Introduction
What is Seismology?

- Seismology is the study of elastic waves in the solid earth.
- Seismic wave theory, observation and inference are the key components of modern seismology.
- Seismology is motivated by our ability to record ground motion caused by the passage of seismic waves.
A Propagating Seismic Wave
E-W component of a seismic wavetrain recorded in Shanghai, China from a magnitude 6.7 earthquake in the New Britain region, PNG on 6th February 2000.
Seismology is the most powerful indirect method for studying the Earth’s interior.

The existence of Earth’s crust, mantle, liquid outer core and solid inner core were inferred from seismograms many decades ago.

- **1890**: Liquid core identified by Oldham
- **1909**: Base of crust identified by Mohorovičić
- **1932**: Inner core inferred by Lehmann
Seismology can be used to reveal the location of economic resources like hydrocarbon and mineral deposits.
Seismic imaging can be performed at a variety of scales (metres to 1,000s km) to reveal information about structure, composition and dynamic processes.

[From Graeber & Asch, 1999]
Tomographic Imaging

\[ \frac{\delta \phi}{\phi} \quad \frac{\delta \beta}{\beta} \]

Perturbation [%]

\[ -1.5 \quad 0 \quad 1.5 \]
Seismograms are used in:

- earthquake location
- source mechanism studies
- estimating deformation
- measuring plate tectonic processes
- hazard assessment
- nuclear monitoring
Seismic Hazard

Peak Acceleration (%g) with 10% Probability of Exceedance in 50 Years

site: NEHRP B-C boundary

November 1996

U.S. Geological Survey
National Seismic Hazard Mapping Project
In this course, we work with vector and tensor functions that describe elastic deformation in solids. To do so, we need to use the vector operators grad, div and curl.
At each point in a continuum, we can define a scalar function that varies with position \( \mathbf{x} \) and time \( t \), and describes some property of the medium e.g. temperature \( T(\mathbf{x}, t) \).

Similarly, we can define a vector function that varies with \( (\mathbf{x}, t) \) e.g. velocity \( \mathbf{v}(\mathbf{x}, t) = (v_x, v_y, v_z) \).

Each component of a vector can itself be a vector, in which case we can define a tensor function. Examples of \( 3 \times 3 \) tensor functions include stress, strain and strain rate.

\[
\mathbf{\sigma} = \begin{bmatrix}
\sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_{zz}
\end{bmatrix}
\]
The gradient operator $\nabla$ is a vector containing three partial derivatives $[\partial/\partial x, \partial/\partial y, \partial/\partial z]$. When applied to a scalar, it produces a vector, when applied to a vector, it produces a tensor.

$$\nabla T = [\partial T/\partial x, \partial T/\partial y, \partial T/\partial z]$$

$$\nabla \mathbf{v} = \begin{bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} \partial v_x/\partial x & \partial v_y/\partial x & \partial v_z/\partial x \\ \partial v_x/\partial y & \partial v_y/\partial y & \partial v_z/\partial y \\ \partial v_x/\partial z & \partial v_y/\partial z & \partial v_z/\partial z \end{bmatrix}$$

The gradient vector of a scalar quantity defines the direction in which it increases fastest; the magnitude equals the rate of change in that direction.
The Gradient Operator

Gradient of a scalar field

\[ f(x,y) \]

\[ \nabla f \]

\[ f(x,y) \]

\[ x \]

\[ y \]
The divergence operator $\nabla \cdot$ has the same form as $\nabla$, but has the opposite effect on the rank of the quantity on which it operates. Applied to a vector it produces a scalar; applied to a tensor it produces a vector.

$$\nabla \cdot \mathbf{v} = \left[ \frac{\partial}{\partial x} \ \frac{\partial}{\partial y} \ \frac{\partial}{\partial z} \right] \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

The divergence of a vector field may be thought of as the local rate of expansion of the vector field.

$$\nabla \cdot \mathbf{v} = \lim_{V \to 0} \left[ \frac{1}{V} \iiint v \cdot \mathbf{\hat{n}} \, dS \right]$$
The Divergence Operator

This formula describes an integral over surface $S$ of a small element with volume $V$. $\hat{n}$ is the unit outward normal on the surface.

The physical significance of the divergence of a vector field is the rate at which "density" exits a given region of space. For example, if $\mathbf{u}$ is the velocity of an incompressible fluid, then $\nabla \cdot \mathbf{u} = 0$ - fluid particles cannot "bunch up".

Divergence of a 2-D vector field $V_n(x,z)$
The other important vector operator is curl, $\nabla \times$. It can be represented as a matrix operating on a vector field.

$$\nabla \times \mathbf{v} = \text{det} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix} = \begin{bmatrix} \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \\ \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \end{bmatrix}$$

The curl may be thought of as the local curvature of the vector field.

$$(\nabla \times \mathbf{v})_i = \lim_{A \to 0} \left[ \frac{\hat{n}_i}{A} \oint_{\partial A} \mathbf{v} \cdot d\mathbf{l} \right]$$
The integral is taken around the perimeter of the small area element $A$ which is perpendicular to $\hat{n}$, the unit normal vector to $\nabla \times \mathbf{v}$. Since there are three orthogonal orientations for the area element, the curl has three components.

The physical significance of the curl of a vector field is the amount of "rotation" or angular momentum of the contents of a given region of space.
Scalar function \( f = \sqrt{(x - 1)^2 + (y - 1)^2} \) in the interval \( 0 \leq x \leq 2, 0 \leq y \leq 2 \).
Example 1

$$\nabla f = \left[ \frac{x - 1}{\sqrt{(x - 1)^2 + (y - 1)^2}}, \frac{y - 1}{\sqrt{(x - 1)^2 + (y - 1)^2}} \right]$$
\[ \nabla \cdot \nabla f = \frac{1}{\sqrt{(x - 1)^2 + (y - 1)^2}} = \nabla^2 f = \text{Laplacian} \]
Example 1

\[ \nabla \times \nabla f = 0,0, \frac{\partial}{\partial x} \left( \frac{y-1}{\sqrt{(x-1)^2+(y-1)^2}} \right) - \frac{\partial}{\partial y} \left( \frac{x-1}{\sqrt{(x-1)^2+(y-1)^2}} \right) = 0 \]

For any scalar functions \( f, g \) and any vectors \( \mathbf{u}, \mathbf{v} \)

\[ \nabla \times \nabla f = 0 \quad \nabla \cdot \nabla \times \mathbf{v} = 0 \]

\[ \nabla (fg) = f \nabla g + g \nabla f \quad \nabla (f/g) = (g \nabla f - f \nabla g)/g^2 \]

\[ \nabla \cdot (f \mathbf{v}) = f \nabla \cdot \mathbf{v} + \mathbf{v} \cdot \nabla f \quad \nabla \cdot (f \nabla g) = f \nabla^2 g + \nabla f \cdot \nabla g \]

\[ \nabla \times (f \mathbf{v}) = \nabla f \times \mathbf{v} + f \nabla \times \mathbf{v} \]

\[ \nabla \cdot (\mathbf{u} \times \mathbf{v}) = \mathbf{v} \cdot \nabla \times \mathbf{u} - \mathbf{u} \cdot \nabla \times \mathbf{v} \]
Scalar function $f = (x - 1) \exp[-(x - 1)^2 - (y - 1)^2]$ in the interval $0 \leq x \leq 2, 0 \leq y \leq 2$. 
\[ \nabla f = \exp[-(x-1)^2-(y-1)^2] \left\{ 1 - 2(x-1)^2, -2(x-1)(y-1) \right\} \]
\[ \nabla \cdot \nabla f = 4(x-1) \exp[-(x-1)^2-(y-1)^2] \left\{ (x-1)^2 + (y-1)^2 - 2 \right\} \]
Vector function $\mathbf{v} = [y \cos x, x \cos y]$ in the interval $0 \leq x \leq 2, 0 \leq y \leq 2$. 
Example 3

\[ \nabla \cdot \mathbf{v} = -y \sin x - x \sin y \]
Example 3

$$\nabla \times \mathbf{v} = [0, 0, \cos y - \cos x]$$