnetic response parameters for two parts of the Australian continent remote from electrical conductivity anomalies. The parameters are based on measurements of vertical-field and horizontal-field spatial gradient, and three different methods are followed in their computation. The response parameters are checked for consistency with a compilation of globally-determined Earth apparent resistivities, and are then interpreted for one-dimensional conductivity structure in the two different parts of the continent. There is evidence that the rise in electrical conductivity from 10^{-1} S m^{-1} to 10^{0} S m^{-1} which occurs at a depth of order 500 km beneath central Australia may occur at a substantially shallower depth of order 230 km beneath southeast Australia.

1. Introduction

The electromagnetic response of a horizontally stratified Earth may be determined by measuring magnetic (and electric) fluctuations at its surface, and reducing these to give a variety of parameters. One such parameter, to be used in this paper, will be denoted by c, where

$$c = Z/(\partial X/\partial x + \partial Y/\partial y) \qquad (1)$$

In eq. 1, x and y are distances in the directions of geographic north and east respectively, and X, Y and Z are the scalar components of the magnetic fluctuation field in the north, east, and downwards directions. All of X, Y, Z and c are frequency-dependent and generally complex, having in-phase and quadrature parts. Early papers on determining electromagnetic response by measuring spatial derivatives are those by Berdichevsky et al. (1969), Schmucker (1970), and Kuckes (1973).

Estimation of c using eq. 1 is subject to the difficulty of determining the spatial derivatives, and indeed, especially at shorter periods, it has proved to be technically more feasible to measure the magnetotelluric ratios A/Y and B/X where A and B are the horizontal electric fields at the surface of the ground in the north and east directions respectively. Then, for true one-dimensional conductivity structure,

$$\frac{A}{Y} = -\frac{B}{X} = i\omega c$$

(Lilley 1975a), where ω is the frequency of magnetic variation being considered, and $i = (-1)^{1/2}$. Thus there are many determinations of conductivity profiles made by the magnetotelluric method, and relatively few by the spatial gradient method. However, at long periods the telluric signals A and B are weak and subject to diurnal temperature effects; also, at all periods, telluric measurements are vulnerable to local 'current channelling' phenomena (Wescott and Hessler, 1962). Hence this paper pursues the possibility of measuring the electromagnetic response of the Earth at long peri-

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ods by making estimates of c using eq. 1 above, applying this particularly to measurements of magnetic fluctuations made with arrays of instruments in Australia.

The first application of the method to Australian data was that of Lilley and Sloane (1976), whose exploratory determinations showed that the method could produce reasonable results, especially when a favourable source-field, such as the quiet daily variation above Australia, was used. Further determinations were then made using the quiet daily variation by Woods and Lilley (1979) for central Australia. This present paper now expands on both these previous works; it compares the continental induction responses with global responses, and it also fits simple layered models to the data, to demonstrate that basic differences may be present between the central and southeast blocks of the Australian continent.

The results in this paper are all based on records of magnetic quiet daily variation, and thus are in a period range bounded by the main harmonics of 24 h. These c-values are considered to be the most secure estimates for Australia so far obtained with the method, though it should be noted that Woods (1979) reports exploratory determinations made at periods both shorter and longer than those given here. It is hoped that it may be possible to confirm Woods' expanded frequency range in future work.

2. Data

The magnetic quiet daily variations analysed to give the results of this paper can be classified in three data sets, which will now be described in the historical order of their occurrence.

2.1. Data set 1

Data set 1 comes from the records of the 1971 array of 26 stations, the area of which is shown in Fig. 1. The data set comprises the three quiet days of 24–26 April 1971 as reported by Bennett and Lilley (1973). For the purposes of the present paper, only records from stations in the north and west of the 1971 array area, which are remote from coastlines, have been used. The records are digitized at half-hour intervals.

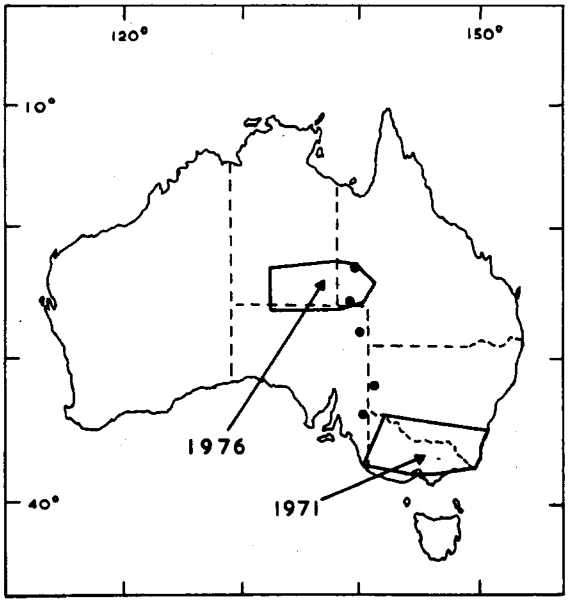

Fig. 1. Map of Australia, showing the recording areas of the 1971 and 1976 magnetometer arrays. The heavy dots mark the stations set up at the conclusion of the 1976 array and used in the "long-line" analysis in the present paper.

2.2. Data set 2

Data set 2 comprises records from the 1976 array of 21 stations, the area of which is also shown in Fig. 1. Woods and Lilley (1979) reported results from analysis of two quiet days (12 and 13 August) from the 1976 array, but note now that the number of days analysed has been expanded to seven, (not all consecutive): 26 July, 8 August, and 11–15 August 1976. The records of these days are shown in Fig. 2. The magnetogram records have been digitized at half-hour intervals.

2.3. Data set 3

Data set 3 is taken from the "long-line" of five stations which are shown as heavy dots on Fig. 1, and extend south from the eastern end of the 1976 array area. Data set 3 is inferior to data set 2 in its number of stations, but has the particular advantage of a long north–south span over which to

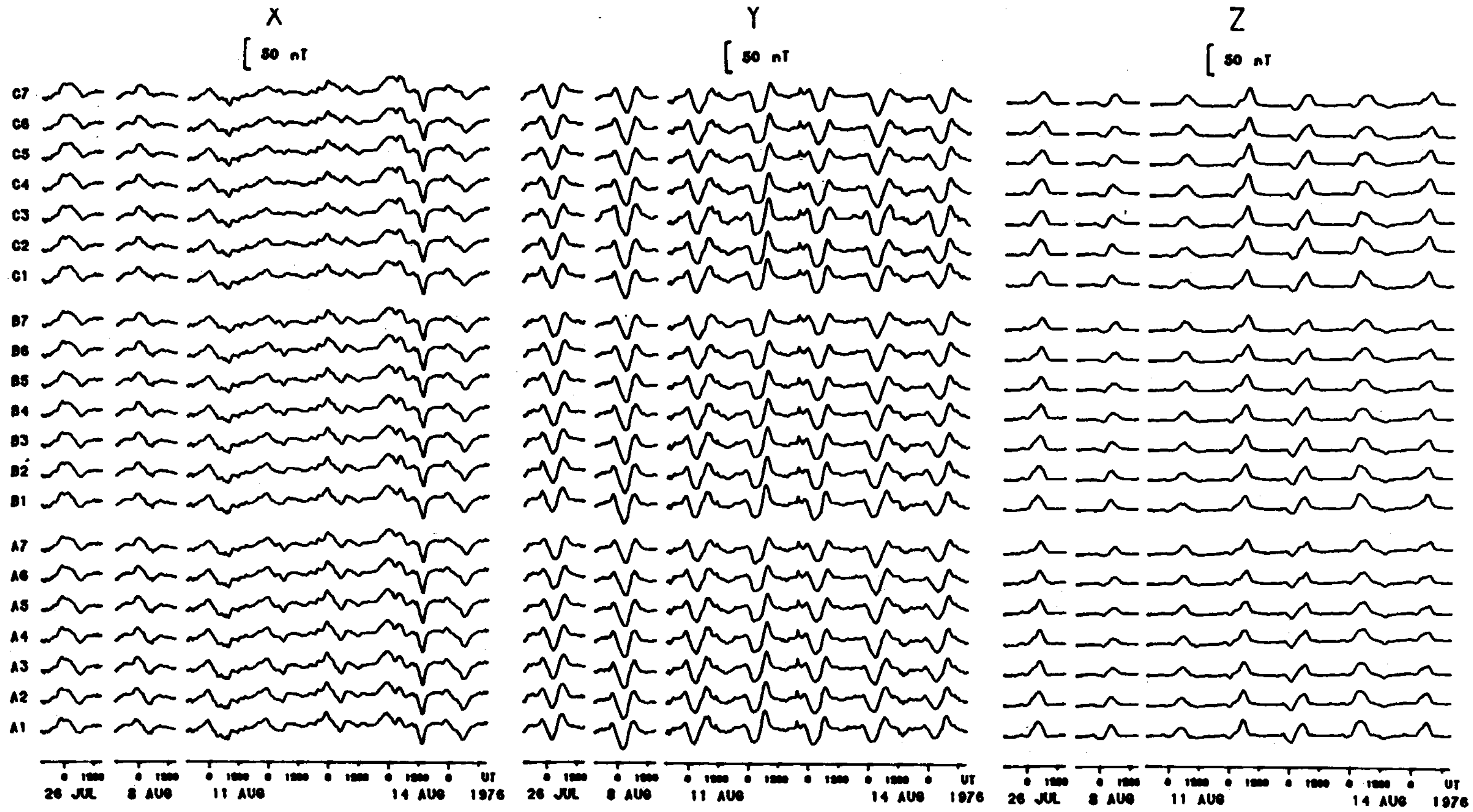

Fig. 2. Records of the magnetic variations of seven quiet days (not all consecutive) recorded by the 1976 array and referred to in this paper as "data set 2".

estimate the north–south gradient of the horizontal field (i.e. the $\partial X/\partial x$ term in eq. 1). Data set 3 comprises three quiet days, (12–14 October 1976), digitized at half-hour intervals.

3. Data reduction and methods of computation

The estimates of the c-values have been made on the basis of variometer data first reduced, in the manner described by Reitzel et al. (1970), to maps of amplitude and phase (or in some cases to maps of cosine and sine transform values). As quiet days are involved, some of the considerations of Lilley (1975b) have been taken into account. Given, then, values of amplitude and phase over an area, the computation of c-values using eq. 1 has been achieved using three distinct methods of computation, which are listed below.

3.1. Method 1

The simple "diamond" method described by Lilley and Sloane (1976). No new calculations have been made by this method: the present paper simply quotes values given in the 1976 paper.

3.2. Method 2

The polynomial surface method of Woods and Lilley (1979), described in that paper and more fully in Woods (1979). In this method polynomial surfaces are fitted to the contour maps of the cosine and sine transforms of the components of the magnetic field variations. The polynomial surfaces have a smoothing effect, and also enable the horizontal spatial derivatives to be calculated directly over the array area, thus allowing c-values to be determined over the entire area to which the surfaces have been fitted. Ranges in c-values are thus obtained, and are carried forward for interpretation.

3.3. Method 3

The "ideal phase" method. This method has been developed specifically for the analysis of quiet daily variation data, as it utilizes the fact that under ideal conditions a constant quiet daily variation source field travels across the Earth in an east to west direction at a speed corresponding to the rotation of the Earth. Then, in eq. 1, the gradient part of the $\partial Y/\partial y$ term arises from the rotation of the Earth only, and so $\partial Y/\partial y$ can be calculated given observations of Y at a single station of known latitude. In practice, values of Y from a number of stations have been used to determine a mean value of Y at some particular latitude, and then the y-derivative of this mean value has been calculated on the basis of Earth rotation.

The $\partial X/\partial x$ term in eq. 1 is calculated by first correcting all stations to some particular local solar time, taking their known geographic longitudes into account. Implicit in this correction is again the assumption that phases will be controlled by the rotation of the Earth. Once longitude has been corrected for, calculation proceeds by plotting values of X against (known) station latitude, and finding lines of best fit for both amplitude and phase. From the lines of best fit, the derivative $\partial X/\partial x$ is calculated directly. A value of Z is then determined by taking the mean value of Z over some appropriate area, and thus knowledge of the parameters necessary to calculate c using eq. 1 is complete.

4. Results obtained for c-values

The results of the c-values thus determined are shown in Fig. 3 for both central and south-east Australia. A great deal of computation is represented in the figure, and in all cases ranges of values, represented by vertical bars, are plotted instead of discrete points. The ranges of values have arisen from the procedure followed of applying the methods of computation to the data sets and extracting the results in a variety of different ways, to obtain some indication of the precision of the results. The ranges of values thus obtained have been plotted directly as the bars on Fig. 3.

Also shown on Fig. 3 are the same c-value results converted to and plotted as the parameter known as "apparent resistivity", here denoted ρ_a,

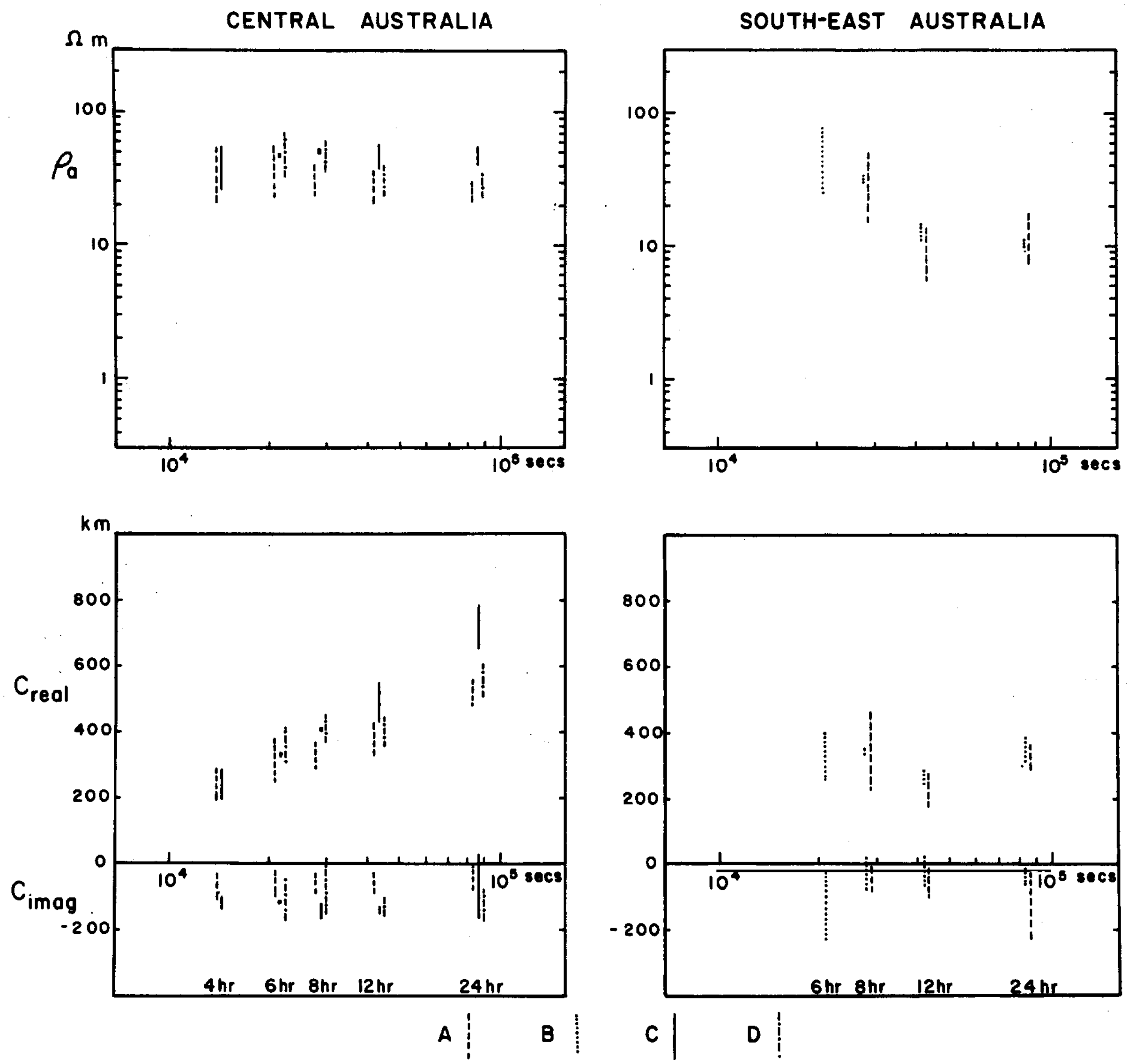

Fig. 3. Summary of results for central and southeast Australia, plotted as a function of period on the horizontal axis, presented both as c-values obtained (below) and apparent resistivity values (ρ_a, above) corresponding to the c-values. Bars are plotted, corresponding to the ranges in values obtained when using a particular computational method on a particular data set. *A* values: given by method 2 used with data set 2 for central Australia, and with data set 1 for southeast Australia. *B* values: given by method 1 and data set 1, (after Lilley and Sloane, 1976). *C* values: given by method 3 and data set 2. *D* values: given by method 3 and data set 3. Values have not been estimated at 4 h periods for southeast Australia due to the low amplitude there of the 4 h harmonic of the magnetic daily variation. Other values are omitted where a particular method has shown itself to be unsuited to a particular data set.

and related to c by

$$\rho_a = 0.4\pi\omega|c|^2$$

(Lilley and Sloane, 1976), where ρ_a is in ohm-m, ω is in rad s^{-1}, and c is in km. Apparent resistivity is a useful and common parameter by which to express the results of Earth electromagnetic induction measurements, and the apparent resistivity ranges of Fig. 3 are transferred to Fig. 4 for comparison there with a summary of results of

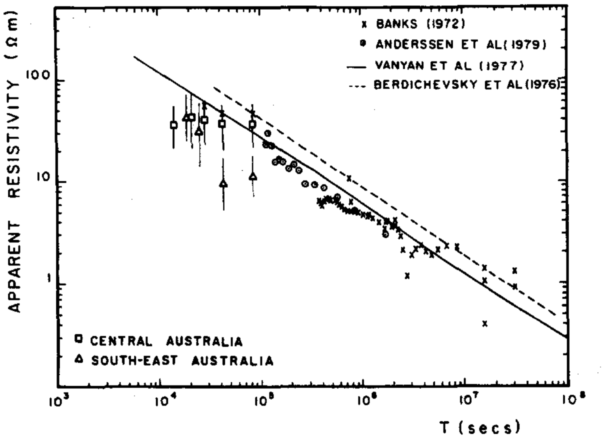

Fig. 4. The ranges of apparent resistivity determined for central and southeast Australia taken from Fig. 3 and plotted on a global compilation of apparent resistivity values.

different "global" analyses of geomagnetic induction. The summary of global results in Fig. 4 is based on compilations by Banks (1972), Berdichevsky et al. (1976), and Vanyan et al. (1977), and also incorporates recent results by Anderssen et al. (1979). The global results in Fig. 4 have generally come from analyses which have used records from stations all over the Earth, and which have assumed the Earth to have a spherically symmetric structure.

5. Interpretation and discussion

The Australian apparent resistivity results plotted on the global compilation of Fig. 4 show that the results obtained are in general agreement with the global results, which, as has been noted, have generally come from quite different analyses of quite different data sets. The figure demonstrates the usefulness of such a global compilation as a basis for comparison of new results.

Having observed that the Australian results are reasonably consistent with the global results, the next step is to examine where the results from the centre and southeast of the continent differ from each other, and to determine differences in the electrical conductivity profiles of the two continental regions which will account for the differences in the c-value results. To this end, simple model-fitting has been carried out, using a trial-and-error forward modelling process based on the well-known theory for electromagnetic induction in a layered half-space, and made efficient by modern computing equipment.

The parameters to be fitted by the models have been taken as the ranges of the c-values in Fig. 3. These ranges are shown in Fig. 5, together with the responses of models found to fit them. In model fitting the philosophy has been followed of keeping the models as simple as possible, so that no complications are introduced which are not required by the data. Thus, in Fig. 5, a two-layer model is shown for southeast Australia as one has been found compatible with the data; whereas for central Australia it has been found necessary to introduce some intermediate layers to obtain a model fit.

A well-known difficulty with forward modelling of this type is the ambiguity involved: how many other (quite different?) models would fit the data equally well? In the present case an enquiry into this question has been possible directly, within the restriction of simple models, as with the computer process, a wide range of models can be tested quickly, in an informed way. It is found that the present data set (possibly fortuitously, as noted below) tightly constrains any simple models which are to be compatible with the ranges of the c-values shown. All acceptable models share the characteristics of, for central Australia, a pronounced rise in electrical conductivity at a depth of order 500 km, and, for southeast Australia, a comparable rise at the more shallow depth of order 230 km.

It may be appropriate to add a comment on the claim that the c-values in Fig. 5 place tight constraints upon acceptable models. The situation is demonstrated in Fig. 5 by the responses of the central Australia model, which can be seen to pass along the very top of the c (imaginary) values, and rather obliquely through the c (real) values. It might be expected that the responses of an optimum model would pass more evenly near the centre points of all the c-values, but for the c-values of Fig. 5 forward modelling soon establishes that models with such responses are physically impossible: thus, wide as the bars for the c-values are on

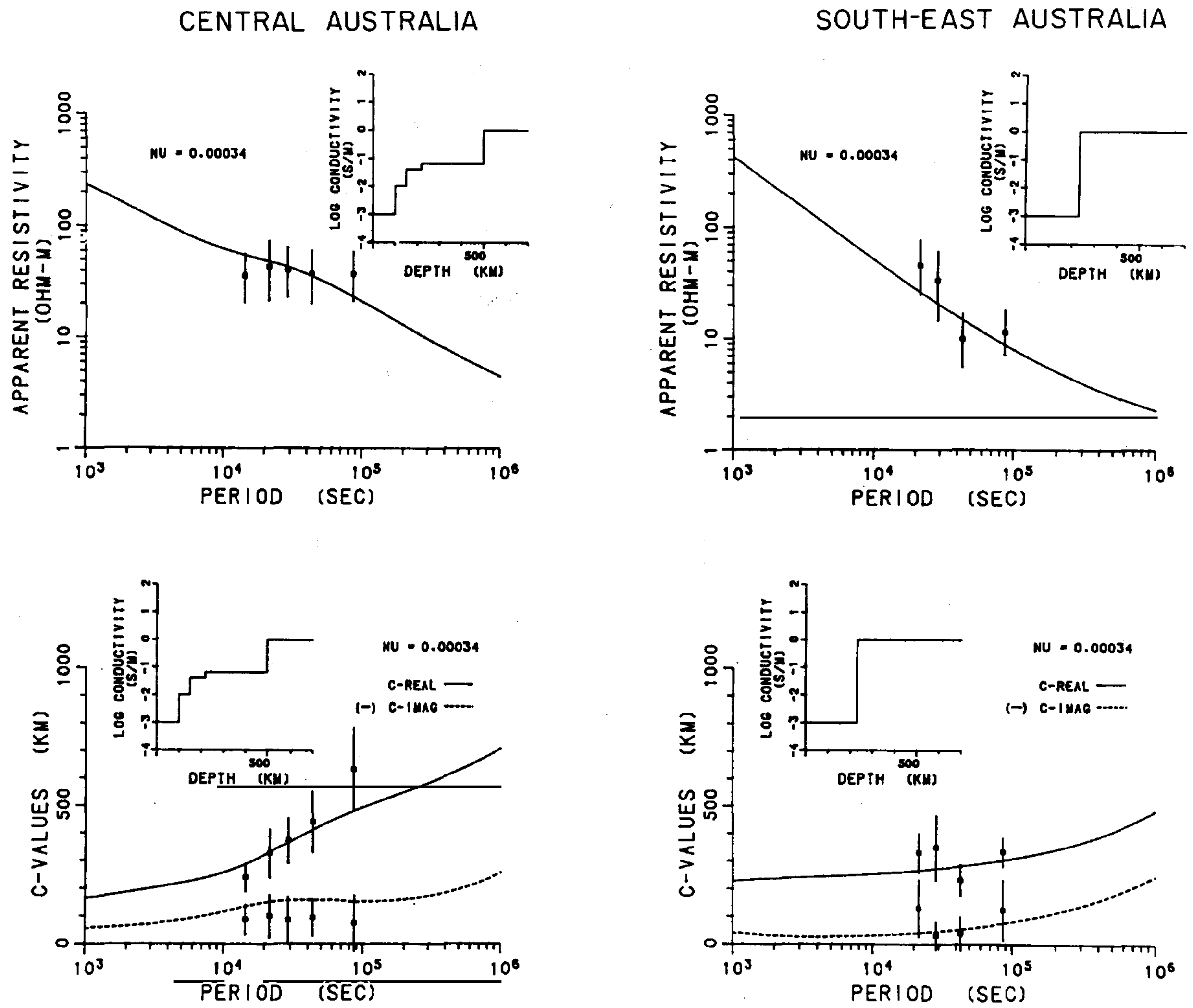

Fig. 5. Two simple models which fit the *c*-values determined for central and southeast Australia, and their theoretical response curves. Surface conducting layers (for example of thickness 1 km and conductivity 0.1 S m^{-1}) could be added and would have little effect on the fit of the response curves to the data at the long periods in question.

Fig. 5, both sets only just permit a physical model to be compatible with them all. While this situation is superficially comforting in that it reduces the ranges of acceptable models, it also emphasizes that the errors and approximations entering into the computations of the *c*-values are not yet fully understood: thus, possibly, the error bars should be even wider, and the range of acceptable models consequently more varied.

6. Conclusions

The *c*-values and models fitted to them in Fig. 5 are considered to represent an improvement on the previous results of Lilley and Sloane (1976) and Woods and Lilley (1979), and an advance is claimed for the application of the spatial gradient method to array data for Australia. Because the 1971 array shows clear coast effects on its eastern and southern edges, while the 1976 array is placed centrally in the continent, the central Australian results may be firmer than those for southeast Australia. Both interpretations would benefit greatly from *c*-values determined over a much expanded period range (as has been explored by Woods 1979); to obtain such data necessitates the use of source-fields other than that of the quiet daily variation (with its favourable horizontal non-uniformity), and so requires further development of data analysis techniques.

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