A property of the determinant of a 2x2 tensor relevant to magnetotellurics

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ABSTRACT

Where a 2x2 tensor relates two geophysical quantities which are measured in the field as time-varying vectors, its determinant is shown to be positive when, for two different events occurring at different times, a step clockwise in the direction of one vector from the first event to the second is accompanied by a step clockwise in the direction of the other vector from the first event to the second. The determinant would be negative were the azimuths of the two field vectors to "counter step", i.e. change in opposite directions. In magnetotellurics the technique known as phase tensor analysis brings a focus on the determinant signs of both MT impedance tensors and telluric distortion tensors, and the azimuth-stepping rules should apply to both categories. Any determinants which may be negative are then seen in the light that they imply a counter-stepping of the azimuths of the two relevant field vectors.

INTRODUCTION

Usually the analysis of magnetotelluric data proceeds with both the real (in-phase) and imaginary (out-of-phase) parts of the impedance tensor having positive determinant values, at all frequencies. If the tensor is strongly anisotropic such determinant values will be small, even zero within error. Errors may also cause small (but otherwise positive) determinant values to present as false negative values. While negative determinant values otherwise arise rarely, the interpreter needs to be alert to them, especially in applying the technique known as "phase-tensor analysis" (Caldwell et al., 2004).

This letter draws attention to a property of the determinant of a 2x2 tensor which expresses one vector quantity in terms of another. The sign of the determinant is shown to correspond to whether a step clockwise in the direction (or "azimuth") of the first vector, from a first event to a second event, is accompanied by a step clockwise or anti-clockwise in the azimuth of the second vector. The behavior of "same-sense stepping" is a property of positive determinants, while "counter stepping" (in opposite directions) is a property of negative determinants.

EXAMPLE OF A TENSOR RELATING TWO FIELD VECTORS

The property is derived first for the case of two telluric stations some distance apart: one termed the base station, and one termed the local station. Two such stations, juxtaposed, are shown in Figure 1 below.

Consistent with traditional telluric prospecting (Sheriff, 1991) the electric signal $\mathbf{E^m}$ at the local station may be related to the electric signal $\mathbf{E^b}$ at the base station by a tensor D:

$$\mathbf{E^m} = D.\mathbf{E^b} \tag{1}$$

Generally D is frequency-dependent and complex, and in this letter all quantities will be considered to apply to signals at a particular frequency. The real and imaginary parts of D will be denoted $\check{D}$ and $\acute{D}$. Thus $D = \check{D} + i\,\acute{D}$, giving

$$\check{\mathbf{E}}^{\mathbf{m}} + i\,\acute{\mathbf{E}}^{\mathbf{m}} = (\check{D} + i\,\acute{D}).(\check{\mathbf{E}}^{\mathbf{b}} + i\,\acute{\mathbf{E}}^{\mathbf{b}}) \tag{2}$$

Taking the elementary case for which $\mathbf{E^b}$ is linearly polarized with its imaginary part zero, and so defines the phase reference, then $\acute{\mathbf{E}}^{\mathbf{b}} = \mathbf{0}$, giving for the in-phase component of $\mathbf{E^m}$ the equation

$$\check{\mathbf{E}}^{\mathbf{m}} = \check{D}.\check{\mathbf{E}}^{\mathbf{b}} \tag{3}$$

and, for the out-of-phase component of $\mathbf{E^m}$, the equation

$$\acute{\mathbf{E}}^{\mathbf{m}} = \acute{D}.\check{\mathbf{E}}^{\mathbf{b}} \tag{4}$$

where again $\check{\mathbf{E}}^{\mathbf{m}}$ and $\acute{\mathbf{E}}^{\mathbf{m}}$ are linearly polarized.

Following geomagnetic convention, observing-axes notation is adopted of x to the north, and y to the east. If these axes are rotated, the elements of D change in a well-understood way. However the determinant of D is an invariant of axes rotation, as are $\det\check{D}$ and $\det\acute{D}$, individually.

The in-phase case

First taking the in-phase part of equation 1 as in equation 3, for an electric signal $\check{\mathbf{E}}^{\mathbf{b}}$ at the base station there will be, at the local station, an in-phase electric signal $\check{\mathbf{E}}^{\mathbf{m}}$ which is generally of a different amplitude and in a different direction. Equation 3 may be expanded

$$\begin{bmatrix} \check{E}_x^m \\ \check{E}_y^m \end{bmatrix} = \begin{bmatrix} \check{D}_{xx} & \check{D}_{xy} \\ \check{D}_{yx} & \check{D}_{yy} \end{bmatrix} . \begin{bmatrix} \check{E}_x^b \\ \check{E}_y^b \end{bmatrix} \tag{5}$$

and the determinant of $\check{D}$ then found in terms of two postulated telluric events, occurring distinct from each other at different times. In the first event, an electric signal of unit amplitude occurring with azimuth ψ_1 at the base station is accompanied by an electric signal at the local station, the in-phase part of which has amplitude E_1 and is at azimuth χ_1. The second event, occurring later, is similar but has different specifications, notably ψ_2, E_2 and χ_2. The situation is as shown in Figure 1.

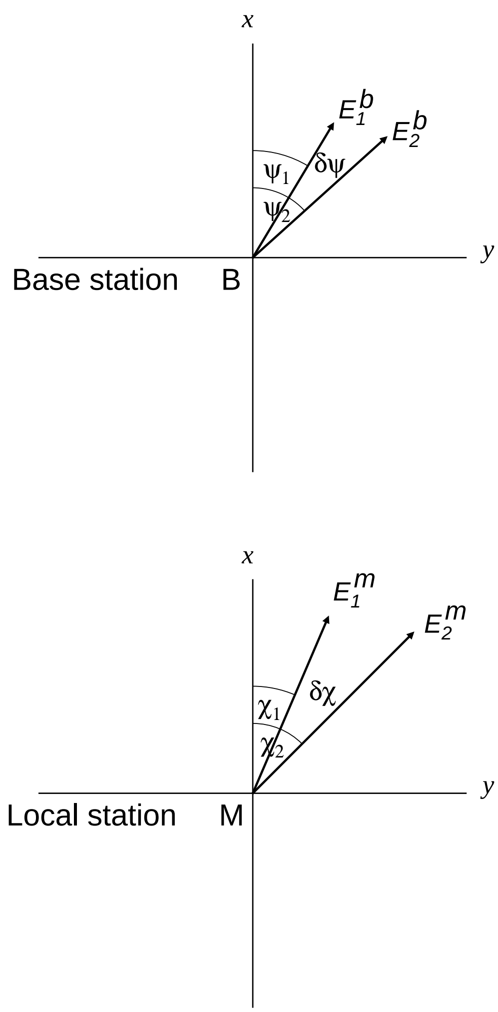

Figure 1: Map-view of the general case of two separate telluric events as described in the text. Note $\delta\psi = \psi_2 - \psi_1$ and $\delta\chi = \chi_2 - \chi_1$.

With then the simplifications that $\check{E}_1^b = \check{E}_2^b = 1$, $\check{E}_1^m = E_1$ and $\check{E}_2^m = E_2$, the first event may be described

$$\begin{bmatrix} E_1 \cos\chi_1 \\ E_1 \sin\chi_1 \end{bmatrix} = \begin{bmatrix} \check{D}_{xx} & \check{D}_{xy} \\ \check{D}_{yx} & \check{D}_{yy} \end{bmatrix} \cdot \begin{bmatrix} \cos\psi_1 \\ \sin\psi_1 \end{bmatrix} \tag{6}$$

and the second event similarly, as

$$\begin{bmatrix} E_2 \cos\chi_2 \\ E_2 \sin\chi_2 \end{bmatrix} = \begin{bmatrix} \check{D}_{xx} & \check{D}_{xy} \\ \check{D}_{yx} & \check{D}_{yy} \end{bmatrix} \cdot \begin{bmatrix} \cos\psi_2 \\ \sin\psi_2 \end{bmatrix} \tag{7}$$

Equations 6 and 7 give four equations in the unknowns $\check{D}_{xx}$, $\check{D}_{xy}$, $\check{D}_{yx}$ and $\check{D}_{yy}$, with solutions

$$\check{D}_{xx} = (E_2 \cos\chi_2 \sin\psi_1 - E_1 \cos\chi_1 \sin\psi_2)/\sin(\psi_1 - \psi_2) \tag{8}$$

$$\check{D}_{xy} = (E_2 \cos\chi_2 \cos\psi_1 - E_1 \cos\chi_1 \cos\psi_2)/\sin(\psi_2 - \psi_1) \tag{9}$$

$$\check{D}_{yx} = (E_2 \sin\chi_2 \sin\psi_1 - E_1 \sin\chi_1 \sin\psi_2)/\sin(\psi_1 - \psi_2) \tag{10}$$

$$\check{D}_{yy} = (E_2 \sin\chi_2 \cos\psi_1 - E_1 \sin\chi_1 \cos\psi_2)/\sin(\psi_2 - \psi_1) \tag{11}$$

from which the following expression for the determinant $(\check{D}_{xx}\check{D}_{yy} - \check{D}_{yx}\check{D}_{xy})$ may be derived:

$$\det \check{\boldsymbol{D}} = E_1 E_2 \frac{\sin(\chi_2 - \chi_1)}{\sin(\psi_2 - \psi_1)} \tag{12}$$

If $(\psi_2 - \psi_1)$ and $(\chi_2 - \chi_1)$ are denoted $\delta\psi$ and $\delta\chi$ respectively, then

$$\det \check{\boldsymbol{D}} = E_1 E_2 \frac{\sin\delta\chi}{\sin\delta\psi} \tag{13}$$

and the azimuths ψ_1 and ψ_2 of the two events can be chosen so that the quantity $\delta\psi$ is small. Then for finite E_1, E_2 and $\det \check{\boldsymbol{D}}$, the quantity $\delta\chi$ will also then be small. (The case of $\det \check{\boldsymbol{D}}$ not finite but zero is treated below.) For $\delta\psi$ and $\delta\chi$ both small, $\sin\delta\psi$ and $\sin\delta\chi$ may be approximated by $\delta\psi$ and $\delta\chi$ respectively, to give

$$\det \check{\boldsymbol{D}} = E_1 E_2 \frac{\delta\chi}{\delta\psi} \tag{14}$$

Because E_1 and E_2 are both positive, the sign of $\det \check{\boldsymbol{D}}$ depends only on how the azimuth of the local-station signal responds to a change in azimuth of the base-station signal. If a step clockwise at the base station is accompanied by a step clockwise at the local station, then $\delta\chi/\delta\psi$ is positive and so $\det \check{\boldsymbol{D}}$ is positive. However a step clockwise at the base station accompanied by a step anti-clockwise at the local station would give a negative $\delta\chi/\delta\psi$, and so demand a negative $\det \check{\boldsymbol{D}}$.

The case of a zero determinant

The case of $\det \check{D} = 0$ corresponds to $\check{D}$ being singular (Strang, 2005). There is then only one azimuth at which an electric signal occurs at the local station, and χ has a fixed value which does not change (though the amplitude of the electric signal changes with changing ψ). The value of $\delta\chi$ is always zero, consistent in equation 14 with $\det \check{D} = 0$.

In the singular case the $\check{\mathbf{E}}^{\mathbf{m}}$ signal may reverse direction along its azimuth, in the sense of changing its phase by half a cycle. When it does so the change in azimuth χ is $180°$. In equation 12 the term $\sin(\chi_2 - \chi_1)$ is then zero, again consistent with $\det \check{D} = 0$.

The out-of-phase case

Similar analysis of equation 4 gives $\det \acute{D} = E_1 E_2 \frac{\delta\chi}{\delta\psi}$, where now E_1, E_2, and $\frac{\delta\chi}{\delta\psi}$ refer to the signals at station M observed out-of-phase with a unit signal at station B. As before, when the electric signal at the base station steps clockwise, the out-of-phase electric signal at the local station steps clockwise for a positive $\det \acute{D}$; but steps counter-clockwise for a negative $\det \acute{D}$.

APPLICATIONS OF EQUATION (14)

The MT tensor

The result for the tensor linking two telluric stations applies equally to the magnetotelluric impedance tensor Z, where now the electric (**E**) and magnetic (**H**) signals are recorded at the same station, M. The magnetotelluric equation (Chave, 2012) is

$$\mathbf{E} = \boldsymbol{Z}.\mathbf{H} \tag{15}$$

For $\mathbf{E} = \check{\mathbf{E}} + i\,\acute{\mathbf{E}}$, $\boldsymbol{Z} = \check{\boldsymbol{Z}} + i\,\acute{\boldsymbol{Z}}$, and $\mathbf{H} = \check{\mathbf{H}} + i\,\acute{\mathbf{H}}$, equation 15 gives

$$\check{\mathbf{E}} + i\,\acute{\mathbf{E}} = (\check{\boldsymbol{Z}} + i\,\acute{\boldsymbol{Z}}).(\check{\mathbf{H}} + i\,\acute{\mathbf{H}}) \tag{16}$$

and, as in equations 3 and 4, taking **H** to be linearly polarized and to define the phase reference so that $\acute{\mathbf{H}} = 0$ gives

$$\check{\mathbf{E}} + i\,\acute{\mathbf{E}} = (\check{\boldsymbol{Z}} + i\,\acute{\boldsymbol{Z}}).\check{\mathbf{H}} \tag{17}$$

Two distinct magnetotelluric events are now postulated to occur at different times at station M, as shown in Figure 2. When each event has a magnetic-field amplitude of unity, similar analysis to that above gives

$$\det \check{\boldsymbol{Z}} = E_1 E_2 \frac{\delta\chi}{\delta\phi} \tag{18}$$

for the real part of Z, where E_1 and E_2 are the amplitudes of the in-phase electric signals; and also

$$\det \acute{\boldsymbol{Z}} = E_1 E_2 \frac{\delta\chi}{\delta\phi} \tag{19}$$

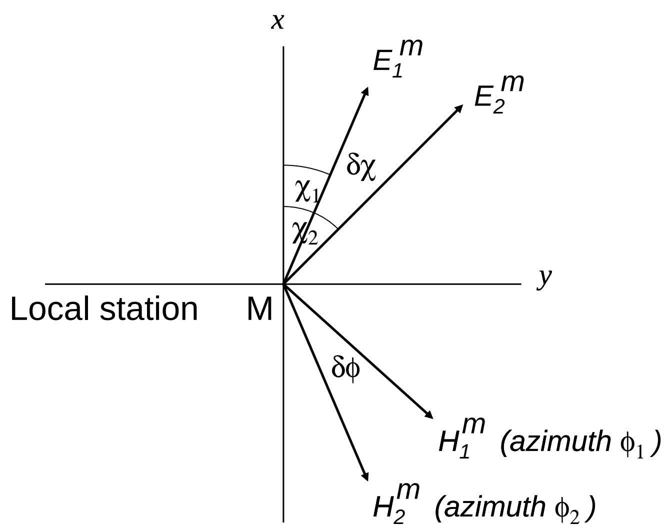

Figure 2: Two distinct magnetotelluric events are shown, both occurring at local station M.
a. The first event: magnetic signal H_1^m at azimuth ϕ_1 is accompanied by an electric signal, the in-phase part of which is E_1^m at azimuth χ_1.
b. The second event: magnetic signal H_2^m at azimuth ϕ_2 is accompanied by an electric signal, the in-phase part of which is E_2^m at azimuth χ_2.
Note $\delta\chi = \chi_2 - \chi_1$ and $\delta\phi = \phi_2 - \phi_1$.

for the imaginary part of Z (the symbols on the right-hand side of equation 19 now referring to the out-of-phase electric signals).

Then a positive value for $\det \check{Z}$ means same-sense stepping of $\check{\mathbf{E}}$ and $\check{\mathbf{H}}$, while a negative value would require counter stepping. Similarly a positive $\det \acute{Z}$ means same-sense stepping of $\acute{\mathbf{E}}$ and $\check{\mathbf{H}}$, and a negative value requires counter stepping.

In this context, particularly distinctive would appear to be any case where the determinants of $\check{Z}$ and $\acute{Z}$ are of opposite sign, so that the in-phase and out-of-phase electric signals have azimuths which respond in opposite ways to an azimuth change of the magnetic field; i.e., the azimuths of the in-phase and out-of-phase electric signals themselves step in opposite directions.

The telluric distortion tensor

The telluric distortion tensor, as introduced by Larsen (1975), Bahr (1988) and Jones (2012), generally is not envisaged as being measured in the field. It involves firstly one set of MT field measurements, the electric field parts of which are assumed to be affected by a local conductive structure. This structure is then imagined to be removed and, the same magnetic fields being imagined to be repeated, a second set of MT measurements is taken. The original electric measurements (defined as “distorted”) are then related to the second set of electric measurements (defined as “undistorted”) by a tensor called the telluric distortion tensor. This tensor, taken to be pure real, is modelled as being due simply to galvanic distortion of electric current by local geologic structure. The situation is shown in Figure 3.

The results above can now be applied to both situations. Consider initially $\mathbf{E}^{\mathrm{m}}$ and $\mathbf{E}^{\mathrm{u}}$ to be the in-phase parts of the electric fields involved, and Z to be the real part of the MT tensor. Then consider that the distorted case is given by

$$\mathbf{E}^{\mathrm{m}} = C\,\mathbf{Z}.\mathbf{H}^{\mathrm{m}} \tag{20}$$

and the second (undistorted) case by

$$\mathbf{E}^{\mathrm{u}} = Z.\mathbf{H}^{\mathrm{m}} \tag{21}$$

with distortion theory linking $\mathbf{E}^{\mathrm{m}}$ and $\mathbf{E}^{\mathrm{u}}$ by a (real) distortion tensor C, as

$$\mathbf{E}^{\mathrm{m}} = C.\mathbf{E}^{\mathrm{u}} \tag{22}$$

The similarity of equation 22 to equation 1 suggests immediately that the same stepping results will apply to C, however it is relevant to follow through the details.

Remembering from Strang (2005) that $\det(CZ) = \det C \,.\, \det Z$, take first the case where $\det C$ and $\det Z$ are both positive. For the situation represented by equation 20, $\det(CZ)$ will be positive and $\mathbf{E}^{\mathrm{m}}$ and $\mathbf{H}^{\mathrm{m}}$ will step in the same direction. Also, for the situation represented by equation 21, $\mathbf{E}^{\mathrm{u}}$ and $\mathbf{H}^{\mathrm{m}}$ will step in the same direction. Thus $\mathbf{E}^{\mathrm{m}}$ and $\mathbf{E}^{\mathrm{u}}$ will step in the same direction.

Considering similarly the other possibilities for positive and negative $\det C$ and $\det Z$ produces the table:

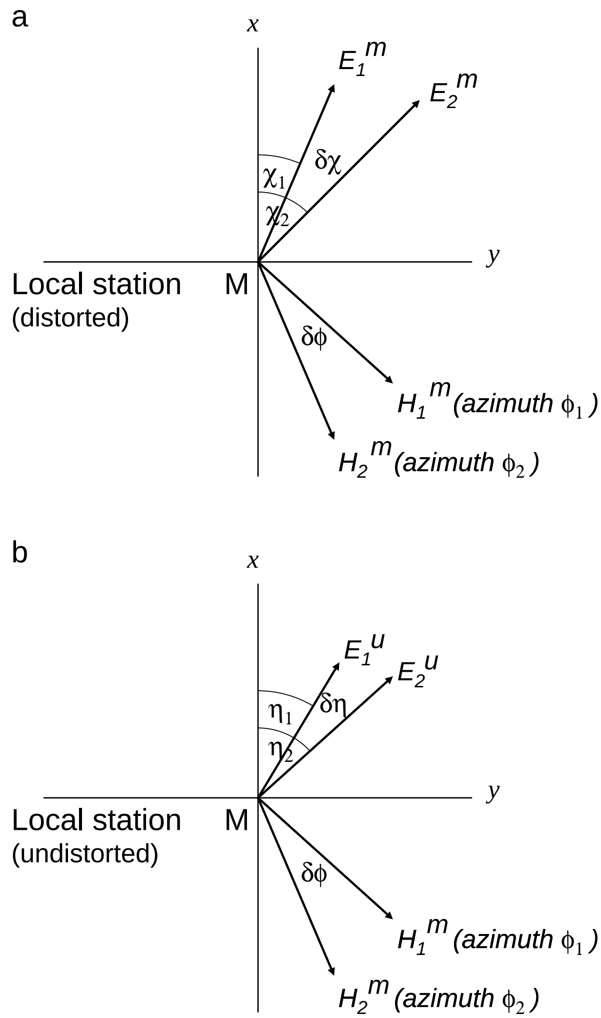

Figure 3: The telluric distortion tensor. Two different recording circumstances are shown, both for station M.
a. The first circumstance, with the local distorting structure in place. At different times, two distinct magnetic events H_1^m (at azimuth ϕ_1) and H_2^m (at azimuth ϕ_2) are accompanied by electric signals, the in-phase parts of which are E_1^m (at azimuth χ_1) and E_2^m (at azimuth χ_2) respectively.
b. The second circumstance, with the distorting structure absent. The two magnetic events H_1^m (at azimuth ϕ_1) and H_2^m (at azimuth ϕ_2) are now accompanied by electric signals with in-phase parts E_1^u (at azimuth η_1) and E_2^u (at azimuth η_2) respectively.
Note $\delta\chi = \chi_2 - \chi_1$, $\delta\eta = \eta_2 - \eta_1$, and $\delta\phi = \phi_2 - \phi_1$.

$\det C$	$\det Z$	$\det(CZ)$	$\mathbf{E^m}$, $\mathbf{H^m}$	$\mathbf{E^u}$, $\mathbf{H^m}$	$\mathbf{E^m}$, $\mathbf{E^u}$
+	+	+	same-sense step	same-sense step	same-sense step
-	+	-	counter step	same-sense step	counter step
+	-	-	counter step	counter step	same-sense step
-	-	+	same-sense step	counter step	counter step

Thus a positive $\det C$ means same-sense stepping for $\mathbf{E^m}$ and $\mathbf{E^u}$ independent of the sign of $\det Z$, while a negative $\det C$ would require counter stepping of $\mathbf{E^m}$ and $\mathbf{E^u}$.

Repeating the analysis above with $\mathbf{E^m}$ and $\mathbf{E^u}$ representing electric fields out of phase with $\mathbf{H^m}$, and Z representing the imaginary part of the MT tensor, again gives the result that a positive $\det C$ means same-sense stepping for $\mathbf{E^m}$ and $\mathbf{E^u}$, while a negative value requires counter stepping.

EXAMPLE OF DETERMINANT BEHAVIOR SHOWN BY A NUMERICALLY-MODELLED DISTORTION TENSOR

The example discussed in this section is that described by Ichihara and Mogi (2009). A telluric distortion tensor C (allowed to be complex) is calculated for a realistic model. To give both situations "a" (distorted) and "b" (undistorted) as in Figure 3, a highly conducting block is moved relative to a major regional structure. Values are determined for the four elements of C and these values, scaled from Fig. 3 of Ichihara and Mogi (2009), are shown in Figure 4 (this letter). In terms of the direction of the telluric field furthest from the distorting structure, inspection of the results of Ichihara and Mogi (2009) in their Fig. 4 shows the distorted field to be substantially reversed.

Further to the information from Ichihara and Mogi (2009), Figure 4 (this letter) also now shows values calculated for the determinants of the real and imaginary parts of C.

The example shows several relevant points. Firstly, at short periods (the blue points) the distortion tensor is essentially simply a pure real identity matrix, $[1, 0; 0, 1]$. There is no distortion occurring, and $\det Re(\boldsymbol{C}) = 1$ with $\det Im(\boldsymbol{C}) = 0$. Secondly, at intermediate periods the distortion takes effect, but in a complex way; see notably the non-zero values of $Im(C_3)$ and $Im(C_4)$. Such distortion at mid periods thus does not accord with the "pure real" distortion model of equation 22.

However thirdly, at the longest periods (the red points), the imaginary parts of C have reduced to zero, and the distortion occurs in the real parts of C only. Thus it is at these long periods only that the distortion accords with models which postulate "pure real" distortion effects.

The example also demonstrates how as the distortion tensor becomes more strongly anisotropic with increasing period, the value of the determinant of its real part reduces in value. Note, however, that this quantity remains positive.

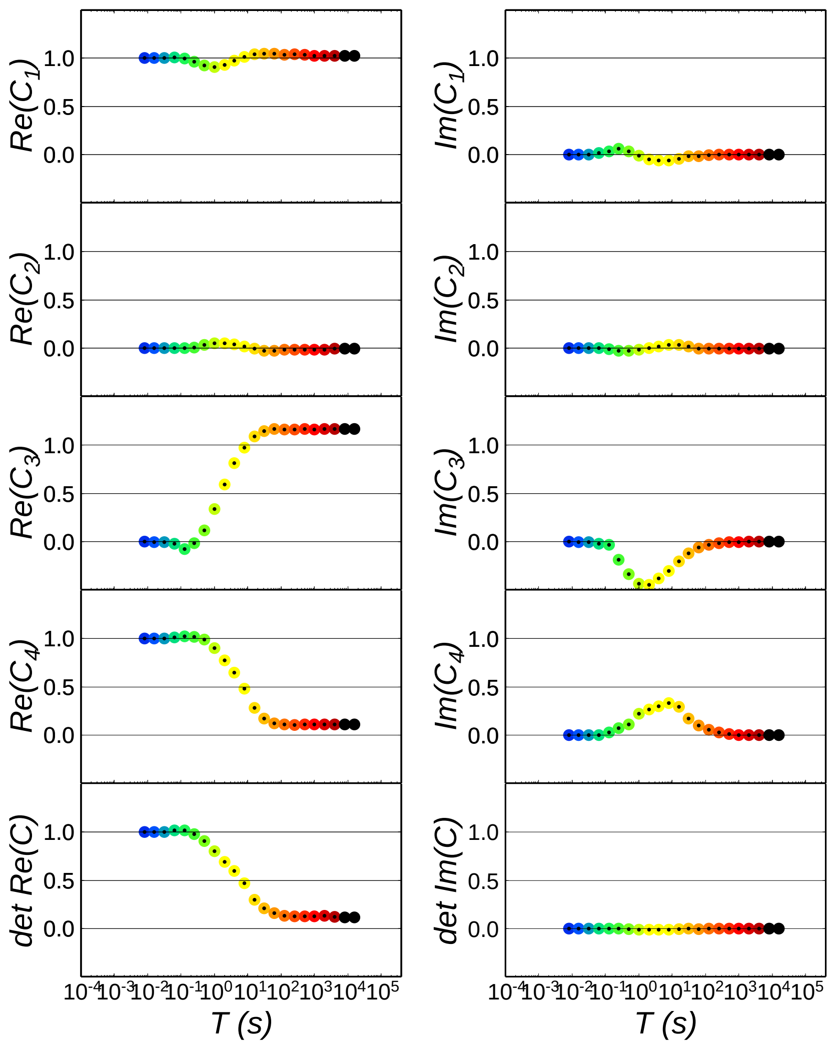

Figure 4: The distortion tensor elements of Ichihara and Mogi (2009) as functions of period T in sec, with determinant values $\det Re(C)$ and $\det Im(C)$ now also calculated and shown in the bottom panels. Points are plotted in color to help the reader compare the period-dependent behavior of the different graphs.

DISCUSSION

The case above is an example where distortion is characterized by a positive determinant. Other examples are the distortion tensors calculated by Larsen (1975), Groom and Bailey (1991), and Chakridi et al. (1992). These cases are reviewed by Jones (2012), and noted to have positive determinants by Lilley (2016).

As in the above example, generally magnetotelluric interpretation proceeds unconcerned by negative determinants. The distortion analyses of Bahr (1988), Groom and Bailey (1989), and Smith (1995) all model an observed MT tensor by a distortion tensor pre-multiplying a regional 2D MT tensor. In each case the distortion tensor model has a positive determinant, so that for a regional 2D MT tensor with a positive determinant, as is generally the case, a resulting MT tensor with a positive determinant is assured.

However the above methods, developed to treat data with implicitly positive determinants, leave unaddressed the possibility of data with negative determinants, and suggest a more complete understanding of negative determinants is desirable. Booker (2014) contributes to this understanding, in discussing in his review a number of distinctive case histories (Key and Constable, 2011; Selway et al., 2012) which, in the context of this letter, may be expected to have negative determinants.

The field data and interpretative model of Selway et al. (2012), for example, indicate that the “TE” negative phase values reported will cause negative determinant values for the imaginary part of the MT tensor. The determinant values for the real part of the MT tensor however will be everywhere positive. The real part of the MT tensor will therefore be generally invertible, and phase-tensor calculations can be made at different periods across the transition of the TE phase from negative to positive.

However phase-tensor ellipses become ambiguous for negative determinants, because the same ellipse shape will also be given by a (different) phase tensor with a positive determinant. Seeing that such results may not be meaningful, Selway et al. (2012, p. 948) abstain from including them with their other plotted data.

This particular case, typified as an ocean edge effect by Key and Constable (2011), is thus evidently one where the in-phase and out-of-phase electric signals counter step, in response to a magnetic field signal with which the in-phase electric field steps in the same sense. It is intended to give this matter further attention elsewhere.

CONCLUSION

Magnetotelluric field observations are most basically of a frequency-dependent 2x2 tensor, the MT tensor, which is complex with real and imaginary parts. This tensor is often taken to be the product of another 2x2 tensor, the distortion tensor, multiplying a “pre-distortion” MT tensor. Distortion tensors are commonly taken to be pure real.

Generally, whether the sign of the determinant of a 2x2 tensor is positive or negative indicates contrasting behavior. Where the tensor relates one vector field to another and the determinant is positive, a change or step clockwise in the direction of one vector field results in a “same-sense” step in the other vector field. This situation is the usual case in magnetotellurics, both for distortion tensors and also for the real part of an observed MT tensor. It is also the common case for the imaginary part of an observed MT tensor, with the exception of the ocean edge effect as described above.

That a determinant should be negative is a mathematical circumstance. The stepping model (rotation) discussed in this paper gives a physical model of that mathematical circumstance, and may make the implications of that mathematical condition easier to visualize.

A practical point is that strongly distorted MT data with near-zero determinants are not unusual, and in such cases phase-tensor computations, which require the in-phase part of an MT tensor to be invertible, may become unstable. In these situations winnowing out any MT data with negative determinants may help in the quality control of observed data, where there is justification in suspecting them as being originally positive (or zero) but now rendered negative due to error.

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